THEORY

OF THE

MOON'S MOTION.

DEDUCED FROM THE LAW OF UNIVERSAL GRAVITATION.

BY

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TO

THE HONORED MEMORY

OF

LEONARD CASE,

THIS WORK,

DUE TO HIS ENCOURAGEMENT AND AID,

IS

Gratefully Inscribed.

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THEORY OF THE MOON'S MOTION.

INTRODUCTION.

THE motion of the moon around the earth has in all ages excited the wonder and the curiosity of mankind. In the study of its motion, the first and most important element to be determined would obviously be the interval of time between two successive new or full moons. By means of long-continued observations, the mean or average length of time between two successive phases of the moon would be accurately determined; and the knowledge thus obtained would serve as a basis for predicting the position of the moon in the heavens at any future time. The most important and interesting question which would arise would evidently be in regard to the time of new moon. To a careless observer it would seem to move always at the same distance, and with a uniform velocity, through the celestial sphere; but a more careful and attentive observer would very soon detect a variation in its apparent size at different times, which would show that its distance from the earth was variable; and he would also perceive that its motion in the heavens was subject to very considerable irregularities. Thus, in attempting to predict the time of new moon after having obtained the mean time of its revolution, he would notice that his predictions were seldom or never exactly verified by observation, but that the time of new moon would happen sometimes earlier, and sometimes later, than the computed time, but would never be in error to a greater extent than about one half a day. A long-continued series of observations would, however, enable him to deduce the law of development which the errors of prediction would follow; and he would thus be enabled to apply a correction to the time obtained on the supposition of a uniform motion of the moon. An observer would notice an inseparable relation between the apparent size of the moon and its velocity in the sky, both elements being a maximum or a minimum at the same time. Thus, when the apparent size of the moon is the greatest, its velocity is also the greatest; and when its apparent size is the smallest, its velocity is the least. But the apparent

magnitude of the moon is greatest when its distance from the earth is the least; whence it would follow that there is a relation between the velocity of the moon and its distance from the earth. Now, in comparing the observed times of new moon with the times predicted on the supposition that it moves with a uniform angular velocity, we should notice that the predicted and observed times were always the same when the apparent size of the moon was either the greatest or the least, and that the difference between the computed and observed times was the greatest when the apparent size was the mean between the extreme values. In other words, the observed and calculated places of the moon are the same at the extremities of the greater axis of its orbit; and the difference between the calculated and observed places is the greatest when the moon is at its mean distance from the earth. The correction which it is necessary to apply in order to obtain the true place of the moon when its mean place is given is called the *equation of the centre.* The moon's equation of the centre was found by means of observations of the times of new and full moon to be about *five degrees;* and, if we accept the claims of the HINDOOS, this correction to the moon's mean place was known to that people *more than three thousand years before* CHRIST; but if we reject their claim, it would seem to follow that this correction escaped the knowledge of mankind until the time of HIPPARCHUS, or about three thousand years after the HINDOOS claim to have employed it in their tables of the moon's motion.

By means of the equation of the centre which we have just explained, astronomers were enabled to predict the time of new or full moon with all desirable precision; but in attempting to calculate the place of the moon about the time of the quadratures, a remarkable deviation was sometimes found to take place. HIPPARCHUS first noticed a remarkable discrepancy between the calculated and observed places of the moon at the time of the quadratures, and executed a series of observations for the purpose of finding its value; but the law according to which the inequality was developed escaped detection during a period of *three hundred years* after its existence was made known to astronomers. It was reserved for PTOLEMY, who lived in the second century of our era, to give a complete explanation of the inequality of the moon's longitude at the quadratures. By means of a long series of observations he discovered that the inequality vanished when the *apsides* of the moon's orbit were also in quadrature, and attained a maximum value when the *apsides* were in the *syzygies.* A relation was thus shown to exist between the moon's longitude and the position of the greater axis of the orbit with respect to the sun. According to PTOLEMY, the

coefficient of the inequality was variable, and equal to 2° 40′ multiplied by the *cosine* of the elongation of the moon's perigee from the sun. The coefficient thus found being multiplied by the *sine* of the moon's elongation from the sun, gave the required correction to the moon's longitude in any part of the orbit, and represented the observations at the quadratures with all desirable precision. This second inequality in the moon's longitude has been called by the name of *evection.* It is plain that the evection always vanishes at the time of new or full moon, and becomes a maximum in the quadratures when the *apsides* of the moon's orbit are in the *syzygies*; it vanishes also in the quadratures when the *apsides* are in quadrature.

By means of the equation of the centre and the evection, astronomers were enabled to calculate the moon's place in the heavens at four equidistant points with all the precision required by the observations; and it was more than *eight hundred years* after the evection was discovered, and its laws ascertained, before astronomers suspected the existence of a *third* inequality in the moon's motion. Toward the end of the tenth century, ABOUL WEFA, an *Arabian* astronomer, by comparing the observations of the moon with the calculated place which was found by applying the equation of the centre and the evection, noticed that the observed and computed places differed considerably at the octants, and remarked the necessity of a *third* correction to the moon's longitude; but he failed to detect the law by which the inequality was developed, and it was neglected and forgotten by succeeding astronomers until after the revival of science in Europe, when it was rediscovered, and its law of development ascertained by TYCHO BRAHE, toward the close of the sixteenth century. This third inequality in the moon's longitude has received the name of the *variation;* and its maximum value was found to be about 36′, or *six-tenths* of a degree. If we multiply this quantity by the *sine* of *twice the moon's elongation from the sun*, we shall obtain the value of the inequality for the corresponding part of the orbit.

TYCHO BRAHE also remarked the existence of a fourth inequality in the moon's longitude; but he failed to determine its amount or to assign the law of its variation. It has since been found to amount to about 11′, and it varies in proportion to the *sine* of the sun's anomaly. Since its period is about one year, it has received the name of the *annual* equation.

An inequality in the moon's latitude was also discovered by TYCHO BRAHE. It was very similar to the evection in its form, but its magnitude depended on the position of the *node*, instead of the *perigee* with respect to the sun. The coefficient of the inequality was variable, and equal to 22′ multiplied by the *cosine*

of the distance of the moon's node from the sun. If the coefficient thus found were multiplied by the *sine* of the moon's elongation from the sun, we should obtain the value of the inequality in the latitude. The inequality therefore vanishes at the time of new or full moon; it also vanishes in quadrature when the node is in quadrature, but it is a maximum at the quadrature if the node is in conjunction or opposition with the sun.

The preceding inequalities in the moon's motion are the only ones that were discovered by observation during all the ages of antiquity, and until near the close of the seventeenth century. Indeed it is not probable that any additional ones would have been discovered till the present time, had not a physical cause of the inequalities been found in the law of universal gravitation. By means of the four inequalities, the *equation of the centre*, the *evection*, the *variation*, and the *annual equation*, astronomers were enabled to predict the moon's longitude for *eight points* of the orbit, at any time of the year, with all the precision required by the existing state of astronomy. But for intermediate points of the orbit, and even for those eight points which had been specially examined, considerable residuals sometimes still remained, thus showing that additional equations were necessary in order to make the calculations agree with the observations. However, since the remaining residuals were due to a multitude of equations of unknown form, together with small errors in the values of the four equations which had been discovered, it was plainly impossible to disentangle the separate equations from the confused mass of outstanding errors. But, on the discovery of the principle of universal gravitation, astronomers were enabled to refer all the inequalities in the motion of the moon to a physical cause; and the development of all the consequences resulting from the operation of this cause has been one of the most important and interesting problems which have engaged the attention of mathematicians from the time of NEWTON to the present day. Before the discovery of the principle of universal gravitation, it was necessary to reason from phenomena to their causes; and the phenomena which had been discovered by astronomers in the solar system, and at the surface of the earth itself, were sufficient to reveal the causes which produced them. Thus KEPLER, by careful and long-continued study of the motions of the planets observed by TYCHO BRAHE, was enabled to discover the laws and determine the centres of their motions; and these laws, when viewed in connection with the principles of mechanical science as made known at the surface of the earth, enabled NEWTON to discover the *law* of universal gravitation. The laws of the planetary motions which KEPLER deduced from the celestial phenomena are as follows:

FIRST.—*The orbits of the planets are ellipses, of which the sun occupies one of the foci;*

SECOND.—*The radius vector of each planet sweeps over equal areas in equal times;* and

THIRD.—*The squares of the times of revolution of the different planets are in the same proportion as the cubes of their mean distances from the sun.*

These are KEPLER'S three laws of the planetary motions; they were discovered early in the seventeenth century, and from them may be deduced the law of universal gravitation. Thus, from the law that the radius vector describes equal areas in equal times, NEWTON draws this conclusion: *that the force acting on the planets is directed toward the centre of the sun.* From the law that the orbits of the planets are ellipses, of which the sun occupies one of the foci, he concludes, *that the force acting on the planet varies in the inverse ratio of the square of the distance of their centres from that of the sun.* Lastly, from the law that the squares of the times of revolution are proportional to the cubes of the mean distances from the sun, he concludes, *that the force is proportional to the mass.*

The same laws of motion are found to prevail in the systems of the satellites, by comparing their times of revolution with their mean distances from their primaries; and it may be demonstrated for the moon by a comparison of her motion with that of projectiles on the surface of the earth. We may therefore consider the gravitation of the heavenly bodies toward each other as a general law of the universe.

Immediately on the deduction of the law of universal gravitation, NEWTON attempted to apply it in order to explain the inequalities in the motion of the moon by reason of the disturbing influence of the sun. Were the moon attracted only by the earth, she ought to move in an elliptical orbit around the earth, and describe equal areas in equal times; but, being also attracted by the sun, her motion ought to deviate a little from the elliptical path which she would otherwise exactly follow. And hence has arisen the famous "problem of three bodies," in which it is required to determine all the circumstances of the motion of one body when it is subjected to the attraction of two other bodies, whose masses and positions are given as the data of the problem. The rigorous solution of this problem surpasses the power of analysis; but approximate solutions of various degrees of excellence, depending on the skill and patience of the calculator, have been obtained by nearly all the great mathematicians since the time of NEWTON.

In NEWTON'S day modern methods of analysis were in their infancy or wholly undeveloped; and he was obliged to employ the ancient methods of geometrical synthesis in his investigations of the moon's perturbations. In this attempt he not only showed his complete mastery of the method, but also exhausted its capabilities, since no succeeding mathematician has been able to extend his solution beyond the point where he left it. Indeed it has been suspected that NEWTON first obtained his solution by the analytical method, and afterward translated his results into the language of the geometrical method; but this seems doubtful, since, were this the case, the wonder were all the greater that he failed to obtain other considerable inequalities of the lunar motions which wholly escaped him. We shall now give some account of the results at which he arrived.

Among the inequalities of the moon's longitude which were discovered by observation, three only, the *evection*, the *variation*, and the *annual equation*, are found to arise from the disturbing influence of the sun. Of these three inequalities, the last two were investigated by NEWTON; and he obtained a very close approximation to their true values. He also showed that the sun's disturbing force caused the moon's nodes to retrograde upon the ecliptic, and also produced a periodic variation in the inclination of the moon's orbit. There are also a number of other inequalities in the moon's motion and in the motion of the node, which he explained in a general way, without attempting to determine their numerical values. But in his calculation of the motion of the moon's perigee he obtained only half the motion indicated by observation. According to the historians of science, this was the only failure of NEWTON'S efforts in the lunar theory. It is manifest, however, that his method of solution, even in his own hands, was not equal to the emergency, for he not only failed in computing the motion of the perigee, but he was wholly unable to explain the evection, which is by far the largest of the moon's inequalities. NEWTON'S investigations in the lunar theory were therefore chiefly valuable as a first attempt to deduce the inequalities of the moon's motion from a strictly physical cause; and his success, although not complete, was sufficient to inspire confidence in the adequacy of the principle of universal gravitation to account for all the lunar motions.

NEWTON'S theory of the moon's inequalities is given in his *Principia*, which was published in the year 1687; and no attempt was made to verify or extend his development beyond the point where he left it, until more than half a century had elapsed. Toward the middle of the eighteenth century several of the most distinguished mathematicians of Europe occupied themselves with the subject

of the lunar motions, and CLAIRAUT succeeded in completely accounting for the motion of the moon's perigee in 1749. The explanation of the motion of the moon's perigee by CLAIRAUT inspired new confidence in the theory of gravitation, and it was subsequently applied with success to account for the irregularities of the planetary motions.

Since NEWTON's time the theory of the moon's motion has been one of the most complicated problems of astronomy. In 1747, CLAIRAUT gave a new solution of the "problem of three bodies," which he applied to the motion of the moon. By means of this solution he fully explained the *evection*, together with the *variation* and *annual equation*, and also made known the existence of several other inequalities which had not been discovered by observation. All the inequalities which had been discovered by observation were therefore completely explained by the theory; and the theory had also indicated several new ones whose existence was subsequently confirmed by observations. Two years later CLAIRAUT succeeded in accounting for the motion of the moon's perigee; and thus all the inequalities of the lunar motions, which had so long perplexed astronomers, were shown to be necessary results of the law of universal gravitation. The results at which CLAIRAUT arrived were subsequently confirmed by the investigations of EULER and D'ALEMBERT, although the theory was not further improved or extended by their labors.

The next person who contributed materially toward the improvement of the physical theory of the moon's motion was LA PLACE, who explained the origin of the secular inequality in the moon's longitude, which was first suspected by HALLEY, and fully confirmed by subsequent investigations. He afterward demonstrated that the motions of the nodes and perigee were also subjected to secular inequalities; and that the moon's latitude and longitude were affected by sensible inequalities arising from the oblateness of the earth. And by carrying on his approximations to terms of a higher order of magnitude than CLAIRAUT had done, he made known the existence of several new inequalities of sensible magnitude in the moon's motion which had not before been suspected by astronomers. But, notwithstanding these elaborate investigations, the moon's calculated place in the heavens continued to deviate from the observed place, and the predicted times of occultations and eclipses often differed considerably from the observed times.

Before the year 1820, the coefficients of the inequalities of the moon's motion were obtained by the discussion of great numbers of observations, the arguments

of the equations being alone derived from theory. They were therefore to a great extent empirical; and as this circumstance was considered derogatory to the dignity of physical astronomy, LA PLACE induced the Academy of Sciences of Paris to propose for the subject of its mathematical prize for the year 1820 the formation by theory alone of lunar tables equally perfect with those which had been obtained from theory and observation combined. This action of the French Academy was the origin of a number of very elaborate works on the lunar theory, the most complete being that of PLANA, which was published in three large quarto volumes in the year 1832. In this work PLANA has computed *one hundred and thirty-three* different inequalities in the moon's longitude, *eighty-three* of which have coefficients of less than one second of arc. Tables of the moon's motion containing some empirical terms were developed from PLANA'S theory by the late Professor PEIRCE, in 1853; and from these tables the ephemeris of the moon in the *American Ephemeris and Nautical Almanac* is computed. The place of the moon as determined from PLANA'S theory agrees somewhat better with observation than those in previous use by astronomers; but the predicted time of an eclipse or occultation is still uncertain by almost a minute; thus indicating a considerable imperfection in the theory of her motion.

In the development of the differential equations of the moon's motion, PLANA employed the true longitude as the independent variable; or, in other words, he determined the *time* in functions of the *true longitude* of the moon. He then obtained, by inverting the series, the true longitude in functions of the time as the independent variable. PLANA'S work on the lunar theory was followed, in the year 1846, by PONTÉCOULANT'S *Théorie Analytique du Système du Monde*, which contains an elaborate theory of the moon's motion, in which he integrates the differential equations by employing the *time* as the independent variable. He obtains very nearly the same results as PLANA had already found by an entirely different method.

The next person who contributed by his labors to the improvement of the lunar theory was HANSEN. In the year 1857 he published very extensive tables of the moon's motion, which were based on a new solution of the problem of three bodies. According to HANSEN'S very refined conception of the problem, the *elements* of the moon's orbit may be regarded as constant, and the *time* alone be subject to perturbation; and his formulæ, being derived by means of a different form of development are not easily comparable with those of PLANA and PONTÉCOULANT. HANSEN'S tables of the moon have, however, been used in the computation of the moon's ephemeris in the English Nautical Almanac since the year

1857; but the comparison of the ephemerides with the observations since made does not indicate any very decided improvement over the tables based on PLANA's theory.

The latest important work on the lunar theory is, however, due to DELAUNAY. This mathematician has given a very elaborate development of the theory of the moon's motion in two large quarto volumes which were published in the years 1860 and 1867. This work is a development of the theory of the variation of the elements, and gives very nearly the same results as PLANA's, although the approximations are carried much further. He finds *four hundred and seventy-nine equations* of the moon's *longitude, four hundred and thirty-six* for the *latitude*, and *one hundred* for the *parallax*. Of these various equations, only about *fifty* for the longitude and *forty* for the latitude exceed one second of arc. His theory of the moon's motion has not, however, been compared with observation; but the close agreement of his equations with those of PLANA leads to the belief that the computed place of the moon would not differ very materially from the place given by PLANA's theory.

We may therefore sum up the progress of the theory in modern times as follows: Before the time of TYCHO BRAHE the best lunar tables gave the moon's place only by a distant approximation; TYCHO's discovery of the *variation* and *annual equation* reduced the errors of the tables to less than *one-quarter* of a degree; the labors of NEWTON and HALLEY brought the errors within about *one-eighth* of a degree; MAYER, by the aid of theory and observations reduced the errors to about the *thirtieth* part of a degree; and lastly, MASON and BURG, according to the authority of LA PLACE, reduced the errors of the tables to less than *one-quarter of a minute of arc*. This brings us to the epoch of pure theory. In this department LA PLACE made the first important advance, and computed the value of upward of *thirty* equations of the longitude, *fifteen* of the latitude, and *twenty-seven* of the parallax. LA PLACE was followed by PLANA and PONTÉCOULANT, who have given about *one hundred and thirty* equations of the longitude, and nearly *one hundred* in the latitude and parallax; and these were followed by HANSEN aud DELAUNAY, the latter of whom has given upward of *four hundred and seventy* equations of the longitude, *four hundred and thirty* for the latitude, and *one hundred* for the parallax. These various theories all agree in assigning very nearly the same numerical values to the coefficients of the different inequalities which are common to them; and this agreement would seem to preclude the possibility of any accidental errors having crept into the calculations so as to vitiate the results to any great extent. And yet, notwith-

standing the vast development which the mathematical theory has received, the most recently constructed lunar tables fail to give the moon's place with much, if any, greater precision than the tables in use at the beginning of the present century. It is hence easy to conclude that the mathematical developments of the lunar theory during the last fifty years have not been attended with any corresponding degree of improvement.

Nearly all the lunar inequalities which DELAUNAY has calculated, *in addition to those calculated by* PLANA, are utterly insignificant in comparison with the outstanding errors of theory. It therefore seems a waste of time and labor to attempt to reconcile the theory with observations by simply pushing the approximations to terms of a still higher order than DELAUNAY has already done; and it still remains a problem of the highest importance to construct a lunar theory which will represent the observations. But the problem of the moon's motion has an irresistible charm, aside from its intrinsic importance; and I have therefore devoted the little leisure at my command during several years past to its careful and systematic development. The results of my labors are given in this volume; and, notwithstanding its small size in comparison with the great works of PLANA and DELAUNAY, I have much confidence in its general correctness. I shall therefore now give a somewhat detailed comparison of the present theory with the theories already mentioned in this Introduction.

In Chapter I. we have given the general differential equations of motion, in which the variable quantities are expressed in terms of both rectangular and polar co-ordinates. The differential equations which are expressed in terms of the polar co-ordinates, have then been so combined that they may be directly integrated. The constants introduced by the integrations give the elements of the moon's orbit, a, e, γ, ω, and Ω, together with the longitude of the moon at a given epoch. By putting the disturbing function equal to nothing we have obtained the equations of the elliptical motion, or the expressions of the three co-ordinates r, v, and θ in terms of the time. The whole theory of the moon's motion is contained in equations (104), (119), and (124) when we neglect the consideration of the effects of the disturbing forces. The general differential equations of the variations of the elements of the elliptical motion are also given in this chapter.

Chapter II. is wholly devoted to the development of the functions and forces which enter into the differential equations of the co-ordinates; and Chapter III. contains the development of that part of the perturbations which depends on the first power of the disturbing force. In these investigations I have carried the

developments only so far as to include terms of the fourth order depending on the eccentricities and inclinations; while PLANA and DELAUNAY have carried the development to terms of the sixth order depending on these quantities. And since the confidence inspired in a new method of analysis depends very much on the facility with which it can be applied to the determination of well-established results, we shall now give a comparison of the numerical values of some of the inequalities of the moon's motion with the values obtained by other investigators. This comparison will be restricted to those inequalities in which the agreement is almost perfect, and to those in which they are most widely different.

We will first compare the coefficients of those inequalities which are mainly independent of the eccentricities and inclinations of the orbits. The inequalities of this kind depend on the argument $2\,(nt-n't)$ and its multiples. If we substitute the value of $\frac{\overline{m}^2}{\mu}$, which is given by equation (716), in the coefficients of $\sin 2\,(nt-n't)$ in equations (307), (400), (470), and (551), omitting the terms depending on e, e', and γ, we shall obtain the complete value of the coefficient as follows:

$$2111''.841-5''.562-0''.030-0''.0007=2106''.248.$$

According to DELAUNAY'S development, this coefficient is composed of the following terms:

$$1586''.888+424''.447+80''.091+12''.769+1''.809+0''.223+0''.021=2106''.248.$$

These two results are identically equal to each other. But a most important distinction between them is the convergency of the series by which they are determined. The four terms of my development are more accurate than the seven terms of DELAUNAY'S, since the seventh term of the latter series is thirty times greater than the fourth term of the former.

If we now compare the coefficients of $\sin 4\,(nt-n't)$, we shall find from equations (400), (470), and (551) the following terms:

$$8''.789-0''.056-0''.0001=8''.733,$$

while DELAUNAY gives

$$5''.070+2''.612+0''.813+0''.196+0''.060=8''.751.$$

These two coefficients, though practically equal to each other, show the same remarkable difference in the convergency of the series by which they are determined; the second term of my development being smaller than the fifth of DELAUNAY'S.

For the equation whose argument is $6(nt - n't)$ we find from equations (470) and (551),

$$0''.0493 - 0''.0005 = 0''.0488,$$

while DELAUNAY gives

$$0''.0218 + 0''.0167 = 0''.0385.$$

This coefficient of DELAUNAY'S is about one-fourth part too small, since he has not carried the approximation to terms of so high an order as in the two former cases. To show, however, that my coefficient is correct, I would observe that the *Monthly Notices of the Royal Astronomical Society* for November, 1877, contains a paper by Prof. J. C. ADAMS, which purports to give the coefficients of the equations we have been comparing, with extreme accuracy. If we reduce his coefficient of $\sin 6(nt - n't)$ to seconds of arc, we obtain $0''.0490$ for this coefficient—a value almost identical with my own. For the coefficient of $\sin 8(nt - n't)$, I find from equation (551) $0''.00034$, while according to Prof. ADAMS it is $0''.00031$.

According to my development, the coefficient of the parallactic inequality is composed of the following terms, which are the coefficients of $\sin(nt - n't)$ in equations (307), (400), (470), and (551):

$$84''.523 + 26''.801 + 10''.280 + 3''.872 = 125''.476,$$

while DELAUNAY gives the following series of terms:

$$74''.023 + 34''.330 + 11''.885 + 4''.428 + 1''.862 + 0''.712 + 0''.381 = 127''.621.$$

The coefficient of this inequality is one of the most troublesome to be determined by the theory, and the four terms above given are the only ones I have yet computed. If we estimate the sum of the remaining terms by induction from those already calculated, we should increase the preceding coefficient by $2''.10$, which would make it equal to $127''.58$. DELAUNAY'S coefficient ought also to be increased, for the same reason, by about $0''.38$, which would make it amount to $128''.00$. These coefficients correspond to a solar parallax of about $8''.75$. According to my computations, the eccentricity and inclination of the orbits would diminish the above coefficient by $2''.11$; and if we assume the mass of the moon to be *one-eightieth* of the earth's mass, the perturbations of the earth by the moon would diminish it by $2''.10$ more. The theoretical coefficient would therefore be equal to $123''.37$. Were the exact value of the coefficient of this inequality determined from observation, we might, by comparing it with the theoretical coefficient, determine the correction to our assumed solar parallax.

The preceding inequalities are the principal ones in which the coefficients of the different theories are directly comparable with each other. For those inequalities in which the eccentricity and inclination enter as factors, the value of the coefficient depends, to a certain extent, on the manner in which the arguments of the different equations are measured.

In DELAUNAY'S theory the anomalies are measured on the plane of the orbit, while the longitudes are measured on the plane of the ecliptic. I find, however, that it conduces to greater simplicity to measure the mean anomalies along the plane of the ecliptic from the point of projection of the perigee of the orbit on the same plane. However, in order to show the rapid convergency of the series which determine the principal periodic inequalities depending on the eccentricity and inclination of the orbit, I here give the two terms of the coefficient of the *evection*, which I have computed. The first two terms depending on the first power of the eccentricity are as follows:

$$4280''.9 + 122''.0.$$

These terms are found in equations (307) and (400) with the argument $\sin(nt - 2n't + \omega)$. According to DELAUNAY, the corresponding coefficient of the evection is made up of the following terms:

$$3176''.4 + 1041''.5 + 297''.5 + 72''.3 + 15''.6 + 3''.3 + 0''.8 + 0''.3.$$

It is evident that the first series converges about ten times as rapidly as the second.

The preceding comparison is sufficient to show the correctness and value of the method which I have employed in the problem of the moon's motion, and the facility of its application is apparent from the work itself. I shall now mention a few cases in which my results are wholly different from what other calculators have found for the same inequalities.

Before doing so, however, we may observe that there are certain fundamental and axiomatic conditions which ought to be satisfied by the results arrived at, whatever may be the method of analysis which we may employ. In the present case the condition to be satisfied is simply, *That all the inequalities introduced into the expressions of the co-ordinates by the disturbing function ought to disappear when the disturbing function is put equal to nothing.* In other words, *the perturbations ought to be functions of the forces which produce them.* It is, however, a remarkable fact in connection with the lunar theory that there are several terms of considerable magnitude in the theories of LA PLACE, PLANA, PONTÉCOU-

LANT, and DELAUNAY which are not functions of the disturbing force. From this circumstance it seems legitimate to conclude that there must be something seriously wrong in the published theories, notwithstanding their intricacy and refinement.

According to the methods of development employed by LA PLACE, PLANA, and PONTÉCOULANT, the general differential equation of the radius vector is of the following form:

$$\frac{d^2r}{dt^2} + N^2r + m^2\cos(it - \epsilon) = 0; \qquad [1]$$

in which N^2 differs from unity by quantities of the order m^2. Now, m^2 denotes the disturbing function, and $m^2 \cos(it - \epsilon)$ is the general term of its development. The general integral of the preceding equation is

$$r = \frac{m^2}{i^2 - N^2}\cos(it - \epsilon), \qquad [2]$$

which is the part of r arising from this term.

Now, it is evident that if i^2 differs from unity by a quantity of the order m, the term $m^2 \cos(it - \epsilon)$ acquires by integration a divisor of that order, which increases the term considerably; so that it will be of the order $n - 1$ in the integral if it be of the order n in the differential equation. LA PLACE remarks that the greatness of the *evection* arises from this circumstance. But if i^2 differs from unity but by a quantity of the order m^2, then the expression of r becomes

$$r = \frac{m^2}{m^2}\cos(it - \epsilon), \qquad [3]$$

and thus becomes of the form $0 \div 0$ when the disturbing force vanishes. The general integral therefore assumes the indeterminate form in all those cases in which the quantity i^2 differs from unity but by quantities of the order m^2. It is evident, however, that the value of r must vanish when $m = 0$, and that it can never become independent of m.

Now, there are several terms in the development of the disturbing function, in the lunar theory, in which i^2 is unity, or differs from it but by quantities of the order m^2; and some of the most remarkable cases of perturbation which have hitherto been supposed to affect the moon's motion have really no existence in nature. The two most important equations of this character having a short period are those depending on the arguments $nt + \omega - 2\Omega$ and $nt - \omega'$ of the present theory. PLANA has given two independent determinations of the coef-

ficient of the first of these inequalities. One of these solutions is obtained by means of the preceding equation, in which he obtains a value independent of the disturbing force by merely cancelling the factor $m^2 + m^2$; and the other, by means of the variation of the elements. I shall now show that his solution, depending on the variation of the elements, is also obtained by means of an indeterminate equation; and that the true value of the variations, instead of being independent of the disturbing force, should really be equal to nothing.

PLANA's solution is obtained in the following manner: In equation (89) we have given the value of the moon's mean longitude nt; and we may suppose that

$$nt = v - 2e \sin (v - \omega), \qquad [4]$$

by neglecting the squares of e and γ. If we suppose that e and ω are variable, the corresponding variation of nt will be given by the equation

$$\delta(nt) = -2\delta e \sin (v - \omega) + 2e\delta\omega \cos (v - \omega). \qquad [5]$$

Now, PLANA gives the differential variations of e and ω on page 97 of Volume I. of his theory, as follows:

$$\left.\begin{aligned} \frac{d\delta e}{dv} &= \tfrac{21}{8} m^2 e\gamma^2 \sin (2\omega - 2\Omega), \\ \frac{d\delta\omega}{dv} &= \tfrac{21}{8} m^2\gamma^2 \cos (2\omega - 2\Omega); \end{aligned}\right\} \qquad [6]$$

in which I have changed θ into Ω, in order to conform to the notation of the present work. In finding the integrals of these equations we must observe that ω and Ω are variable. If we suppose that

$$\omega = \omega_0 + (1 - c)v, \qquad \Omega = \Omega_0 - (g - 1)v, \qquad [7]$$

we shall find,

$$\left.\begin{aligned} \frac{d\delta e}{dv} &= \tfrac{21}{8} m^2 e\gamma^2 \sin (2gv - 2cv + 2\omega_0 - 2\Omega_0), \\ \frac{d\delta\omega}{dv} &= \tfrac{21}{8} m^2\gamma^2 \cos (2gv - 2cv + 2\omega_0 - 2\Omega_0). \end{aligned}\right\} \qquad [8]$$

These equations give by integration, after substituting ω and Ω in the integrals,

$$\left.\begin{aligned} \delta e &= -\tfrac{21}{8} \frac{m^2 e\gamma^2}{2g - 2c} \cos (2\omega - 2\Omega), \\ \delta\omega &= \tfrac{21}{8} \frac{m^2\gamma^2}{2g - 2c} \sin (2\omega - 2\Omega). \end{aligned}\right\} \qquad [9]$$

Now the motions of the perigee and node are of the order m^2, and we have very nearly

$$2g - 2c = 3m^2;$$

and the values of δe and $\delta\omega$ become

$$\left.\begin{aligned} \delta e &= -\tfrac{7}{8}\frac{m^2}{m^2}e\gamma^2\cos(2\omega - 2\Omega), \\ \delta\omega &= +\tfrac{7}{8}\frac{m^2}{m^2}\gamma^2\sin(2\omega - 2\Omega). \end{aligned}\right\} \qquad [10]$$

Now, if we cancel the factor which is common to the numerator and denominator of these expressions, the resulting values of δe and $\delta\omega$ would seem to be independent of the disturbing force, and the law of their variation only, would be determined by it. If we then substitute the values of δe and $\delta\omega$ in the value of $\delta(nt)$ we shall find,

$$\delta(nt) = \tfrac{7}{4}e\gamma^2\sin(nt + \omega - 2\Omega); \qquad [11]$$

which agrees with the value found by PLANA on page 98 of his theory. But this conclusion is not satisfactory; and I shall now show that if we neglect the square of the disturbing force we may suppose the elements to be constant in the differential equations, and that we shall then have

$$\delta e = 0, \qquad \delta\omega = 0. \qquad [12]$$

For this purpose let us put, for brevity,

$$2\omega_0 - 2\Omega_0 = \beta; \qquad [13]$$

and let the variation of β arising from the disturbing forces be denoted by $\delta\beta$. The differential equation which determines δe will then become

$$\frac{d\delta e}{dv} = \tfrac{21}{8}m^2e\gamma^2\{\sin(\beta + \delta\beta) = \sin\beta + \delta\beta\cos\beta\}. \qquad [14]$$

Now, $\delta\beta$ being of the order m^2, we may neglect the term $m^2\delta\beta\cos\beta$, since it is of the order of the square of the disturbing force; and we shall have

$$\frac{d\delta e}{dv} = \tfrac{21}{8}m^2e\gamma^2\sin\beta. \qquad [15]$$

To integrate this equation we shall suppose that β is a function of v, and that we have

$$\beta = \beta_0 + \kappa v. \qquad [16]$$

We shall then have

$$\frac{d\delta e}{dv} = \tfrac{21}{8} m^2 e\gamma^2 \sin(\beta_0 + \varkappa v); \qquad [17]$$

and this gives by integration,

$$\delta e = -\tfrac{21}{8} \frac{m^2 e\gamma^2}{\varkappa} \cos(\beta_0 + \varkappa v) + C; \qquad [18]$$

C being an arbitrary constant quantity. If we determine C so that δe may vanish when $v = 0$, we shall have

$$C = \tfrac{21}{8} \frac{m^2 e\gamma^2}{\varkappa} \cos\beta_0, \qquad [19]$$

and the complete integral becomes

$$\delta e = -\tfrac{21}{8} \frac{m^2}{\varkappa} e\gamma^2 \cos(\beta_0 + \varkappa v) + \tfrac{21}{8} \frac{m^2}{\varkappa} e\gamma^2 \cos\beta_0. \qquad [20]$$

In the case of constant elements we shall have $\varkappa = 0$, and then the value of δe will become

$$\delta e = 0. \qquad [21]$$

In the same way we may show that $\delta\omega = 0$; and if we substitute these values of δe and $\delta\omega$ in the value of $\delta(nt)$ we shall obtain

$$\delta(nt) = 0. \qquad [22]$$

The effect of this term on the lunar tables is to add $-84''.8$ to the coefficient of the inequality depending on the elliptical motion, thus changing it from $+45''.4$ to $-39''.4$. Its importance in the theory of the moon's motion is therefore very apparent.

The two equations which I have mentioned above are the most important ones of short period in the lunar theory which are affected by the conditions of indetermination, and become thereby apparently independent of the forces which produce them.

The preceding are examples of inequalities of short period in the moon's motion; but there is a very important equation of long period depending on the argument $\omega - \omega'$, or the angular distance between the perigee of the sun and that of the moon, and which has a period of about nine years. This inequality was not computed by La Place; but in the theories of Plana, Pontécoulant, and Delaunay it appears as an inequality which is independent of the disturbing force, although it is wholly due to perturbation. Only two terms of

the series which give the value of the coefficient were computed by PLANA and PONTÉCOULANT, but DELAUNAY has calculated four terms of the series, the first two terms of which agree with PLANA. According to DELAUNAY, the sum of the first two terms is $+0''.39$, and the sum of the third and fourth terms amounts to $+0''.49$, which make the whole coefficient amount to $0''.88$; while according to the theory here given the coefficient of this inequality is $108''.5$.

The inequalities of long period, or those which are independent of the co-ordinates of the sun and moon, and arise from the variation of the elements of the elliptical motion, are determined with great facility in the present theory. For if we designate the forces of long period by the equations,

$$\left.\begin{aligned}\left(\frac{dR}{dr}\right) &= h\cos(\alpha t-\beta),\\ \left(\frac{dR}{dv}\right) &= h\sin(\alpha t-\beta),\end{aligned}\right\} \qquad [23]$$

in which h is a constant coefficient depending on the masses, mean distances, and eccentricities of the disturbed and disturbing bodies, and α is a coefficient depending on the variation of the elements of these bodies, but independent of their distances and configuration, the whole effect of these forces on the longitude of disturbed body will be given by the equation,

$$\delta v = h\left\{2\frac{n}{\alpha}-3\frac{n^2}{\alpha^2}\right\}\sin(\alpha t-\beta). \qquad [24]$$

In this equation the first term of the second member gives the whole perturbation arising from the variation of the central force, and the second term gives the whole effect of the tangential force; and since the factor $\frac{n}{\alpha}$ denotes the period of the argument, *it follows that the inequalities of long period arising from the variation of the central force are proportional to the products of the forces by the periods of their arguments; while the similar inequalities arising from the tangential forces are proportional to the products of the forces by the squares of the periods of their arguments.*

It is remarkable that the magnitude of the inequalities of long period arising from the two classes of forces which produce them should follow the same law as the acquired velocity and the space passed over by falling bodies at the surface of the earth, the one being proportional to the time, and the other to the square of the time.

But the most interesting problem in regard to the moon, aside from the immediate requirements of practical astronomy, is unquestionably in relation to the secular acceleration of its motion. The principal cause of its acceleration was made known by LA PLACE nearly a century ago; and the amount of the acceleration, as calculated by him, was found to satisfy, very nearly, the ancient and modern observations. I have given a new solution of this interesting problem, from which it appears that the secular equation of the longitude arises from a secular diminution of the moon's radius vector, instead of a secular variation of the mean distance. Indeed, if we neglect the square of the disturbing force, the moon's mean distance is subject only to periodical inequalities depending on the mutual configuration of the three bodies—the earth, the moon, and the sun; and in this respect it conforms to the same law of perturbation as that which prevails in the planetary system. The principal part of the secular acceleration arises from the diminution of the eccentricity of the earth's orbit; but I have discovered that there is also a small secular equation of the longitude arising from the oblateness of the earth. It has been known since the time of NEWTON that the attraction of a spheroidal body on a point without its surface is different from that of a sphere having the same mass. If the spheroid be one of revolution, like the earth, the attraction depends not only on the distance of the attracted point from the earth's centre, but also on its distance from the equator. If the attracted point were situated in the plane of the earth's equator, the attraction of the earth upon it would be greater at a given distance than if the earth were spherical. The attraction would also be greater either north or south of the equator until we reached the parallel of about 35° 16′, at which points the attraction of the earth would be nearly independent of its spheroidal form. For all points situated beyond the parallels of 35° 16′ the attraction of the earth is less than it would be if it were spherical.

From these general considerations we may draw the following conclusions, which are confirmed by analysis: *First,* A body would revolve round the earth, at a given distance from its centre, in less time if it moved in the plane of the equator than it would if the earth were spherical; and its motion would be uniform. *Second,* The time of revolution would be increased if the body moved in a plane inclined to the equator, and its motion would not be uniform, on account of the redundancy or deficiency of matter beneath the different parts of its course. Now, since the inclination of the moon's orbit to the equator is always less than 35° 16′, it follows that the earth's attraction on the moon is always greater than it would be if the earth were spherical. But since the incli-

nation varies between the limits of about 18° 19′ and 28° 35′ during a period of about nineteen years, it follows that the earth's attraction undergoes sensible variations; and hence the moon's place at any given time requires to be corrected on account of the varying inclination of its orbit to the equator. All these varying inequalities in the forces would accurately compensate each other during each revolution of the moon's node, provided the mean inclination of the lunar orbit to the equator always retained the same value. But the mean inclination of the moon's orbit to the equator is the same as the inclination of the ecliptic to the same plane; and since the inclination of the ecliptic to the equator is slowly becoming less, it follows that the plane of the moon's orbit is gradually approaching the plane of the equator; and hence its mean motion must be increasing.

The equations of long period and the secular equations are investigated in Chapter IX. The secular equation arising from the eccentricity of the earth's orbit is given in a finite form, instead of being developed in an infinite series depending on the different powers of the time. If we reduce it to the form of an infinite series, we shall find that it is represented by the following equation, in which we have retained only the first term of the development:

$$\delta v = -111108''.5\int\{e'^2 - e_0'^2\}\,dt = +7''.90\,i^2, \qquad [25]$$

in which i denotes the number of centuries elapsed since the epoch of 1850. It is, however, more accurate to use the tabular value of the integral which is given on page 362, since that includes all the powers of the time. The corresponding secular equation arising from the oblateness of the earth is found on page 363 to be equal to $0''.1979\,i^2$. If we add this to the value of δv arising from the variation of the eccentricity, we find that the secular equation is equal to

$$\delta v = 8''.10\,i^2. \qquad [26]$$

In Chapter VIII. I have investigated the inequalities of the moon's motion arising from the oblateness of the earth; and the results which I have obtained agree in all respects with the deductions of previous investigators, with the single exception of the equation in longitude, which I have found to be less than the value assigned to it by La Place, Plana, and Pontécoulant in the ratio 12 to 19.

The periodic inequalities in the moon's motion arising from the oblateness of the earth, and which are independent of the sun's attraction, are entirely insensible; and we have an interesting example of the manner in which a

disturbing force which is too feeble to produce any sensible perturbations is made wonderfully effective by means of an independent element of disturbance. This independent element which gives effectiveness to the earth's spheroidal form, as a disturbing force, is the motion of the moon's node, which arises from the sun's attraction; and it is interesting to trace the manner in which this modification of the earth's attractive force is produced. For this purpose we shall suppose the moon's orbit to be circular, and it is evident that the earth's attraction on the moon is then at a maximum whenever the moon is in the celestial equator, or twice during each revolution. We will now suppose that the longitude of the moon's ascending node is 90°, and examine into the consequences that must take place while it retrogrades through a semi-circumference, or from $+90°$ to $-90°$.

When the longitude of the node is equal to $+90°$, the plane of the moon's orbit intersects the equator at a distance of 12° 45′ to the eastward of the vernal equinox; and since the node retrogrades on the ecliptic about 1° 27′ during each sidereal revolution of the moon, it follows that the moon will arrive at the equator at a point a little to the westward of its previous crossing. In other words, the moon will make a complete revolution with respect to the centre of force in a period somewhat shorter than the sidereal revolution. At the end of 9.8 years the longitude of the node will be $-90°$, and the orbit will intersect the equator at a distance of 12° 45′ to the westward of the vernal equinox. Now, the moon performs 124.3256 sidereal revolutions while the node is retrograding through an arc of 180°. But 124.3256 sidereal revolutions correspond to 124.3256 revolutions $+23° 21'$ with respect to the equator. Whence it appears that while the node is retrograding from $+90°$ to $-90°$ the time of revolution with respect to the equator is shorter, on an average, by $20^m 32^s$ than the sidereal revolution. It is plain that while the node is retrograding from $-90°$ through the autumnal equinox to $+90°$, the point of intersection of the orbit and equator will advance from $-12° 45'$ to $+12° 45'$, and the time of revolution of the moon with respect to the equator will exceed the time of the sidereal revolution by the same amount that it fell short of that quantity while retrograding through the other half of the orbit. It is evident that the inclination of the orbit to the equator increases while the equatorial node is approaching the vernal equinox, at which point it is a maximum, and diminishes while it is receding from it. The constant retrograde motion of the ecliptic node of the moon's orbit, therefore, gives rise to a merely oscillatory motion of the equatorial node; and it is this pendulum-like motion of the equatorial node that produces

all the sensible inequalities in the moon's motion depending on the oblateness of the earth.

In the *American Journal of Science* for January, 1880, I have attempted to explain the origin of the discordance between the results found by LA PLACE and those obtained in the present work. I find, however, by re-examining the subject after an interval of a year, that the explanation there given does not cover the whole ground; and I shall now give what seems to be an entirely satisfactory explanation. For this purpose I shall here examine in detail the whole of LA PLACE's calculation of the inequalities which affect the radius vector and the longitude. In this investigation it has been found convenient to use BOWDITCH's translation of the *Mécanique Céleste*, as the facilities for referring to any part of the work by means of the marginal numbers are much better than in the original. I shall also change the notation somewhat, putting ϵ for λ, ☊ for θ, and shall also put f equal to unity. The numbers here enclosed in brackets refer to the corresponding marginal numbers of the *Mécanique Céleste*.

The expression of the force R [5362], which is used by LA PLACE, is the same as the fourth term of the value of R given in equation (613), since we may put $\sin\theta\cos\theta = \tan\theta = s$ by neglecting the third power of the inclination. If we put $\mu = 1$, and

$$\beta = m \sin \epsilon \cos \epsilon, \tag{27}$$

m being given by equation (612), the value of R will become

$$R = 2a^2 \frac{\beta}{r^3} s \sin v. \tag{28}$$

This value of R gives the following values of the partial differential coefficients:

$$\left(\frac{dR}{dr}\right) = -6a^2 \frac{\beta}{r^4} s \sin v, \tag{29}$$

$$\left(\frac{dR}{dv}\right) = 2a^2 \frac{\beta}{r^3} s \cos v, \tag{30}$$

$$\left(\frac{dR}{ds}\right) = 2a^2 \frac{\beta}{r^3} \sin v. \tag{31}$$

The equations which determine the values of δr and δv are [5361] and [5367], which are here repeated for convenience of reference:

$$\frac{d^2 r\delta r}{dt^2} + \frac{r\delta r}{r^3} + 2\int \mathrm{d}R + r\left(\frac{dR}{dr}\right) = 0, \tag{32}$$

$$d\delta v = 3\frac{dt^2}{r^2 dv}\int \mathrm{d}R + 2\frac{dt^2}{r^2 dv}\, r\left(\frac{dR}{dr}\right) \qquad [33]$$

Now, the values of $\mathrm{d}R$ and s are given by the equations

$$\mathrm{d}R = \left(\frac{dR}{dr}\right) dr + \left(\frac{dR}{dv}\right) dv + \left(\frac{dR}{ds}\right) ds. \qquad [34]$$

$$s = \gamma \sin(gv - \Omega). \qquad [35]$$

Equation [35] gives by differentiation,

$$ds = g\gamma dv \cos(gv - \Omega). \qquad [36]$$

If we neglect the eccentricity of the orbit, we shall have $dr = 0$; consequently, the term $\left(\frac{dR}{dr}\right) dr$ will disappear from the value of $\mathrm{d}R$, and we shall have, by substituting the values of s and ds,

$$\left.\begin{aligned} \left(\frac{dR}{dv}\right) dv &= a^2\beta\gamma dv \{\sin(gv + v - \Omega) + \sin(gv - v - \Omega)\} \\ \left(\frac{dR}{ds}\right) ds &= a^2\beta\gamma g dv \{\sin(gv + v - \Omega) - \sin(gv - v - \Omega)\} \end{aligned}\right\}. \qquad [37]$$

If we substitute these values in equation [34], we shall find, by retaining only the term depending on $\sin(gv - v - \Omega)$,

$$\mathrm{d}R = -a^2\frac{\beta}{r^3}\gamma(g - 1)\, dv \sin(gv - v - \Omega). \qquad [38]$$

This gives by integration,

$$\int \mathrm{d}R = a^2\frac{\beta}{r^3}\gamma \cos(gv - v - \Omega); \qquad [39]$$

and this is the value found by La Place in [5364]. In these equations $g - 1$ denotes the ratio of the retrograde motion of the nodes to that of the moon; and in the case of constant elements we should have $g = 1$.

If we substitute the value of s in equation [29], we shall obtain the following term, after multiplying by r:

$$r\left(\frac{dR}{dr}\right) = -3a^2\frac{\beta}{r^3}\gamma \cos(gv - v - \Omega). \qquad [40]$$

Therefore, equation [32] will become

$$\frac{d^2r\delta r}{dt^2} + \frac{r\delta r}{r^3} - a^2\frac{\beta}{r^3}\gamma\cos(gv - v - \Omega) = 0. \qquad [41]$$

Now, since $dt^2 = a^3dv^2$, and $r = a$, when we neglect the eccentricity of the orbit, equation [41] will become

$$\frac{d^2\delta r}{dv^2} + \delta r - a\beta\gamma\cos(gv - v - \Omega) = 0. \qquad [42]$$

The integral of this equation is

$$\delta r = a\beta\gamma\cos(gv - v - \Omega). \qquad [43]$$

This is the value of δr corresponding to LA PLACE'S method, and it is only *one-third* of the value which I have found for the same term in equation (647). I shall now explain the origin of this discordance, and the explanation thus found will also completely account for the discordance of the inequality in the longitude.

For this purpose it is necessary to observe that LA PLACE has given in Book II., Chapter V., of the *Mécanique Céleste,* a general method for integrating the differential equations of the second order; and that the method followed by him in his investigation of the effect of the earth's oblateness does not seem to be in accordance with it. In explaining this method at the commencement of Chapter VIII., he says: "These differential equations being of the second order, *their finite integrals, and also their integrals of the first order, will be the same as if the ellipses were invariable; so that we may take the differentials of the finite equations of the elliptical motions, supposing the elements of these motions to be constant.*"

Now, if we suppose the elements of the elliptical motion to be constant, we must put $g = 1$ in taking the differential of the value of s, and we shall thus obtain

$$ds = \gamma dv\cos(gv - \Omega); \qquad [44]$$

and if we use this value of ds in the formation of equation [34], we shall find

$$\mathrm{d}R = 0. \qquad [45]$$

This value of $\mathrm{d}R$ agrees with the value found in equation (659'); and if we substitute it and the value of $r\left(\frac{dR}{dr}\right)$ in equation [32], it will give

$$\delta r = 3a\beta\gamma\cos(gv - v - \Omega), \qquad [46]$$

which is the same as the corresponding term found in equation (647).

If we substitute $dR=0$, and the value of $r\left(\frac{dR}{dr}\right)$ given by equation [40] in equation [33], it will become

$$d\delta v = -6\frac{a^2}{r^3}\frac{dt^2}{r^2dv}\beta\gamma\cos(gv - v - \Omega). \qquad [47]$$

This is twice the value found by La Place in [5368], but it agrees perfectly with the value of the corresponding term given by equation (652) of this work.

La Place has, however, given a second term depending on the same argument. This second term arises from the variation of the sun's disturbing force, which is due to the variation of the moon's latitude produced by the earth's oblateness. The expression of this force is given in equation [5372], and is as follows:

$$\delta R = \tfrac{3}{2}m'u'^3r^2s\delta s. \qquad [48]$$

I shall now show that this value of δR is the same as the value of R given by equation [5362], except that it has a contrary sign.

According to [5374], we have

$$\tfrac{3}{2}\,m'u'^3r^3 = \tfrac{3}{2}\frac{m^2}{r^3} = 2\frac{g-1}{r}, \qquad [49]$$

and if we substitute this in [48], it becomes

$$\delta R = 2\frac{g-1}{r}s\delta s. \qquad [50]$$

But since the value of δs given by [5376], when reduced to the notation here used, is

$$\delta s = -a^2\frac{\beta}{(g-1)r^2}\sin v, \qquad [51]$$

its substitution in [50] will reduce it to

$$\delta R = -2a^2\frac{\beta}{r^3}s\sin v. \qquad [52]$$

This is the same as equation [28] or [5362] of *Mécanique Céleste*, except that it has a contrary sign. This force is therefore the reaction of the force expended by the sun in giving motion to the moon's nodes, which in turn produces the inequality in the moon's latitude.

B

But in this second part of his work La Place seems to have committed a grave oversight, for he has treated his equation [5372] in the construction of [5373] as though δs were constant; whereas it is a function of both r and v, according to [5376], which he afterward uses in his reductions. However, since I have just shown that equation [5372] is the same as [5362], and has a contrary sign, it is unnecessary to pursue this part of the inquiry further, since it is evident that the whole value of δv must be derived from the value of R in [5362].

La Place has given the complete value of $d\delta v$ corresponding to the plane of the orbit in [5379]; and he gives a correction in [5385] to reduce it to the plane of the ecliptic. It is apparent, however, that this correction is not required, for La Place has shown in [923'], etc., where this subject is first treated, that this correction is of the order of the square of the disturbing force; and as terms of that order have not been considered, it is evident that the value of that correction which he has given in [5385] is erroneous.

In Chapter VII. we have investigated the variations of the elements of the moon's orbit. The computation of the motion of the moon's perigee baffled the efforts of the best mathematicians during more than half a century after the discovery of the law of universal gravitation. The variations of all the elements are here obtained with great facility; but since a knowledge of these variations is not essential for the purposes of the present work, it has not been thought necessary to continue the approximations to terms of a higher order than the square of the disturbing force; and the computations have been introduced more for the purpose of illustrating the use of the various formulæ, and showing the rapid convergency of the successive approximations, than for the completeness of the results already obtained.

Chapter X. contains the investigation of the inequalities arising from the perturbations of the sun's motion. The only bodies that disturb the sun's apparent motion, which we have here considered, are the *moon* and *Venus;* but the inequalities resulting from this cause are of very little importance—the parallactic equation being the only one of sensible magnitude.

Lastly, in Chapter XI. we have reduced the inequalities contained in the preceding chapters to numbers. The numerical values of the perturbations of the radius vector, the longitude and the latitude of the moon, are given in equations (724), (725), and (726) respectively; and a comparison of the numbers

arising from the different approximations shows the rapidity of the convergence of the series by which the complete values of the inequalities are determined. In general, the terms of a higher order than we have computed are of very little importance in the present state of the lunar theory; but there are a few terms, the most important of which are the annual equation and those whose arguments are the *sum* and the *difference* of the mean anomalies of the moon and sun, in which it is necessary to continue the approximations to terms of a higher order. This is what we hope to do hereafter, in a Supplementary Chapter.

We see, by this exposition, how very intricate is the mechanical problem of the moon's motion, and what laborious and long-continued efforts have been necessary in order to attain a near approximation to its correct solution. Beginning in the remotest ages, the most gifted geniuses among mankind have successively grappled with it, and many have made important contributions to our knowledge of her motions. The earliest observers, however, directed their efforts more to the discovery of important and interesting cycles of change in the motions of the sun and moon, rather than to the discovery of the inequalities which affect her motion among the stars. Thus, the *Saros* or cycle of eclipses, dates from an unknown antiquity; and the *Metonic* cycle antedates PTOLEMY, the discoverer of the *evection*, by nearly six hundred years. After the time of PTOLEMY our knowledge of the moon's motion was not sensibly increased during a period of fifteen hundred years. At the close of this long stationary period the people of Western Europe had acquired a taste for science, and the intellectual energies which had been slumbering during so many ages suddenly awoke to a consciousness of their power, and immediately aspired to a higher destiny. The invention of *logarithms* and the discovery of the *Infinitesimal Calculus* supplied the means of estimating the effect of the mechanical forces of nature; and the fortunate discovery of the law of universal gravitation presented a most inviting field for the exercise of their powers. We have seen what rapid advances the development of the lunar theory began to take about the middle of the last century, and how bright were the prospects for its immediate and complete solution. But the complete solution of the problem was like the conquest of a great empire, in which the first assault of the invading army was too successful, and further advances made before the capitulation of important strongholds which lay along its path, thus leaving obstacles in the rear which neutralize all the advantages gained by the subsequent advance. This appears to have been the case with the lunar theory; for, if the computations of the

present work are correct, astronomers have carried their approximations to terms of the *fifth*, *sixth*, and *seventh* orders of magnitude, before those of the *third* and *fourth* had been correctly computed. This seems to be a sufficient reason for the nearly stationary condition of the lunar theory during the past three quarters of a century, notwithstanding the great efforts which have been made to perfect its solution. Its advancement has been blocked by the obstacles thrown in its path by analysis itself; and we may therefore reasonably hope for substantial improvement in the theory and tables when they are no longer embarrassed with equations which have no existence in nature.

We have thus completed the development of the physical theory of the moon's motion to the extent which we originally proposed; and this elaborate investigation has fully confirmed the conclusions at which we arrived by means of a comparison of the published ephemerides of the moon with the observations; namely, that the existing theories, instead of being correct to terms of the seventh order, are really erroneous in terms of the third order. Now, it is evident that the elements of the moon's orbit, deduced from the observations by means of a theory which was so imperfect, must necessarily partake of its imperfections; and hence it is that empiricism has been so frequently invoked to supply the defects or neutralize the errors of existing theories. It is, however, perhaps too much to expect that the present work is wholly free from errors, either systematic or accidental; but great care has been taken, by means of duplicate computations, to avoid accidental errors, and it is confidently believed that none of any considerable magnitude will be found in this investigation. And if the present work is free from systematic errors, it will serve as a very good basis for the extension of the theory to terms of a higher order than has here been undertaken, should such extension at any time be thought necessary or desirable.

It now remains to compare the present theory with observations; but this requires too much detail for the limits of the present work; for it is evident that any comparison of sufficient extent to be useful would require a vastly greater expenditure of time and labor than the development of the theory itself. In order to do this properly it would be necessary to first determine the elements of the moon's orbit corresponding to the form of development given in the present work, and then by means of these elements and the general equations of the perturbations to compute the place of the moon corresponding to the times of the observations. This is a work of too great magnitude for private enterprise, and can only be properly done by means of organized effort. However, should mathe-

maticians and astronomers find that my criticism of existing theories is correct, I feel that the labor and expense incurred in the preparation of this work has not been wholly wasted, since it indicates the direction which future efforts must take in order to remedy the defects of our present tables of the moon's motion.

CLEVELAND, *February 11, 1881.*

CHAPTER I.

GENERAL DIFFERENTIAL EQUATIONS OF THE MOON'S MOTION, WITH THE THEORY OF HER ELLIPTICAL MOTION, AND THE VARIATION OF THE ARBITRARY CONSTANTS.

1. For this purpose let us take the general differential equations of the motion of a body which is acted upon by any number of gravitating forces, which equations are as follows (*Mécanique Céleste* [499], BOWDITCH's translation):

$$\frac{ddx}{dt^2} = \left(\frac{dQ}{dx}\right), \qquad \frac{ddy}{dt^2} = \left(\frac{dQ}{dy}\right), \qquad \frac{ddz}{dt^2} = \left(\frac{dQ}{dz}\right). \tag{A}$$

In these equations x, y and z denote the rectangular co-ordinates, and the function Q represents all the forces which act upon the body whose motion is required. The time is denoted by t, whose element dt is supposed to be constant. We may put equations (A) under a more convenient form for computation in the following manner. If we denote the sum of the masses of the moon and earth by μ, and all the other forces which act upon the moon by R, the function Q will be given by the equation

$$Q = \frac{\mu}{r} - R. \tag{1}$$

The co-ordinates, x, y and z, of the moon will also be given by the equations

$$x = r\cos\theta\cos v, \qquad y = r\cos\theta\sin v, \qquad z = r\sin\theta. \tag{2}$$

If we substitute these values of x, y, z and Q in equations (A), they will become

$$\left.\begin{aligned}
&\frac{ddr - rdv^2\cos^2\theta - rd\theta^2}{dt^2} + \frac{\mu}{r^2} = -\left(\frac{dR}{dr}\right), && (3)\\
&\frac{2rdrdv\cos^2\theta - 2r^2\sin\theta\cos\theta\, dvd\theta + r^2\cos^2\theta\, ddv}{dt^2} = -\left(\frac{dR}{dv}\right), && (4)\\
&\frac{2rdrd\theta + r^2\sin\theta\cos\theta\, dv^2 + r^2dd\theta}{dt^2} = -\left(\frac{dR}{d\theta}\right). && (5)
\end{aligned}\right\} \quad (A')$$

The integrals of these equations will be the polar co-ordinates r, v and θ of the body whose motion is required.

If we multiply equation (4) by dt, and integrate, we shall find

$$r^2\cos^2\theta\frac{dv}{dt} = c - \int\left(\frac{dR}{dv}\right)dt, \tag{6}$$

c being an arbitrary constant quantity.

If we now multiply equation (4) by $\tan\theta \sin v dt$, and equation (5) by $\cos v dt$, and take the sum of their products, we shall get, after putting $2\sin^2\theta = 1 - \cos^2\theta + \sin^2\theta$,

$$\left.\begin{array}{l}\dfrac{1}{dt}\left\{\begin{array}{l}2rdrdv\sin\theta\cos\theta\sin v + r^2dv^2\sin\theta\cos\theta\cos v + r^2ddv\sin\theta\cos\theta\sin v \\ + r^2dvd\theta\cos^2\theta\sin v + 2rdrd\theta\cos v - r^2dvd\theta\sin v + r^2dd\theta\cos v \\ - r^2dvd\theta\sin^2\theta\sin v\end{array}\right\} \\ \qquad\qquad = -dt\left\{\left(\dfrac{dR}{dv}\right)\tan\theta\sin v + \left(\dfrac{dR}{d\theta}\right)\cos v\right\}\end{array}\right\}. \quad (7)$$

Equation (7) gives by integration,

$$\left.\begin{array}{l}\dfrac{1}{dt}\{r^2dv\sin\theta\cos\theta\sin v + r^2\,d\theta\cos v\} \\ \quad = c' - \int dt\left\{\left(\dfrac{dR}{dv}\right)\tan\theta\sin v + \left(\dfrac{dR}{d\theta}\right)\cos v\right\}\end{array}\right\}. \quad (8)$$

In like manner, if we multiply equation (4) by $-\tan\theta\cos v dt$, and (5) by $\sin v dt$, we shall get, by integrating the sum of their products,

$$\left.\begin{array}{l}\dfrac{1}{dt}\{r^2d\theta\sin v - r^2dv\sin\theta\cos\theta\cos v\} \\ \quad = c'' + \int dt\left\{\left(\dfrac{dR}{dv}\right)\tan\theta\cos v - \left(\dfrac{dR}{d\theta}\right)\sin v\right\}\end{array}\right\}. \quad (9)$$

c' and c'' being arbitrary constant quantities.

If we now multiply equations (A′) by $r^2\{dv\cos\theta\sin v + d\theta\sin\theta\cos v\}$, $dr\dfrac{\sin v}{\cos\theta} + 2rdv\cos\theta\cos v$, and $dr\sin\theta\cos v + 2d\theta\cos\theta\cos v$, respectively, and take the sum of the products, we shall obtain

$$\left.\begin{array}{l}\dfrac{1}{dt^2}\left\{\begin{array}{l}3r^2drd\theta^2\cos\theta\cos v + 2r^3d\theta dd\theta\cos\theta\cos v - r^3d\theta^3\sin\theta\cos v \\ -r^3d\theta^2dv\cos\theta\sin v + 3r^2drdv^2\cos^3\theta\cos v + 2r^3dvddv\cos^3\theta\cos v \\ -3r^3dv^2d\theta\sin\theta\cos^2\theta\cos v - r^3dv^3\cos^3\theta\sin v + 2rdr^2dv\cos\theta\sin v \\ + r^2ddrdv\cos\theta\sin v + r^2drddv\cos\theta\sin v - r^2drdvd\theta\sin\theta\sin v \\ + r^2drdv^2\cos\theta\cos v + 2rdr^2d\theta\sin\theta\cos v + r^2ddrd\theta\sin\theta\cos v \\ + r^2drdd\theta\sin\theta\cos v + r^2drd\theta^2\cos\theta\cos v - r^2drdvd\theta\sin\theta\sin v\end{array}\right\} \\ \quad + \mu\{\cos\theta\sin v dv + \sin\theta\cos v d\theta\} \\ = -r^2\{dv\cos\theta\sin v + d\theta\sin\theta\cos v\}\left(\dfrac{dR}{dr}\right) - \left\{dr\dfrac{\sin v}{\cos\theta} + 2rdv\cos\theta\cos v\right\}\left(\dfrac{dR}{dv}\right) \\ \qquad\qquad - \{dr\sin\theta\cos v + 2rd\theta\cos\theta\cos v\}\left(\dfrac{dR}{d\theta}\right)\end{array}\right\}. \quad (10)$$

Multiplying equations (A′) respectively by $r^2\{d\theta\sin\theta\sin v - dv\cos\theta\cos v\}$,

$-dr\frac{\cos v}{\cos\theta}+2rdv\cos\theta\sin v$, and $dr\sin\theta\sin v+2rd\theta\cos\theta\sin v$, the sum of their products will give

$$\left.\begin{array}{l}\frac{1}{dt^2}\left\{\begin{array}{l}3r^2drd\theta^2\cos\theta\sin v+2r^3d\theta dd\theta\cos\theta\sin v-r^3d\theta^3\sin\theta\sin v\\ +r^3dvd\theta^2\cos\theta\cos v+3r^2drdv^2\cos^3\theta\sin v+2r^3dvddv\cos^3\theta\sin v\\ -3r^3dv^2d\theta\sin\theta\cos^2\theta\sin v+r^3dv^3\cos^3\theta\cos v-2rdr^2dv\cos\theta\cos v\\ -r^2ddrdv\cos\theta\cos v+rdrdvd\theta\sin\theta\cos v+r^2drdv^2\cos\theta\sin v\\ -r^2drddv\cos\theta\cos v+2rdr^2d\theta\sin\theta\sin v+r^2ddrd\theta\sin\theta\sin v\\ +r^2drdd\theta\sin\theta\sin v+r^2drd\theta^2\cos\theta\sin v+r^2drdvd\theta\sin\theta\cos v\end{array}\right\}\\ +\mu\{d\theta\sin\theta\sin v-dv\cos\theta\cos v\}\\ =r^2\{dv\cos\theta\cos v-d\theta\sin\theta\sin v\}\left(\frac{dR}{dr}\right)+\{dr\frac{\cos v}{\cos\theta}-2rdv\cos\theta\sin v\}\left(\frac{dR}{dv}\right)\\ \qquad-\{dr\sin\theta\sin v+2rd\theta\cos\theta\sin v\}\left(\frac{dR}{d\theta}\right)\end{array}\right\}. \quad (11)$$

If we now multiply equations (A′) respectively by $-r^2d\theta\cos\theta$, $2rdv\sin\theta$, and $-dr\cos\theta+2rd\theta\sin\theta$, the sum of their products will give

$$\left.\begin{array}{l}\frac{1}{dt^2}\left\{\begin{array}{l}-2rdr^2d\theta\cos\theta-r^2ddrd\theta\cos\theta-r^2drdd\theta\cos\theta+r^2drd\theta^2\sin\theta\\ +3r^2drd\theta^2\sin\theta+r^3dv^2d\theta\cos^3\theta+2r^3d\theta dd\theta\sin\theta+r^3d\theta^3\cos\theta\\ +3r^2drdv^2\sin\theta\cos^2\theta+2r^3dvddv\sin\theta\cos^2\theta-2r^3dv^2d\theta\sin^2\theta\cos\theta\end{array}\right\}\\ -\mu\cos\theta\, d\theta\\ =r^2d\theta\cos\theta\left(\frac{dR}{dr}\right)-2rdv\sin\theta\left(\frac{dR}{dv}\right)+\{dr\cos\theta-2rd\theta\sin\theta\}\left(\frac{dR}{d\theta}\right)\end{array}\right\}. \quad (12)$$

Lastly, if we multiply equations (A′) respectively by $2dr$, $2dv$ and $2d\theta$, the sum of their products will give

$$\left.\begin{array}{l}\frac{1}{dt^2}\left\{\begin{array}{l}2drddr+2rdrdv^2\cos^2\theta+2rdrd\theta^2+2r^2d\theta dd\theta\\ +2r^2dvddv\cos^2\theta-2r^2dv^2d\theta\sin\theta\cos\theta\end{array}\right\}+2\mu\frac{dr}{r^2}\\ \qquad=-2\left\{\left(\frac{dR}{dr}\right)dr+\left(\frac{dR}{dv}\right)dv+\left(\frac{dR}{d\theta}\right)d\theta\right\}\end{array}\right\}. \quad (13)$$

If we now take the integrals of equations (10), (11), (12) and (13), we shall obtain

$$\left.\begin{array}{l}\left.\begin{array}{l}\frac{1}{dt^2}\{r^2d\theta^2\cos\theta\cos v+r^2dv^2\cos^3\theta\cos v+r^2drdv\cos\theta\sin v+r^2drd\theta\sin\theta\cos v\}\\ -\mu\cos\theta\cos v-f\end{array}\right\}\\ =-\int\left\{r^2\{dv\cos\theta\sin v+d\theta\sin\theta\cos v\}\left(\frac{dR}{dr}\right)+\{dr\frac{\sin v}{\cos\theta}+2rdv\cos\theta\cos v\}\left(\frac{dR}{dv}\right)\right.\\ \qquad\left.+\{dr\sin\theta\cos v+2rd\theta\cos\theta\cos v\}\left(\frac{dR}{d\theta}\right)\right\}\end{array}\right\}; \quad (14)$$

$$\left.\begin{aligned}&\frac{1}{dt^2}\{r^2d\theta^2\cos\theta\sin v+r^2dv^2\cos^2\theta\sin v-r^2drdv\cos\theta\cos v+r^2drd\theta\sin\theta\sin v\}\\&\qquad-\mu\cos\theta\sin v-f'\\&=-\int\left\{r^2\{d\theta\sin\theta\sin v-dv\cos\theta\cos v\}\left(\frac{dR}{dr}\right)+\{2rdv\cos\theta\sin v-\frac{\cos v}{\cos\theta}dr\}\left(\frac{dR}{dv}\right)\right.\\&\qquad\qquad\left.+\{dr\sin\theta\sin v+2rd\theta\cos\theta\sin v\}\left(\frac{dR}{d\theta}\right)\right\}\end{aligned}\right\}. \quad (15)$$

$$\left.\begin{aligned}&\frac{1}{dt^2}\{r^2d\theta^2\sin\theta+r^2dv^2\sin\theta\cos^2\theta-r^2drd\theta\cos\theta\}-\mu\sin\theta-f''\\&=\int\left\{r^2\,d\theta\cos\theta\left(\frac{dR}{dr}\right)-2rdv\sin\theta\left(\frac{dR}{dv}\right)-\{2rd\theta\sin\theta-dr\cos\theta\}\left(\frac{dR}{d\theta}\right)\right\}\end{aligned}\right\}; \quad (16)$$

$$\left.\begin{aligned}&\frac{1}{dt^2}\{dr^2+r^2d\theta^2+r^2dv^2\cos^2\theta\}-\frac{2\mu}{r}+\frac{\mu}{a}\\&\qquad=-2\int\left\{\left(\frac{dR}{dr}\right)dr+\left(\frac{dR}{dv}\right)dv+\left(\frac{dR}{d\theta}\right)d\theta\right\}\end{aligned}\right\}; \quad (17)$$

f, f', f'' and $\frac{\mu}{a}$ being arbitrary constant quantities to complete the integrals. Equations (6), (8), (9), (14), (15), (16) and (17) are the polar equivalents of equations (P), *Mécanique Céleste* [572].

If we now multiply equation (8) by $\cos v$, and equation (9) by $\sin v$, the sum of their products will give

$$\left.\begin{aligned}\frac{d\theta}{dt}&=\frac{c'\cos v+c''\sin v}{r^2}\\&+\frac{\sin v}{r^2}\int dt\left\{\left(\frac{dR}{dv}\right)\tan\theta\cos v-\left(\frac{dR}{d\theta}\right)\sin v\right\}\\&-\frac{\cos v}{r^2}\int dt\left\{\left(\frac{dR}{dv}\right)\tan\theta\sin v+\left(\frac{dR}{d\theta}\right)\cos v\right\}\end{aligned}\right\}. \quad (18)$$

Equation (6) gives

$$\frac{dv}{dt}=\frac{c}{r^2\cos^2\theta}-\frac{1}{r^2\cos^2\theta}\int\left(\frac{dR}{dv}\right)dt. \quad (19)$$

If we now multiply equations (14) and (15) by $\sin v$ and $-\cos v$ respectively, and divide the sum of their products by $r^2\cos\theta\frac{dv}{dt}$, we shall get

$$\left.\begin{aligned}
\frac{dr}{dt} &= \frac{f\sin v - f'\cos v}{r^2\cos\theta}\cdot\frac{dt}{dv} \\
&+ \frac{\cos v\,dt}{r^2\cos\theta\,dv}\int\left\{\begin{aligned}
&r^2\{d\theta\sin\theta\sin v - dv\cos\theta\cos v\}\left(\frac{dR}{dr}\right) \\
&+\left\{2r\,dv\cos\theta\sin v - dr\frac{\cos v}{\cos\theta}\right\}\left(\frac{dR}{dv}\right) \\
&+\{2r\,d\theta\cos\theta\sin v + dr\sin\theta\sin v\}\left(\frac{dR}{d\theta}\right)
\end{aligned}\right\} \\
&- \frac{\sin v\,dt}{r^2\cos\theta\,dv}\int\left\{\begin{aligned}
&r^2\{d\theta\sin\theta\cos v + dv\cos\theta\sin v\}\left(\frac{dR}{dr}\right) \\
&+\left\{2r\,dv\cos\theta\cos v + dr\frac{\sin v}{\cos\theta}\right\}\left(\frac{dR}{dv}\right) \\
&+\{2r\,d\theta\cos\theta\cos v + dr\sin\theta\cos v\}\left(\frac{dR}{d\theta}\right)
\end{aligned}\right\}
\end{aligned}\right\}. \tag{20}$$

The integrals of equations (18), (19) and (20) will give the latitude, longitude and radius vector of the moon at any time t. To determine the integrals of these equations it is however necessary to know the values of the constant quantities c, c', c'', f, f', f'' and a which were introduced by the integrations, when the function R is equal to nothing, and we shall now proceed to determine them.

2. For this purpose we shall suppose that $R=0$, which gives $\left(\frac{dR}{dr}\right)=0$, $\left(\frac{dR}{dv}\right)=0$, and $\left(\frac{dR}{d\theta}\right)=0$; and equations (6), (8), (9), (14), (15), (16) and (17) will become

$$r^2\cos^2\theta\frac{dv}{dt} = c; \tag{21}$$

$$r^2\sin\theta\cos\theta\sin v\frac{dv}{dt} + r^2\cos v\frac{d\theta}{dt} = c'; \tag{22}$$

$$r^2\sin v\frac{d\theta}{dt} - r^2\sin\theta\cos\theta\cos v\frac{dv}{dt} = c''; \tag{23}$$

$$\frac{1}{dt^2}\left\{\begin{aligned}&r^3d\theta^2\cos\theta\cos v + r^3dv^2\cos^3\theta\cos v\\ &+ r^2dr\,dv\cos\theta\sin v + r^2dr\,d\theta\sin\theta\cos v\end{aligned}\right\} - \mu\cos\theta\cos v = f; \tag{24}$$

$$\frac{1}{dt^2}\left\{\begin{aligned}&r^3d\theta^2\cos\theta\sin v + r^3dv^2\cos^3\theta\sin v\\ &- r^2dr\,dv\cos\theta\cos v + r^2dr\,d\theta\sin\theta\sin v\end{aligned}\right\} - \mu\cos\theta\sin v = f'; \tag{25}$$

$$\frac{1}{dt^2}\{r^3d\theta^2\sin\theta + r^3dv^2\sin\theta\cos^2\theta - r^2dr\,d\theta\cos\theta\} - \mu\sin\theta = f''; \tag{26}$$

$$\frac{1}{dt^2}\{dr^2 + r^2 d\theta^2 + r^2 dv^2 \cos^2\theta\} - \frac{2\mu}{r} + \frac{\mu}{a} = 0. \tag{27}$$

Equations (18), (19) and (20) also become

$$\frac{d\theta}{dt} = \frac{c' \cos v + c'' \sin v}{r^2}; \tag{28}$$

$$\frac{dv}{dt} = \frac{c}{r^2 \cos^2\theta}; \tag{29}$$

$$\frac{dr}{dt} = \frac{f \sin v - f' \cos v}{r^2 \cos\theta} \cdot \frac{dt}{dv}. \tag{30}$$

If we substitute the value of dt given by (29) in equation (28) it will become

$$\frac{d\theta}{\cos^2\theta} = \frac{c'}{c} \cos v dv + \frac{c''}{c} \sin v dv. \tag{31}$$

Equation (31) gives by integration

$$\tan\theta = \frac{c'}{c} \sin v - \frac{c''}{c} \cos v. \tag{32}$$

When the moon is at the node $v = \Omega$, and $\theta = 0$, therefore equation (32) will give

$$0 = \frac{c'}{c} \sin \Omega - \frac{c''}{c} \cos \Omega; \tag{33}$$

therefore

$$\tan \Omega = \frac{c''}{c'}. \tag{34}$$

When θ is a maximum it is equal to the inclination of the orbit; and $\frac{d\theta}{dt} = 0$; equations (22) and (23) will therefore give, by substituting the value $\frac{dv}{dt}$,

$$c' = c \tan\theta \sin v, \qquad c'' = -c \tan\theta \cos v. \tag{35}$$

If we square these equations the sum of their squares will give

$$\tan^2\theta = \frac{c'^2 + c''^2}{c^2} = \gamma^2. \tag{36}$$

The quantities c, c' and c'' therefore determine the place of the node and the inclination of the orbit; and if we now put

$$\frac{c'}{c} = \gamma \cos \Omega, \qquad \frac{c''}{c} = \gamma \sin \Omega, \tag{37}$$

equation (32) will become

$$\tan\theta = \gamma\{\sin v \cos \Omega - \cos v \sin \Omega\} = \gamma \sin (v - \Omega), \tag{38}$$

which gives

$$\sin\theta = \frac{\gamma \sin (v - \Omega)}{\sqrt{1 + \gamma^2 \sin^2 (v - \Omega)}}, \qquad \cos\theta = \frac{1}{\sqrt{1 + \gamma^2 \sin^2 (v - \Omega)}}. \tag{39}$$

If we now square the values of c, c' and c'', and put their sum equal to h^2, we shall find

$$h^2 = c^2 + c'^2 + c''^2 = \frac{r^2}{dt^2}\{r^2 \cos^2\theta dv^2 + r^2 d\theta^2\}. \tag{40}$$

This equation gives

$$\frac{r^2 \cos^2\theta dv^2 + r^2 d\theta^2}{dt^2} = \frac{h^2}{r^2}. \tag{41}$$

If we substitute this in equation (27) it will become

$$\frac{dr^2}{dt^2} + \frac{h^2}{r^2} - \frac{2\mu}{r} + \frac{\mu}{a} = 0, \tag{42}$$

At the extremities of the transverse axis, $dr = 0$, and equation (42) will give

$$\mu r^2 - 2\mu a r = -a h^2. \tag{43}$$

This equation gives
$$r = a + a\sqrt{1 - \frac{h^2}{\mu a}}, \tag{44}$$

and
$$r = a - a\sqrt{1 - \frac{h^2}{\mu a}}. \tag{45}$$

The sum of these values of r is equal to the transverse axis of the orbit, and their difference is the double of the eccentricity. Therefore $2a$ denotes the transverse axis of the orbit, and $\sqrt{1 - \frac{h^2}{\mu a}}$ denotes the ratio of the eccentricity to the semitransverse axis or mean distance. If we put

$$e = \sqrt{1 - \frac{h^2}{\mu a}}; \tag{46}$$

equations (44) and (45) will give

$$r = a(1 \pm e), \tag{47}$$

which are the values of r at the extremities of the transverse axis.

If we now denote the longitude of the moon, when r is a minimum, by ω, and also put $dr=0$, and $v=\omega$, in equations (24) and (25), they will become

$$\left[\frac{1}{dt^2}\{r^3d\theta^2\cos\theta+r^3dv^2\cos^3\theta\}-\mu\cos\theta\right]\cos\omega=f; \tag{48}$$

$$\left[\frac{1}{dt^2}\{r^3d\theta^2\cos\theta+r^3dv^2\cos^3\theta\}-\mu\cos\theta\right]\sin\omega=f'. \tag{49}$$

These two equations give

$$\tan\omega=\frac{f'}{f}. \tag{50}$$

If we now take the sum of the squares of the values of f, f' and f'', given by equations (24–26), we shall obtain

$$\left.\begin{aligned}\frac{1}{dt^4}\{r^6(d\theta^2+dv^2\cos^2\theta)^2+r^4dr^2(d\theta^2+dv^2\cos^2\theta)\}\\ -\frac{2\mu}{dt^2}r^3\{d\theta^2+dv^2\cos^2\theta\}\end{aligned}\right\}=f^2+f'^2+f''^2-\mu^2. \tag{51}$$

But equation (41) gives

$$d\theta^2+dv^2\cos^2\theta=h^2\frac{dt^2}{r^4}. \tag{52}$$

Substituting this in equation (51) it becomes

$$\frac{h^4}{r^2}+h^2\frac{dr^2}{dt^2}-\frac{2\mu h^2}{r}=f^2+f'^2+f''^2-\mu^2. \tag{53}$$

If in this equation we put $dr=0$, $r=a(1\pm e)$, and $h^2=a\mu(1-e^2)$, it will become

$$f^2+f'^2+f''^2=\mu^2e^2. \tag{54}$$

If we now substitute $dt=\dfrac{r^2\cos^2\theta dv}{c}$, $r=a(1-e)$, $v=\omega$, $\theta=\theta_0$ and $dr=0$, in equations (24), (25) and (26), they will become

$$\left[\frac{c^2}{a(1-e)}\left\{\frac{d\theta^2}{\cos^4\theta_0dv^2}+\frac{1}{\cos^2\theta_0}\right\}-\mu\right]\cos\theta_0\cos\omega=f, \tag{55}$$

$$\left[\frac{c^2}{a(1-e)}\left\{\frac{d\theta^2}{\cos^4\theta_0dv^2}+\frac{1}{\cos^2\theta_0}\right\}-\mu\right]\cos\theta_0\sin\omega=f', \tag{56}$$

$$\left[\frac{c^2}{a(1-e)}\left\{\frac{d\theta^2}{\cos^4\theta_0dv^2}+\frac{1}{\cos^2\theta_0}\right\}-\mu\right]\sin\theta_0=f''. \tag{57}$$

But equation (38) gives, when $v = \omega$,

$$d\theta = \gamma \cos^2\theta_0 \cos(\omega - \Omega)\, dv, \tag{58}$$

whence we get
$$\frac{d\theta^2}{dv^2 \cos^4\theta_0} = \gamma^2 \cos^2(\omega - \Omega), \tag{59}$$

and
$$\frac{1}{\cos^2\theta_0} = 1 + \gamma^2 \sin^2(\omega - \Omega), \tag{60}$$

whence we get
$$\frac{d\theta^2}{\cos^4\theta_0 dv^2} + \frac{1}{\cos^2\theta_0} = 1 + \gamma^2, \tag{61}$$

and equations (55–57) become

$$\left\{\frac{c^2(1+\gamma^2)}{a(1-e)} - \mu\right\} \cos\theta_0 \cos\omega = f, \tag{62}$$

$$\left\{\frac{c^2(1+\gamma^2)}{a(1-e)} - \mu\right\} \cos\theta_0 \sin\omega = f', \tag{63}$$

$$\left\{\frac{c^2(1+\gamma^2)}{a(1-e)} - \mu\right\} \sin\theta_0 = f''. \tag{64}$$

But equations (36), (40) and (46) give

$$c^2(1+\gamma^2) = c^2 + c'^2 + c''^2 = h^2 = a\mu(1 - e^2), \tag{65}$$

therefore we get
$$\frac{c^2(1+\gamma^2)}{a(1-e)} = \mu(1+e). \tag{66}$$

Substituting this value in equations (62–64), we get finally,

$$\left.\begin{aligned} f &= \mu e \cos\theta_0 \cos\omega, \\ f' &= \mu e \cos\theta_0 \sin\omega, \\ f'' &= \mu e \sin\theta_0. \end{aligned}\right\} \tag{67}$$

Therefore the quantities f, f' and f'' denote the product of the sum of the masses of the moon and earth into the co-ordinates of the centre of the orbit when referred to the focus as the origin.

8. Having thus found the values of the constant quantities introduced by the integrations, if we now substitute them in the differential equations of the co-ordinates r, v and θ, we shall obtain, by means of another integration, the values of these co-ordinates in terms of the time. But as we have already found the value of θ in terms of v, in equation (38), we shall also find the values of r and t

in terms of v. We shall then, by inverting the formulas, be able to find the values of r, v and θ in terms of the time t.

Equation (65) gives

$$c = \frac{\sqrt{\mu a(1-e^2)}}{\sqrt{1+\gamma^2}}. \tag{68}$$

If we substitute this value in equation (29) it will become

$$\frac{dt}{dv} = \frac{\sqrt{1+\gamma^2}}{\sqrt{\mu a(1-e^2)}} \cdot r^2 \cos^2\theta. \tag{69}$$

And if we also substitute these values, and also the values of f and f', in equation (30), it will give

$$\frac{dr}{r^2} = \frac{e(1+\gamma^2)\cos_0}{a(1-e^2)} \cdot \cos^3\theta \sin(v-\omega)\, dv. \tag{70}$$

But we have

$$\cos^3\theta = \frac{1}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{3}{2}}}; \tag{71}$$

which, being substituted in equation (70), gives

$$\frac{dr}{r^2} = \frac{e(1+\gamma^2)\cos\theta_0 \sin(v-\omega)\, dv}{a(1-e^2)\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{3}{2}}}. \tag{72}$$

This equation will give, by integration,

$$\frac{1}{r} = \frac{1}{a(1-e^2)}\left\{1 + \frac{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega) - \frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)}{\sqrt{1+\gamma^2\sin^2(v-\Omega)}}\right\}; \tag{73}$$

which is the equation of an ellipse.

If we now substitute this value of r in equation (69) it will become

$$\frac{dt}{dv} = \frac{a^{\frac{3}{2}}(1-e^2)^{\frac{3}{2}}\sqrt{1+\gamma^2}\cos^2\theta}{\sqrt{\mu}\left\{1 + \frac{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega) - \frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)}{\sqrt{1+\gamma^2\sin^2(v-\Omega)}}\right\}^2}. \tag{74}$$

This equation expresses the rigorous relation which exists between the element

of time dt and the differential of the moon's motion in longitude corresponding to any part of the orbit; and its integral will give the true time t, which is required for the moon to pass through any arc of longitude which is denoted by v. It is not, however, readily integrated in its present form; but we may develop the second member into an infinite series arranged according to the ascending powers of e and γ, which, in the lunar theory, are numerically small quantities, and carry the approximations to any degree of accuracy which may be necessary. The different terms of the equation when thus developed can then be integrated separately, without any analytical difficulty. We shall now attend to this transformation of equation (74), and shall carry the approximation to terms of the seventh order of magnitude depending on the eccentricity and inclination of the orbit. This degree of approximation is greater than is necessary in the theory of the moon's motion; but as the same formula may be applied to the motions of the planets, it was thought best to give it all needful extension for that purpose.

4. If we develop the variable part of the second member of equation (74) by the binomial theorem, it will become as follows:

$$\cos^2\theta\left\{1+\frac{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)}{\sqrt{1+\gamma^2\sin^2(v-\Omega)}}\right\}^{-2}$$

$$\left.\begin{aligned}
&=\cos^2\theta-2\,.\frac{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{1}{2}}}\\
&\qquad+3\,.\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)\}^2}{\{1+\gamma^2\sin^2(v-\Omega)\}^2}\\
&\qquad-4\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)\}^3}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{3}{2}}}\\
&\qquad+5\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)\}^4}{\{1+\gamma^2\sin^2(v-\Omega)\}^3}\\
&\qquad-6\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)\}^5}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{5}{2}}}\\
&\qquad+7\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos_0\theta\cos(v+\omega-2\Omega)\}^6}{\{1+\gamma^2\sin^2(v-\Omega)\}^4}\\
&\qquad-8\frac{\{e(1+\frac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\frac{1}{2}e\gamma^2\cos_0\theta\cos(v+\omega-2\Omega)\}^7}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{7}{2}}}
\end{aligned}\right\}. \qquad (75)$$

We shall now develop each term of this equation separately. Since $\cos\theta = \dfrac{1}{\sqrt{1+\gamma^2\sin^2(v-\Omega)}}$, we shall obtain

$$\cos^2\theta = \{1+\gamma^2\sin^2(v-\Omega)\}^{-1} = 1-\gamma^2\sin^2(v-\Omega)+\gamma^4\sin^4(v-\Omega)-\gamma^6\sin^6(v-\Omega)$$

$$\left.\begin{aligned} &= 1-\tfrac{1}{2}\gamma^2(1-\tfrac{3}{4}\gamma^2+\tfrac{5}{8}\gamma^4)+\tfrac{1}{2}\gamma^2(1-\gamma^2+\tfrac{15}{16}\gamma^4)\cos 2(v-\Omega) \\ &\quad +\tfrac{1}{8}\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos 4(v-\Omega)+\tfrac{1}{32}\gamma^6\cos 6(v-\Omega) \end{aligned}\right\} \qquad (76)$$

We also have

$$\cos\theta_0 = \{1+\gamma^2\sin^2(\omega-\Omega)\}^{-\frac{1}{2}}$$

$$\left.\begin{aligned} &= 1-\tfrac{1}{4}\gamma^2+\tfrac{9}{64}\gamma^4-\tfrac{25}{256}\gamma^6+\tfrac{1}{4}\gamma^2(1-\tfrac{3}{4}\gamma^2+\tfrac{75}{128}\gamma^4)\cos 2(\omega-\Omega) \\ &\quad +\tfrac{3}{64}\gamma^4(1-\tfrac{5}{4}\gamma^2)\cos 4(\omega-\Omega)+\tfrac{5}{512}\gamma^6\cos 6(\omega-\Omega) \end{aligned}\right\}. \qquad (77)$$

We shall therefore obtain the following values of the different terms of the second member of equation (75):

$$\frac{e(1+\tfrac{1}{2}\gamma^2)\cos\theta_0\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos\theta_0\cos(v+\omega-2\Omega)}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{3}{2}}}$$

$$\left.\begin{aligned} &= e(1-\tfrac{1}{2}\gamma^2+\tfrac{21}{64}\gamma^4-\tfrac{31}{128}\gamma^6)\cos(v-\omega)+\tfrac{3}{64}e\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos 3(v-\omega) \\ &\quad +\tfrac{3}{8}e\gamma^2(1-\gamma^2+\tfrac{57}{64}\gamma^4)\cos(3v-\omega-2\Omega)+\tfrac{1}{8}e\gamma^2(1-\gamma^2+\tfrac{57}{64}\gamma^4)\cos(v-3\omega+2\Omega) \\ &\quad -\tfrac{3}{128}e\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos(3v+\omega-4\Omega)+\tfrac{1}{128}e\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos(v+3\omega-4\Omega) \\ &\quad +\tfrac{15}{128}e\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos(5v-\omega-4\Omega)+\tfrac{3}{128}e\gamma^4(1-\tfrac{3}{2}\gamma^2)\cos(v-5\omega+4\Omega) \\ &\quad -\tfrac{5}{512}e\gamma^6\cos(5v+\omega-6\Omega)+\tfrac{1}{512}e\gamma^6\cos(v+5\omega-6\Omega)+\tfrac{35}{1024}e\gamma^6\cos(7v-\omega-6\Omega) \\ &\quad +\tfrac{15}{1024}e\gamma^6\cos(5v-3\omega-2\Omega)+\tfrac{9}{1024}e\gamma^6\cos(3v-5\omega+2\Omega) \\ &\quad +\tfrac{5}{1024}e\gamma^6\cos(v-7\omega+6\Omega) \end{aligned}\right\}; \quad (78)$$

$$\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^2\cos^2\theta_0\{1+\gamma^2\sin^2(v-\Omega)\}^{-2}$$

$$\left.\begin{aligned} &= \tfrac{1}{2}e^2(1-\tfrac{1}{2}\gamma^2+\tfrac{3}{8}\gamma^4)+\tfrac{1}{2}e^2(1-\tfrac{1}{2}\gamma^2+\tfrac{1}{4}\gamma^4)\cos 2(v-\omega)+\tfrac{1}{8}e^2\gamma^2(1-\gamma^2)\cos 2(v-\Omega) \\ &\quad +\tfrac{1}{8}e^2\gamma^2(1-\gamma^2)\cos(2v-4\omega+2\Omega)+\tfrac{1}{32}e^2\gamma^4\cos(2v-6\omega+4\Omega) \\ &\quad +\tfrac{1}{16}e^2\gamma^4\cos 4(v-\omega)+\tfrac{3}{32}e^2\gamma^4\cos(6v-2\omega-4\Omega) \\ &\quad +\tfrac{1}{4}e^2\gamma^2(1-\gamma^2)\cos(4v-2\omega-2\Omega) \end{aligned}\right\}; \quad (79)$$

$$\begin{aligned}&\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^3\cos^3\theta_0\{1+\gamma^2\sin^2(v-\Omega)\}^{-\frac{3}{2}}\\
&=\left.\begin{aligned}&\tfrac{3}{4}e^3(1-\tfrac{1}{2}\gamma^2+\tfrac{11}{32}\gamma^4)\cos(v-\omega)+\tfrac{1}{4}e^3(1-\tfrac{1}{2}\gamma^2+\tfrac{15}{64}\gamma^4)\cos 3(v-\omega)\\
&+\tfrac{15}{256}e^3\gamma^4\cos 5(v-\omega)+\tfrac{3}{16}e^3\gamma^2(1-\gamma^2)\cos(3v-\omega-2\Omega)\\
&-\tfrac{3}{256}e^3\gamma^4\cos(3v+\omega-4\Omega)+\tfrac{1}{16}e^3\gamma^2(1-\gamma^2)\cos(v-3\omega+2\Omega)\\
&+\tfrac{5}{32}e^3\gamma^2(1-\gamma^2)\cos(5v-3\omega-2\Omega)+\tfrac{3}{32}e^3\gamma^2(1-\gamma^2)\cos(3v-5\omega+2\Omega)\\
&-\tfrac{3}{256}e^3\gamma^4\cos(3v+\omega-4\Omega)+\tfrac{1}{256}e^3\gamma^4\cos(v+3\omega-4\Omega)\\
&+\tfrac{3}{512}e^3\gamma^4\cos(v-5\omega+4\Omega)+\tfrac{15}{512}e^3\gamma^4\cos(3v-7\omega+4\Omega)\\
&+\tfrac{35}{512}e^3\gamma^4\cos(7nt-3\omega-4\Omega)\end{aligned}\right\};\quad(80)\end{aligned}$$

$$\begin{aligned}&\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^4\cos^4\theta_0\{1+\gamma^2\sin(v-\Omega)\}^{-2}\\
&=\left.\begin{aligned}&\tfrac{3}{8}e^4(1-\tfrac{1}{2}\gamma^2)+\tfrac{1}{2}e^4(1-\tfrac{1}{2}\gamma^2)\cos 2(v-\omega)+\tfrac{1}{8}e^4(1-\tfrac{1}{2}\gamma^2)\cos 4(v-\omega)\\
&+\tfrac{1}{16}e^4\gamma^2\cos 2(v-\Omega)+\tfrac{3}{32}e^4\gamma^2\cos(2v-4\omega+2\Omega)+\tfrac{3}{16}e^4\gamma^2\cos(4v-2\omega-2\Omega)\\
&+\tfrac{3}{32}e^4\gamma^2\cos(6v-4\omega-2\Omega)+\tfrac{1}{16}e^4\gamma^2\cos(4v-6\omega+2\Omega)\end{aligned}\right\};\quad(81)\end{aligned}$$

$$\begin{aligned}&\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^5\cos\theta_0{}^5\{1+\gamma^2\sin^2(v-\Omega)\}^{-\frac{5}{2}}\\
&=\left.\begin{aligned}&\tfrac{5}{8}e^5(1-\tfrac{1}{2}\gamma^2)\cos(v-\omega)+\tfrac{5}{16}e^5(1-\tfrac{1}{2}\gamma^2)\cos 3(v-\omega)\\
&+\tfrac{1}{16}e^5(1-\tfrac{1}{2}\gamma^2)\cos 5(v-\omega)+\tfrac{15}{128}e^5\gamma^2\cos(3v-\omega-2\Omega)\\
&+\tfrac{5}{128}e^5\gamma^2\cos(v-3\omega+2\Omega)+\tfrac{5}{132}e^5\gamma^2\cos(5v-3\omega-2\Omega)\\
&+\tfrac{3}{32}e^5\gamma^2\cos(3v-5\omega+2\Omega)+\tfrac{7}{128}e^5\gamma^2\cos(7nt-5\omega-2\Omega)\\
&+\tfrac{5}{128}e^5\gamma^2\cos(5v-7\omega+2\Omega)\end{aligned}\right\};\quad(82)\end{aligned}$$

$$\left.\begin{aligned}&\frac{\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^6\cos^6\theta_0}{\{1+\gamma^2\sin^2(v-\Omega)\}^4}\\
&=\tfrac{5}{16}e^6+\tfrac{15}{32}e^6\cos 2(v-\omega)+\tfrac{3}{16}e^6\cos 4(v-\omega)+\tfrac{1}{32}e^6\cos 6(v-\omega)\end{aligned}\right\};\quad(83)$$

$$\left.\begin{aligned}&\frac{\{e(1+\tfrac{1}{2}\gamma^2)\cos(v-\omega)-\tfrac{1}{2}e\gamma^2\cos(v+\omega-2\Omega)\}^7\cos^7\theta_0}{\{1+\gamma^2\sin^2(v-\Omega)\}^{\frac{9}{2}}}\\
&=\tfrac{35}{64}e^7\cos(v-\omega)+\tfrac{21}{64}e^7\cos 3(v-\omega)+\tfrac{7}{64}e^7\cos 5(v-\omega)+\tfrac{1}{64}e^7\cos 7(v-\Omega)\end{aligned}\right\}.\quad(84)$$

If we now multiply equation (78) by -2, (79) by $+3$, (80) by -4, (81) by $+5$, (82) by -6, (83) by $+7$, and (84) by -8; and then add the products to equation (76), we shall get the following development of equation (74):

$$\frac{\sqrt{\mu}}{a^{\frac{3}{2}}(1-e^2)^{\frac{3}{2}}\sqrt{1+\gamma^2}}\cdot\frac{dt}{dv}$$

$$\left.\begin{aligned}
&= 1-\tfrac{1}{2}\gamma^2+\tfrac{3}{2}e^2+\tfrac{3}{8}\gamma^4+\tfrac{15}{8}e^4-\tfrac{5}{16}\gamma^6+\tfrac{35}{16}e^6-\tfrac{3}{4}e^2\gamma^2+\tfrac{9}{16}e^2\gamma^4-\tfrac{15}{16}e^4\gamma^2\\
&\quad -2e\left\{\begin{aligned}&1-\tfrac{1}{2}\gamma^2+\tfrac{3}{2}e^2+\tfrac{21}{64}\gamma^4+\tfrac{15}{8}e^4-\tfrac{3}{4}e^2\gamma^2\\ &-\tfrac{31}{128}\gamma^6+\tfrac{33}{64}e^2\gamma^4+\tfrac{35}{16}e^6-\tfrac{15}{16}e^4\gamma^2\end{aligned}\right\}\cos(v-\omega)\\
&\quad +\tfrac{3}{2}e^2\{1-\tfrac{1}{2}\gamma^2+\tfrac{5}{8}e^2+\tfrac{1}{4}\gamma^4+\tfrac{35}{16}e^4-\tfrac{5}{6}e^2\gamma^2\}\cos 2(v-\omega)\\
&\quad -\{e^3-\tfrac{1}{2}e^3\gamma^2+\tfrac{3}{32}e\gamma^4+\tfrac{15}{8}e^5+\tfrac{15}{64}e^3\gamma^4-\tfrac{9}{64}e\gamma^6+\tfrac{21}{8}e^7-\tfrac{15}{16}e^5\gamma^2\}\cos 3(v-\omega)\\
&\quad +\{\tfrac{5}{8}e^4-\tfrac{5}{16}e^4\gamma^2+\tfrac{3}{16}e^2\gamma^4+\tfrac{21}{16}e^6\}\cos 4(v-\omega)+\tfrac{7}{32}e^6\cos 6(v-\omega)\\
&\quad -\{\tfrac{3}{8}e^5+\tfrac{7}{8}e^7-\tfrac{3}{16}e^5\gamma^2+\tfrac{15}{64}e^3\gamma^4\}\cos 5(v-\omega)-\tfrac{1}{8}e^7\cos 7(v-\omega)\\
&\quad +\{\tfrac{1}{2}\gamma^2-\tfrac{1}{2}\gamma^4+\tfrac{3}{8}e^2\gamma^2+\tfrac{15}{32}\gamma^6-\tfrac{3}{8}e^2\gamma^4+\tfrac{5}{16}e^4\gamma^2\}\cos 2(v-\Omega)\\
&\quad +\tfrac{1}{8}\gamma^4\{1-\tfrac{3}{2}\gamma^2\}\cos 4(v-\Omega)+\tfrac{1}{32}\gamma^6\cos 6(v-\Omega)\\
&\quad -\{\tfrac{3}{4}e\gamma^2-\tfrac{3}{4}e\gamma^4+\tfrac{3}{4}e^3\gamma^2-\tfrac{3}{4}e^3\gamma^4+\tfrac{171}{256}e\gamma^6+\tfrac{45}{64}e^5\gamma^2\}\cos(3v-\omega-2\Omega)\\
&\quad -\{\tfrac{1}{4}e\gamma^2-\tfrac{1}{4}e\gamma^4+\tfrac{1}{4}e^3\gamma^2-\tfrac{1}{4}e^3\gamma^4+\tfrac{57}{256}e\gamma^6+\tfrac{15}{64}e^5\gamma^2\}\cos(v-3\omega+2\Omega)\\
&\quad +\{\tfrac{3}{64}e\gamma^4-\tfrac{9}{128}e\gamma^6+\tfrac{3}{64}e^3\gamma^4\}\cos(3v+\omega-4\Omega)\\
&\quad -\{\tfrac{1}{64}e\gamma^4-\tfrac{3}{128}e\gamma^6+\tfrac{1}{64}e^3\gamma^4\}\cos(v+3\omega-4\Omega)\\
&\quad -\{\tfrac{15}{64}e\gamma^4-\tfrac{45}{128}e\gamma^6+\tfrac{15}{128}e^3\gamma^4\}\cos(5v-\omega-4\Omega)\\
&\quad -\{\tfrac{3}{64}e\gamma^4-\tfrac{9}{128}e\gamma^6+\tfrac{3}{128}e^3\gamma^4\}\cos(v-5\omega+4\Omega)\\
&\quad +\tfrac{5}{256}e\gamma^6\cos(5v+\omega-6\Omega)-\tfrac{1}{256}e\gamma^6\cos(v+5\omega-6\Omega)\\
&\quad -\tfrac{35}{512}e\gamma^6\cos(7nt-\omega-6\Omega)-\tfrac{5}{512}e\gamma^6\cos(v-7\omega+6\Omega)\\
&\quad +\{\tfrac{3}{8}e^2\gamma^2-\tfrac{3}{8}e^2\gamma^4+\tfrac{15}{32}e^4\gamma^2\}\cos(2v-4\omega+2\Omega)\\
&\quad +\{\tfrac{3}{4}e^2\gamma^2-\tfrac{3}{4}e^2\gamma^4+\tfrac{15}{16}e^4\gamma^2\}\cos(4v-2\omega-2\Omega)\\
&\quad +\tfrac{3}{32}e^2\gamma^4\cos(2v-6\omega+4\Omega)+\tfrac{9}{32}e^2\gamma^4\cos(6v-2\omega-4\Omega)\\
&\quad -\{\tfrac{3}{8}e^3\gamma^2-\tfrac{3}{8}e^3\gamma^4+\tfrac{9}{512}e\gamma^6+\tfrac{9}{16}e^5\gamma^2\}\cos(3v-5\omega+2\Omega)\\
&\quad -\{\tfrac{5}{8}e^3\gamma^2-\tfrac{5}{8}e^3\gamma^4+\tfrac{15}{512}e\gamma^6+\tfrac{15}{16}e^5\gamma^2\}\cos(5v-3\omega-2\Omega)\\
&\quad +\tfrac{15}{32}e^4\gamma^2\cos(6v-4\omega-2\Omega)+\tfrac{5}{16}e^4\gamma^2\cos(4nt-6\omega+2\Omega)\\
&\quad -\tfrac{15}{128}e^3\gamma^4\cos(3nt-7\omega+4\Omega)-\tfrac{35}{128}e^3\gamma^4\cos(7v-3\omega-4\Omega)\\
&\quad -\tfrac{21}{64}e^5\gamma^2\cos(7nt-5\omega-2\Omega)-\tfrac{15}{64}e^5\gamma^2\cos(5v-7\omega+2\Omega)
\end{aligned}\right\}. \qquad (85)$$

Now we have

$$(1-e^2)^{\frac{3}{2}}\sqrt{1+\gamma^2}=1-\tfrac{3}{2}e^2+\tfrac{1}{2}\gamma^2+\tfrac{3}{8}e^4-\tfrac{1}{8}\gamma^4-\tfrac{3}{4}e^2\gamma^2+\tfrac{3}{16}e^4\gamma^2+\tfrac{3}{16}e^2\gamma^4+\tfrac{1}{16}e^6+\tfrac{1}{16}\gamma^6. \tag{86}$$

Multiplying equation (85) by this value of $(1-e^2)^{\frac{3}{2}}\sqrt{1+\gamma^2}$, it becomes

$$\left.\begin{aligned}
\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}\cdot\frac{dt}{dv} = {} & 1-2e\{1-\tfrac{3}{64}\gamma^4+\tfrac{3}{128}e^2\gamma^4+\tfrac{3}{64}\gamma^6\}\cos(v-\omega) \\
& +\tfrac{3}{2}e^2\{1+\tfrac{1}{6}e^2+\tfrac{1}{16}e^4-\tfrac{1}{3}\gamma^4\}\cos 2(v-\omega) \\
& -\{e^3+\tfrac{3}{5}e^5+\tfrac{3}{32}e\gamma^4+\tfrac{3}{16}e^7-\tfrac{9}{32}e^3\gamma^4-\tfrac{3}{32}e\gamma^6\}\cos 3(v-\omega) \\
& +\{\tfrac{5}{8}e^4+\tfrac{3}{16}e^2\gamma^4+\tfrac{3}{8}e^6\}\cos 4(v-\omega)+\tfrac{7}{32}e^6\cos 6(v-\omega) \\
& -\{\tfrac{3}{8}e^5+\tfrac{5}{16}e^7+\tfrac{15}{64}e^3\gamma^4\}\cos 5(v-\omega)-\tfrac{1}{8}e^7\cos 7(v-\omega) \\
& +\tfrac{1}{2}\gamma^2\{1-\tfrac{1}{2}\gamma^2-\tfrac{3}{4}e^2+\tfrac{5}{16}\gamma^4-\tfrac{1}{8}e^4+\tfrac{3}{8}e^2\gamma^2\}\cos 2(v-\Omega) \\
& +\tfrac{1}{8}\gamma^4\{1-\gamma^2-\tfrac{3}{2}e^2\}\cos 4(v-\Omega)+\tfrac{1}{32}\gamma^6\cos 6(v-\Omega) \\
& -\tfrac{3}{4}e\gamma^2\{1-\tfrac{1}{2}\gamma^2-\tfrac{1}{2}e^2+\tfrac{17}{64}\gamma^4-\tfrac{3}{16}e^4+\tfrac{1}{4}e^2\gamma^2\}\cos(3v-\omega-2\Omega) \\
& -\tfrac{1}{4}e\gamma^2\{1-\tfrac{1}{2}\gamma^2-\tfrac{1}{2}e^2+\tfrac{17}{64}\gamma^4-\tfrac{3}{16}e^4+\tfrac{1}{4}e^2\gamma^2\}\cos(v-3\omega+2\Omega) \\
& +\tfrac{3}{64}e\gamma^4\{1-\gamma^2-\tfrac{1}{2}e^2\}\cos(3v+\omega-4\Omega)-\tfrac{1}{64}e\gamma^4\{1-\gamma^2-\tfrac{1}{2}e^2\}\cos(v+3\omega-4\Omega) \\
& -\tfrac{15}{64}e\gamma^4\{1-\gamma^2-e^2\}\cos(5v-\omega-4\Omega)+\tfrac{5}{256}e\gamma^6\cos(5v+\omega-6\Omega) \\
& -\tfrac{3}{64}e\gamma^4\{1-\gamma^2-e^2\}\cos(v-5\omega+4\Omega)-\tfrac{1}{256}e\gamma^6\cos(v+5\omega-6\Omega) \\
& +\tfrac{3}{8}e^2\gamma^2\{1-\tfrac{1}{2}\gamma^2-\tfrac{1}{4}e^2\}\cos(2v-4\omega+2\Omega)-\tfrac{35}{512}e\gamma^6\cos(7v-\omega-6\Omega) \\
& +\tfrac{3}{4}e^2\gamma^2\{1-\tfrac{1}{2}\gamma^2-\tfrac{1}{4}e^2\}\cos(4v-2\omega-2\Omega)-\tfrac{5}{512}e\gamma^6\cos(v-7\omega+6\Omega) \\
& -\{\tfrac{3}{8}e^3\gamma^2-\tfrac{3}{16}e^3\gamma^4+\tfrac{9}{512}e\gamma^6\}\cos(3v-5\omega+2\Omega)+\tfrac{3}{32}e^2\gamma^4\cos(2v-6\omega+4\Omega) \\
& -\{\tfrac{5}{8}e^3\gamma^2-\tfrac{5}{16}e^3\gamma^4+\tfrac{15}{512}e\gamma^6\}\cos(5v-3\omega-2\Omega)+\tfrac{9}{32}e^2\gamma^4\cos(6v-2\omega-4\Omega) \\
& -\tfrac{15}{128}e^3\gamma^4\cos(3v-7\omega+4\Omega)-\tfrac{35}{128}e^3\gamma^4\cos(7v-3\omega-4\Omega) \\
& +\tfrac{15}{32}e^4\gamma^2\cos(6v-4\omega-2\Omega)+\tfrac{5}{16}e^4\gamma^2\cos(4v-6\omega+2\Omega) \\
& -\tfrac{21}{64}e^5\gamma^2\cos(7v-5\omega-2\Omega)-\tfrac{15}{64}e^5\gamma^2\cos(5v-7\omega+2\Omega)
\end{aligned}\right\}. \tag{87}$$

If we now put $$n=\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}, \tag{88}$$

equation (87) will give, by integration,

$$
\left.
\begin{aligned}
nt = v &- 2e\left(1 - \tfrac{3}{64}\gamma^4 + \tfrac{3}{128}e^2\gamma^4 + \tfrac{3}{64}\gamma^6\right)\sin(v-\omega) \\
&+ \tfrac{3}{4}e^2\left(1 + \tfrac{1}{6}e^2 + \tfrac{1}{16}e^4 - \tfrac{1}{8}\gamma^4\right)\sin 2(v-\omega) \\
&- \left(\tfrac{1}{3}e^3 + \tfrac{1}{8}e^5 + \tfrac{1}{32}e\gamma^4 + \tfrac{1}{16}e^7 - \tfrac{3}{32}e^3\gamma^4 - \tfrac{1}{32}e\gamma^6\right)\sin 3(v-\omega) \\
&+ \left\{\tfrac{5}{32}e^4 + \tfrac{3}{32}e^6 + \tfrac{3}{64}e^2\gamma^4\right\}\sin 4(v-\omega) + \tfrac{7}{192}e^6\sin 6(v-\omega) \\
&- \left\{\tfrac{3}{40}e^5 + \tfrac{1}{16}e^7 + \tfrac{3}{64}e^3\gamma^4\right\}\sin 5(v-\omega) - \tfrac{1}{56}e^7\sin 7(v-\omega) \\
&+ \tfrac{1}{4}\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\right\}\sin 2(v-\Omega) \\
&- \tfrac{1}{4}\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\right\}\sin 2(\omega-\Omega) \\
&+ \tfrac{1}{32}\gamma^4\left\{1 - \gamma^2 - \tfrac{3}{2}e^2\right\}\sin 4(v-\Omega) + \tfrac{1}{192}\gamma^6\sin 6(v-\Omega) \\
&- \tfrac{1}{32}\gamma^4\left\{1 - \gamma^2 - \tfrac{3}{2}e^2\right\}\sin 4(\omega-\Omega) - \tfrac{1}{192}\gamma^6\sin 6(\omega-\Omega) \\
&- \tfrac{1}{4}e\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\right\}\sin(3v-\omega-2\Omega) \\
&- \tfrac{1}{4}e\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\right\}\sin(v-3\omega+2\Omega) \\
&+ \tfrac{1}{64}e\gamma^4\left\{1-\gamma^2-\tfrac{1}{2}e^2\right\}\sin(3v+\omega-4\Omega) - \tfrac{1}{64}e\gamma^4\left\{1-\gamma^2-\tfrac{1}{2}e^2\right\}\sin(v+3\omega-4\Omega) \\
&- \tfrac{3}{64}e\gamma^4\left\{1 - \gamma^2 - e^2\right\}\sin(5v-\omega-4\Omega) + \tfrac{1}{256}e\gamma^6\sin(5v+\omega-6\Omega) \\
&- \tfrac{3}{64}e\gamma^4\left\{1 - \gamma^2 - e^2\right\}\sin(v-5\omega+4\Omega) - \tfrac{1}{256}e\gamma^6\sin(v+5\omega-6\Omega) \\
&+ \tfrac{3}{16}e^2\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\right\}\sin(2v-4\omega+2\Omega) - \tfrac{5}{512}e\gamma^6\sin(7v-\omega-6\Omega) \\
&+ \tfrac{3}{16}e^2\gamma^2\left\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\right\}\sin(4v-2\omega-2\Omega) - \tfrac{5}{512}e\gamma^6\sin(v-7\omega+6\Omega) \\
&- \left\{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\right\}\sin(3v-5\omega+2\Omega) + \tfrac{3}{64}e^2\gamma^4\sin(2v-6\omega+4\Omega) \\
&- \left\{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\right\}\sin(5v-3\omega-2\Omega) + \tfrac{3}{64}e^2\gamma^4\sin(6v-2\omega-4\Omega) \\
&- \tfrac{5}{128}e^3\gamma^4\sin(3v-7\omega+4\Omega) - \tfrac{5}{128}e^3\gamma^4\sin(7v-3\omega-4\Omega) \\
&+ \tfrac{5}{64}e^4\gamma^2\sin(6v-4\omega-2\Omega) + \tfrac{5}{64}e^4\gamma^2\sin(4v-6\omega+2\Omega) \\
&- \tfrac{3}{64}e^5\gamma^2\sin(7v-5\omega-2\Omega) - \tfrac{3}{64}e^5\gamma^2\sin(5v-7\omega+2\Omega)
\end{aligned}
\right\} \quad (89)
$$

In this equation nt denotes the mean longitude of the moon, and v the true longitude; and the constant introduced by the integrations has been made to satisfy the condition that the *mean* and *true* longitudes shall be equal to each other at the extremities of the transverse axis. By inverting this series we may obtain the value of v in terms of nt.

5. For this purpose we shall observe that if we change the sign of all the terms of the second member except the first, and at the same time change v into

nt in those terms, and calling the sum of the terms thus changed $f(nt)$, we shall have, according to the theorem of LA GRANGE,

$$\left.\begin{aligned} v = nt + f(nt) &+ \tfrac{1}{2}\frac{df(nt)^2}{ndt} + \tfrac{1}{6}\frac{d^2f(nt)^3}{n^2dt^2} + \tfrac{1}{24}\frac{d^3f(nt)^4}{n^3dt^3} \\ &+ \tfrac{1}{120}\frac{d^4f(nt)^5}{n^4dt^4} + \tfrac{1}{720}\frac{d^5f(nt)^6}{n^5dt^5} + \tfrac{1}{5040}\frac{d^6f(nt)^7}{n^6dt^6} \end{aligned}\right\} \tag{90}$$

We shall therefore have

$$\left.\begin{aligned} f(nt) = 2e\,&\{1 - \tfrac{3}{64}\gamma^4 + \tfrac{3}{128}e^2\gamma^4 + \tfrac{3}{64}\gamma^6\}\sin(nt-\omega) \\ &- \tfrac{3}{4}e^2\{1 + \tfrac{1}{6}e^2 + \tfrac{1}{16}e^4 - \tfrac{1}{8}\gamma^4\}\sin 2(nt-\omega) \\ &+ \{\tfrac{1}{3}e^3 + \tfrac{1}{8}e^5 + \tfrac{1}{32}e\gamma^4 + \tfrac{1}{16}e^7 - \tfrac{3}{32}e^3\gamma^4 - \tfrac{1}{32}e\gamma^6\}\sin 3(nt-\omega) \\ &- \{\tfrac{5}{32}e^4 + \tfrac{3}{32}e^6 + \tfrac{3}{64}e^2\gamma^4\}\sin 4(nt-\omega) - \tfrac{7}{192}e^6\sin 6(nt-\omega) \\ &+ \{\tfrac{3}{40}e^5 + \tfrac{1}{16}e^7 + \tfrac{3}{64}e^3\gamma^4\}\sin 5(nt-\omega) + \tfrac{1}{56}e^7\sin 7(nt-\omega) \\ &- \tfrac{1}{4}\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\}\sin 2(nt-\Omega) \\ &+ \tfrac{1}{4}\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\}\sin 2(\omega-\Omega) \\ &- \tfrac{1}{32}\gamma^4\{1 - \gamma^2 - \tfrac{3}{2}e^2\}\sin 4(nt-\Omega) - \tfrac{1}{192}\gamma^6\sin 6(nt-\Omega) \\ &+ \tfrac{1}{32}\gamma^4\{1 - \gamma^2 - \tfrac{3}{2}e^2\}\sin 4(\omega-\Omega) + \tfrac{1}{192}\gamma^6\sin 6(\omega-\Omega) \\ &+ \tfrac{1}{4}e\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\}\sin(3nt-\omega-2\Omega) \\ &+ \tfrac{1}{4}e\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\}\sin(nt-3\omega+2\Omega) \\ &- \tfrac{1}{64}e\gamma^4\{1 - \gamma^2 - \tfrac{1}{2}e^2\}\sin(3nt+\omega-4\Omega) + \tfrac{1}{64}e\gamma^4\{1 - \gamma^2 - \tfrac{1}{2}e^2\}\sin(nt+3\omega-4\Omega) \\ &+ \tfrac{3}{64}e\gamma^4\{1 - \gamma^2 - e^2\}\sin(5nt-\omega-4\Omega) - \tfrac{1}{256}e\gamma^6\sin(5nt+\omega-6\Omega) \\ &+ \tfrac{3}{64}e\gamma^4\{1 - \gamma^2 - e^2\}\sin(nt-5\omega+4\Omega) + \tfrac{1}{256}e\gamma^6\sin(nt+5\omega-6\Omega) \\ &- \tfrac{3}{16}e^2\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\}\sin(2nt-4\omega+2\Omega) + \tfrac{5}{512}e\gamma^6\sin(7nt-\omega-6\Omega) \\ &- \tfrac{3}{16}e^2\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\}\sin(4nt-2\omega-2\Omega) + \tfrac{5}{512}e\gamma^6\sin(nt-7\omega+6\Omega) \\ &+ \{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\}\sin(3nt-5\omega+2\Omega) - \tfrac{3}{64}e^2\gamma^4\sin(2nt-6\omega+4\Omega) \\ &+ \{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\}\sin(5nt-3\omega+2\Omega) - \tfrac{3}{64}e^2\gamma^4\sin(6nt-2\omega-4\Omega) \\ &+ \tfrac{5}{128}e^3\gamma^4\sin(3nt-7\omega+4\Omega) + \tfrac{5}{128}e^3\gamma^4\sin(7nt-3\omega-4\Omega) \\ &- \tfrac{5}{64}e^4\gamma^2\sin(6nt-4\omega-2\Omega) - \tfrac{5}{64}e^4\gamma^2\sin(4nt-6\omega+2\Omega) \\ &+ \tfrac{3}{64}e^5\gamma^2\sin(7nt-5\omega-2\Omega) + \tfrac{3}{64}e^5\gamma^2\sin(5nt-7\omega+2\Omega) \end{aligned}\right\}. \tag{91}$$

From this equation we get the following values of the different powers of $f(nt)$, which are all correct to terms of the seventh order:

$$\left.\begin{aligned}
f(nt)^2 &= 2e^2+\tfrac{9}{32}e^4+\tfrac{1}{16}\gamma^4+\tfrac{43}{288}e^6-\tfrac{1}{16}\gamma^6-\tfrac{7}{32}e^2\gamma^4\\
&-\{\tfrac{3}{2}e^3+\tfrac{1}{2}e^5+\tfrac{1}{8}e\gamma^4+\tfrac{9}{32}e^7-\tfrac{19}{64}e^3\gamma^4-\tfrac{1}{8}e\gamma^6\}\cos(nt-\omega)\\
&-\{2e^2-\tfrac{2}{3}e^4+\tfrac{1}{16}\gamma^4-\tfrac{47}{128}e^6-\tfrac{7}{16}e^2\gamma^4-\tfrac{1}{16}\gamma^6\}\cos 2(nt-\omega)\\
&+\{\tfrac{3}{2}e^3-\tfrac{1}{16}e^5+\tfrac{1}{8}e\gamma^4-\tfrac{3}{20}e^7-\tfrac{73}{128}e^3\gamma^4-\tfrac{1}{8}\gamma^6\}\cos 3(nt-\omega)\\
&-\{\tfrac{91}{96}e^4+\tfrac{31}{160}e^6+\tfrac{7}{32}e^2\gamma^4\}\cos 4(nt-\omega)-\tfrac{1859}{5760}e^6\cos 6(nt-\omega)\\
&+\{\tfrac{9}{16}e^5+\tfrac{1}{4}e^7+\tfrac{35}{128}e^3\gamma^4\}\cos 5(nt-\omega)+\tfrac{29}{160}e^7\cos 7(nt-\omega)\\
&+\{\tfrac{11}{16}e^2\gamma^2-\tfrac{11}{32}e^2\gamma^4-\tfrac{13}{96}e^4\gamma^2+\tfrac{1}{128}\gamma^6\}\{\cos 2(nt-\Omega)+\cos 2(\omega-\Omega)\}\\
&-\{\tfrac{1}{32}\gamma^4-\tfrac{1}{32}\gamma^6-\tfrac{7}{32}e^2\gamma^4\}\{\cos 4(nt-\Omega)+\cos 4(\omega-\Omega)\}\\
&-\tfrac{1}{128}\gamma^6\{\cos 6(nt-\Omega)+\cos 6(\omega-\Omega)\}-\tfrac{1}{128}e\gamma^6\cos(3nt+3\omega-6\Omega)\\
&-\{e\gamma^2-\tfrac{1}{2}e\gamma^4+\tfrac{35}{128}e\gamma^6-\tfrac{1}{8}e^5\gamma^2-\tfrac{3}{8}e^3\gamma^2+\tfrac{3}{16}e^3\gamma^4\}\cos(nt+\omega-2\Omega)\\
&+\{\tfrac{1}{2}e\gamma^2-\tfrac{1}{4}e\gamma^4-\tfrac{5}{6}e^3\gamma^2+\tfrac{5}{12}e^3\gamma^4-\tfrac{9}{128}e^5\gamma^2+\tfrac{7}{64}e\gamma^6\}\cos(nt-3\omega+2\Omega)\\
&+\{\tfrac{1}{2}e\gamma^2-\tfrac{1}{4}e\gamma^4-\tfrac{5}{6}e^3\gamma^2+\tfrac{5}{12}e^3\gamma^4-\tfrac{9}{128}e^5\gamma^2+\tfrac{7}{64}e\gamma^6\}\cos(3nt-\omega-2\Omega)\\
&+\{\tfrac{1}{8}e\gamma^4-\tfrac{1}{8}e\gamma^6-\tfrac{79}{256}e^3\gamma^4\}\{\cos(nt-5\omega+4\Omega)+\cos(5nt-\omega-4\Omega)\}\\
&-\{\tfrac{11}{16}e^2\gamma^2-\tfrac{11}{32}e^2\gamma^4-\tfrac{83}{128}e^4\gamma^2+\tfrac{1}{128}\gamma^6\}\{\cos(2nt-4\omega+2\Omega)+\cos(4nt-2\omega-2\Omega)\}\\
&+\{\tfrac{31}{48}e^3\gamma^2-\tfrac{31}{96}e^3\gamma^4-\tfrac{29}{80}e^5\gamma^2+\tfrac{7}{256}e\gamma^6\}\{\cos(3nt-5\omega+2\Omega)+\cos(5nt-3\omega-2\Omega)\}\\
&-\{\tfrac{1}{8}e\gamma^4-\tfrac{1}{8}e\gamma^6-\tfrac{85}{384}e^3\gamma^4\}\{\cos(3nt+\omega-4\Omega)+\cos(nt+3\omega-4\Omega)\}\\
&+\{\tfrac{1}{16}\gamma^4-\tfrac{1}{16}\gamma^6-\tfrac{3}{64}e^2\gamma^4\}\cos(2nt+2\omega-4\Omega)\\
&-\tfrac{25}{128}e^2\gamma^4\{\cos(6nt-2\omega-4\Omega)+\cos(2nt-6\omega+4\Omega)\}\\
&-\tfrac{197}{384}e^4\gamma^2\{\cos(6nt-4\omega-2\Omega)+\cos(4nt-6\omega+2\Omega)\}\\
&+\tfrac{23}{768}e\gamma^6\{\cos(nt-7\omega+6\Omega)+\cos(7nt-\omega-6\Omega)\}\\
&-\tfrac{5}{192}e\gamma^6\{\cos(nt+5\omega-6\Omega)+\cos(5nt+\omega-6\Omega)\}\\
&+\tfrac{167}{768}e^3\gamma^4\{\cos(7nt-3\omega-4\Omega)+\cos(3nt-7\omega+4\Omega)\}\\
&+\tfrac{237}{640}e^5\gamma^2\{\cos(7nt-5\omega-2\Omega)+\cos(5nt-7\omega+2\Omega)\}\\
&+\tfrac{1}{128}\gamma^6\{\cos(4nt+2\omega-6\Omega)+\cos(2nt+4\omega-6\Omega)\}
\end{aligned}\right\}; \quad (92)$$

nt in those terms, and calling the sum of the terms thus changed $f(nt)$, we shall have, according to the theorem of LA GRANGE,

$$\left.\begin{aligned} v = nt + f(nt) &+ \tfrac{1}{2}\frac{df(nt)^2}{ndt} + \tfrac{1}{6}\frac{d^2f(nt)^3}{n^2dt^2} + \tfrac{1}{24}\frac{d^3f(nt)^4}{n^3dt^3} \\ &+ \tfrac{1}{120}\frac{d^4f(nt)^5}{n^4dt^4} + \tfrac{1}{720}\frac{d^5f(nt)^6}{n^5dt^5} + \tfrac{1}{5040}\frac{d^6f(nt)^7}{n^6dt^6} \end{aligned}\right\} \quad (90)$$

We shall therefore have

$$\left.\begin{aligned} f(nt) = {} & 2e\,\{1 - \tfrac{3}{64}\gamma^4 + \tfrac{3}{128}e^2\gamma^4 + \tfrac{3}{64}\gamma^6\}\sin(nt-\omega) \\ & - \tfrac{3}{4}e^2\{1 + \tfrac{1}{6}e^2 + \tfrac{1}{16}e^4 - \tfrac{1}{8}\gamma^4\}\sin 2(nt-\omega) \\ & + \{\tfrac{1}{3}e^3 + \tfrac{1}{8}e^5 + \tfrac{1}{32}e\gamma^4 + \tfrac{1}{16}e^7 - \tfrac{3}{32}e^3\gamma^4 - \tfrac{1}{32}e\gamma^6\}\sin 3(nt-\omega) \\ & - \{\tfrac{5}{32}e^4 + \tfrac{3}{32}e^6 + \tfrac{3}{64}e^2\gamma^4\}\sin 4(nt-\omega) - \tfrac{7}{192}e^6\sin 6(nt-\omega) \\ & + \{\tfrac{3}{40}e^5 + \tfrac{1}{16}e^7 + \tfrac{3}{64}e^3\gamma^4\}\sin 5(nt-\omega) + \tfrac{1}{56}e^7\sin 7(nt-\omega) \\ & - \tfrac{1}{4}\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\}\sin 2(nt-\text{☊}) \\ & + \tfrac{1}{4}\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{3}{4}e^2 + \tfrac{5}{16}\gamma^4 - \tfrac{1}{8}e^4 + \tfrac{3}{8}e^2\gamma^2\}\sin 2(\omega-\text{☊}) \\ & - \tfrac{1}{32}\gamma^4\{1 - \gamma^2 - \tfrac{3}{2}e^2\}\sin 4(nt-\text{☊}) - \tfrac{1}{192}\gamma^6\sin 6(nt-\text{☊}) \\ & + \tfrac{1}{32}\gamma^4\{1 - \gamma^2 - \tfrac{3}{2}e^2\}\sin 4(\omega-\text{☊}) + \tfrac{1}{192}\gamma^6\sin 6(\omega-\text{☊}) \\ & + \tfrac{1}{4}e\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\}\sin(3nt-\omega-2\text{☊}) \\ & + \tfrac{1}{4}e\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{2}e^2 + \tfrac{17}{64}\gamma^4 - \tfrac{3}{16}e^4 + \tfrac{1}{4}e^2\gamma^2\}\sin(nt-3\omega+2\text{☊}) \\ & - \tfrac{1}{64}e\gamma^4\{1 - \gamma^2 - \tfrac{1}{2}e^2\}\sin(3nt+\omega-4\text{☊}) + \tfrac{1}{64}e\gamma^4\{1-\gamma^2-\tfrac{1}{2}e^2\}\sin(nt+3\omega-4\text{☊}) \\ & + \tfrac{3}{64}e\gamma^4\{1 - \gamma^2 - e^2\}\sin(5nt-\omega-4\text{☊}) - \tfrac{1}{256}e\gamma^6\sin(5nt+\omega-6\text{☊}) \\ & + \tfrac{3}{64}e\gamma^4\{1 - \gamma^2 - e^2\}\sin(nt-5\omega+4\text{☊}) + \tfrac{1}{256}e\gamma^6\sin(nt+5\omega-6\text{☊}) \\ & - \tfrac{3}{16}e^2\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\}\sin(2nt-4\omega+2\text{☊}) + \tfrac{5}{512}e\gamma^6\sin(7nt-\omega-6\text{☊}) \\ & - \tfrac{3}{16}e^2\gamma^2\{1 - \tfrac{1}{2}\gamma^2 - \tfrac{1}{4}e^2\}\sin(4nt-2\omega-2\text{☊}) + \tfrac{5}{512}e\gamma^6\sin(nt-7\omega+6\text{☊}) \\ & + \{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\}\sin(3nt-5\omega+2\text{☊}) - \tfrac{3}{64}e^2\gamma^4\sin(2nt-6\omega+4\text{☊}) \\ & + \{\tfrac{1}{8}e^3\gamma^2 - \tfrac{1}{16}e^3\gamma^4 + \tfrac{3}{512}e\gamma^6\}\sin(5nt-3\omega+2\text{☊}) - \tfrac{3}{64}e^2\gamma^4\sin(6nt-2\omega-4\text{☊}) \\ & + \tfrac{5}{128}e^3\gamma^4\sin(3nt-7\omega+4\text{☊}) + \tfrac{5}{128}e^3\gamma^4\sin(7nt-3\omega-4\text{☊}) \\ & - \tfrac{5}{64}e^4\gamma^2\sin(6nt-4\omega-2\text{☊}) - \tfrac{5}{64}e^4\gamma^2\sin(4nt-6\omega+2\text{☊}) \\ & + \tfrac{3}{64}e^5\gamma^2\sin(7nt-5\omega-2\text{☊}) + \tfrac{3}{64}e^5\gamma^2\sin(5nt-7\omega+2\text{☊}) \end{aligned}\right\} \quad (91)$$

5

$$
\left.\begin{aligned}
f(nt)^4 = {} & 6e^4 + \tfrac{49}{24}e^6 + \tfrac{9}{8}e^2\gamma^4 \\
& - \{6e^5 + \tfrac{201}{64}e^7 + \tfrac{33}{16}e^3\gamma^4\}\cos(nt-\omega) \\
& - \{8e^4 - \tfrac{37}{16}e^6 + \tfrac{3}{2}e^2\gamma^4\}\cos 2(nt-\omega) \\
& + \{9e^5 + \tfrac{57}{64}e^7 + \tfrac{99}{32}e^3\gamma^4\}\cos 3(nt-\omega) \\
& + \{2e^4 - \tfrac{59}{8}e^6 + \tfrac{3}{8}e^2\gamma^4\}\cos 4(nt-\omega) + \tfrac{145}{48}e^6\cos 6(nt-\omega) \\
& - \{3e^5 - \tfrac{307}{64}e^7 + \tfrac{37}{32}e^3\gamma^4\}\cos 5(nt-\omega) - \tfrac{163}{64}e^7\cos 7(nt-\omega) \\
& - \{6e^3\gamma^2 - 3e^3\gamma^4 + \tfrac{11}{16}e^5\gamma^2 + \tfrac{9}{32}e\gamma^6\}\cos(nt+\omega-2\Omega) \\
& + \{4e^3\gamma^2 - 2e^3\gamma^4 - \tfrac{189}{64}e^5\gamma^2 + \tfrac{3}{16}e\gamma^6\}\cos(3nt-\omega-2\Omega) \\
& + \{4e^3\gamma^2 - 2e^3\gamma^4 - \tfrac{189}{64}e^5\gamma^2 + \tfrac{3}{16}e\gamma^6\}\cos(nt-3\omega+2\Omega) \\
& - \{e^3\gamma^2 - \tfrac{1}{2}e^3\gamma^4 - \tfrac{171}{32}e^5\gamma^2 + \tfrac{3}{64}e\gamma^6\}\cos(3nt-5\omega+2\Omega) \\
& - \{e^3\gamma^2 - \tfrac{1}{2}e^3\gamma^4 - \tfrac{171}{32}e^5\gamma^2 + \tfrac{3}{64}e\gamma^6\}\cos(5nt-3\omega-2\Omega) \\
& + \tfrac{9}{8}e^2\gamma^4\cos(2nt+2\omega-4\Omega) - \tfrac{3}{32}e\gamma^6\cos(3nt+3\omega-6\Omega) \\
& - \tfrac{1}{64}e\gamma^6\{\cos(7nt-\omega-6\Omega) + \cos(nt-7\omega+6\Omega)\} \\
& - \tfrac{131}{64}e^5\gamma^2\{\cos(5nt-7\omega+2\Omega) + \cos(7nt-5\omega-2\Omega)\} \\
& - \tfrac{41}{32}e^3\gamma^4\{\cos(3nt+\omega-4\Omega) + \cos(nt+3\omega-4\Omega)\} \\
& - \tfrac{41}{64}e^3\gamma^4\{\cos(3nt-7\omega+4\Omega) + \cos(7nt-3\omega-4\Omega)\} \\
& + \tfrac{3}{16}e^2\gamma^4\{\cos(2nt-6\omega+4\Omega) + \cos(6nt-2\omega-4\Omega)\} \\
& - \tfrac{51}{8}e^4\gamma^2\{\cos(2nt-4\omega+2\Omega) + \cos(4nt-2\omega-2\Omega)\} \\
& + \tfrac{17}{8}e^4\gamma^2\{\cos(4nt-6\omega+2\Omega) + \cos(6nt-4\omega-2\Omega)\} \\
& + \tfrac{123}{64}e^3\gamma^4\{\cos(nt-5\omega+4\Omega) + \cos(5nt-\omega-4\Omega)\} \\
& + \tfrac{1}{16}e\gamma^6\{\cos(nt+5\omega-6\Omega) + \cos(5nt+\omega-6\Omega)\} \\
& - \tfrac{3}{4}e^2\gamma^4\{\cos 4(\omega-\Omega) + \cos 4(nt-\Omega)\} \\
& + \tfrac{17}{4}e^4\gamma^2\{\cos 2(\omega-\Omega) + \cos 2(nt-\Omega)\}
\end{aligned}\right\}; \qquad (94)
$$

$$\left.\begin{aligned}
f(nt)^5 = {} & \{20e^5 + \tfrac{275}{48}e^7 + \tfrac{25}{4}e^3\gamma^4\} \sin (nt - \omega) - \tfrac{75}{4}e^6 \sin 2 (nt - \omega) \\
& - \{10e^5 - \tfrac{205}{16}e^7 + \tfrac{25}{8}e^3\gamma^4\} \sin 3 (nt - \omega) + 15e^6 \cos 4 (nt - \omega) \\
& + \{2e^5 - \tfrac{125}{48}e^7 + \tfrac{5}{8}e^3\gamma^4\} \sin 5 (nt - \omega) - \tfrac{15}{4}e^6 \sin 6 (nt - \omega) \\
& + \tfrac{215}{48}e^7 \sin 7 (nt - \omega) + \tfrac{25}{2}e^4\gamma^2 \{\sin 2 (\omega - \Omega) - \sin 2 (nt - \Omega)\} \\
& + \tfrac{25}{4}e^4\gamma^2 \{\sin (2nt - 4\omega + 2\Omega) + \sin (4nt - 2\omega - 2\Omega)\} \\
& - \tfrac{5}{4}e^4\gamma^2 \{\sin (4nt - 6\omega + 2\Omega) + \sin (6nt - 4\omega - 2\Omega)\} \\
& + \tfrac{25}{8}e^5\gamma^2 \{\sin (5nt - 7\omega + 2\Omega) + \sin (7nt - 5\omega - 2\Omega)\} \\
& + \tfrac{125}{8}e^5\gamma^2 \{\sin (nt - 3\omega + 2\Omega) + \sin (3nt - \omega - 2\Omega)\} \\
& - \tfrac{25}{2}e^5\gamma^2 \{\sin (3nt - 5\omega + 2\Omega) + \sin (5nt - 3\omega - 2\Omega)\} \\
& + \tfrac{25}{8}e^3\gamma^4 \{\sin (3nt + \omega - 4\Omega) - \sin (nt + 3\omega - 4\Omega)\} \\
& - \tfrac{25}{16}e^3\gamma^4 \{\sin (nt - 5\omega + 4\Omega) + \sin (5nt - \omega - 4\Omega)\} \\
& + \tfrac{5}{16}e^3\gamma^4 \{\sin (3nt - 7\omega + 4\Omega) + \sin (7nt - 3\omega - 4\Omega)\}
\end{aligned}\right\}; \qquad (95)$$

$$\left.\begin{aligned}
f(nt)^6 = {} & 20e^6 - \tfrac{45}{2}e^7 \cos (nt - \omega) - 30e^6 \cos 2 (nt - \omega) \\
& + \tfrac{81}{2}e^7 \cos 3 (nt - \omega) + 12e^6 \cos 4 (nt - \omega) - \tfrac{45}{2}e^7 \cos 5 (nt - \omega) \\
& - 2e^6 \cos 6 (nt - \omega) + \tfrac{9}{2}e^7 \cos 7 (nt - \omega) - 30e^5\gamma^2 \cos (nt + \omega - 2\Omega) \\
& + \tfrac{45}{2}e^5\gamma^2 \{\cos (3nt - \omega - 2\Omega) + \cos (nt - 3\omega + 2\Omega)\} \\
& - 9e^5\gamma^2 \{\cos (5nt - 3\omega - 2\Omega) + \cos (3nt - 5\omega + 2\Omega)\} \\
& + \tfrac{3}{2}e^5\gamma^2 \{\cos (7nt - 5\omega - 2\Omega) + \cos (5nt - 7\omega + 2\Omega)\}
\end{aligned}\right\}; \qquad (96)$$

$$\left.\begin{aligned}
f(nt)^7 = {} & 70e^7 \sin (nt - \omega) - 42e^7 \sin 3 (nt - \omega) \\
& + 14e^5 \sin 5 (nt - \omega) - 2e^7 \sin 7 (nt - \omega)
\end{aligned}\right\}. \qquad (97)$$

$$
\frac{1}{2}\frac{df(nt)^2}{ndt} =
$$

$$
\left.\begin{aligned}
&\{\tfrac{3}{4}e^3 + \tfrac{1}{4}e^5 + \tfrac{1}{16}e\gamma^4 + \tfrac{9}{64}e^7 - \tfrac{1}{16}e\gamma^6 - \tfrac{19}{128}e^3\gamma^4\} \sin(nt-\omega) \\
&+ \{2e^2 - \tfrac{2}{3}e^4 + \tfrac{1}{16}\gamma^4 - \tfrac{1}{16}\gamma^6 - \tfrac{47}{128}e^6 - \tfrac{7}{16}e^2\gamma^4\} \sin 2(nt-\omega) \\
&- \{\tfrac{9}{4}e^3 - \tfrac{3}{32}e^5 + \tfrac{3}{16}e\gamma^4 - \tfrac{9}{40}e^7 - \tfrac{3}{16}e\gamma^6 - \tfrac{219}{256}e^3\gamma^4\} \sin 3(nt-\omega) \\
&+ \{\tfrac{91}{48}e^4 + \tfrac{31}{80}e^6 + \tfrac{7}{16}e^2\gamma^4\} \sin 4(nt-\omega) + \tfrac{1859}{1920}e^6 \sin 6(nt-\omega) \\
&- \{\tfrac{45}{32}e^5 + \tfrac{5}{8}e^7 + \tfrac{175}{256}e^3\gamma^4\} \sin 5(nt-\omega) - \tfrac{203}{320}e^7 \sin 7(nt-\omega) \\
&- \{\tfrac{11}{16}e^2\gamma^2 - \tfrac{11}{32}e^2\gamma^4 - \tfrac{13}{96}e^4\gamma^2 + \tfrac{1}{128}\gamma^6\} \sin 2(nt-\Omega) \\
&+ \{\tfrac{1}{16}\gamma^4 - \tfrac{1}{16}\gamma^6 - \tfrac{7}{16}e^2\gamma^4\} \sin 4(nt-\Omega) + \tfrac{3}{128}\gamma^6 \sin 6(nt-\Omega) \\
&+ \{\tfrac{1}{2}e\gamma^2 - \tfrac{1}{4}e\gamma^4 + \tfrac{35}{256}e\gamma^6 - \tfrac{1}{16}e^5\gamma^2 - \tfrac{3}{16}e^3\gamma^2 + \tfrac{3}{32}e^3\gamma^4\} \sin(nt+\omega-2\Omega) \\
&- \{\tfrac{1}{4}e\gamma^2 - \tfrac{1}{8}e\gamma^4 - \tfrac{5}{12}e^3\gamma^2 + \tfrac{5}{24}e^3\gamma^4 - \tfrac{9}{256}e^5\gamma^2 + \tfrac{7}{128}e\gamma^6\} \sin(nt-3\omega+2\Omega) \\
&- \{\tfrac{3}{4}e\gamma^2 - \tfrac{3}{8}e\gamma^4 - \tfrac{5}{4}e^3\gamma^2 + \tfrac{5}{8}e^3\gamma^4 - \tfrac{27}{256}e^5\gamma^2 + \tfrac{21}{128}e\gamma^6\} \sin(3nt-\omega-2\Omega) \\
&+ \{\tfrac{11}{16}e^2\gamma^2 - \tfrac{11}{32}e^2\gamma^4 - \tfrac{83}{128}e^4\gamma^2 + \tfrac{1}{128}\gamma^6\} \sin(2nt-4\omega+2\Omega) \\
&+ \{\tfrac{11}{8}e^2\gamma^2 - \tfrac{11}{16}e^2\gamma^4 - \tfrac{83}{64}e^4\gamma^2 + \tfrac{1}{64}\gamma^6\} \sin(4nt-2\omega-2\Omega) \\
&- \{\tfrac{93}{96}e^3\gamma^2 - \tfrac{93}{192}e^3\gamma^4 - \tfrac{87}{160}e^5\gamma^2 + \tfrac{21}{512}e\gamma^6\} \sin(3nt-5\omega+2\Omega) \\
&- \{\tfrac{155}{96}e^3\gamma^2 - \tfrac{155}{192}e^3\gamma^4 - \tfrac{29}{32}e^5\gamma^2 + \tfrac{35}{512}e\gamma^6\} \sin(5nt-3\omega-2\Omega) \\
&- \{\tfrac{1}{16}\gamma^4 - \tfrac{1}{16}\gamma^6 - \tfrac{3}{64}e^2\gamma^4\} \sin(2nt+2\omega-4\Omega) + \tfrac{3}{256}e\gamma^6 \sin(3nt+3\omega-6\Omega) \\
&- \{\tfrac{1}{16}e\gamma^4 - \tfrac{1}{16}e\gamma^6 - \tfrac{79}{512}e^3\gamma^4\} \sin(nt-5\omega+4\Omega) \\
&- \{\tfrac{5}{16}e\gamma^4 - \tfrac{5}{16}e\gamma^6 - \tfrac{395}{512}e^3\gamma^4\} \sin(5nt-\omega-4\Omega) \\
&+ \{\tfrac{3}{16}e\gamma^4 - \tfrac{3}{16}e\gamma^6 - \tfrac{35}{256}e^3\gamma^4\} \sin(3nt+\omega-4\Omega) \\
&+ \{\tfrac{1}{16}e\gamma^4 - \tfrac{1}{16}e\gamma^6 - \tfrac{35}{768}e^3\gamma^4\} \sin(nt+3\omega-4\Omega) \\
&+ \tfrac{75}{128}e^2\gamma^4 \sin(6nt-2\omega-4\Omega) + \tfrac{25}{128}e^2\gamma^4 \sin(2nt-6\omega+4\Omega) \\
&- \tfrac{1169}{1536}e^3\gamma^4 \sin(7nt-3\omega-4\Omega) - \tfrac{167}{512}e^3\gamma^4 \sin(3nt-7\omega+4\Omega) \\
&- \tfrac{23}{1536}e\gamma^6 \sin(nt-7\omega+6\Omega) - \tfrac{161}{1536}e\gamma^6 \sin(7nt-\omega-6\Omega) \\
&+ \tfrac{5}{384}e\gamma^6 \sin(nt+5\omega-6\Omega) + \tfrac{25}{384}e\gamma^6 \sin(5nt+\omega-6\Omega) \\
&- \tfrac{1659}{1280}e^5\gamma^2 \sin(7nt-5\omega-2\Omega) - \tfrac{237}{256}e^5\gamma^2 \sin(5nt-7\omega+2\Omega) \\
&- \tfrac{1}{64}\gamma^6 \sin(4nt+2\omega-6\Omega) - \tfrac{1}{128}\gamma^6 \sin(2nt+4\omega-6\Omega) \\
&+ \tfrac{591}{384}e^4\gamma^2 \sin(6nt-4\omega-2\Omega) + \tfrac{394}{384}e^4\gamma^2 \sin(4nt-6\omega+2\Omega)
\end{aligned}\right\} ; \qquad (98)
$$

$$\left.\begin{aligned}
f(nt)^5 = {} & \{20e^5 + \tfrac{275}{48}e^7 + \tfrac{25}{4}e^3\gamma^4\} \sin(nt - \omega) - \tfrac{15}{4}e^6 \sin 2(nt - \omega) \\
& - \{10e^5 - \tfrac{205}{16}e^7 + \tfrac{25}{8}e^3\gamma^4\} \sin 3(nt - \omega) + 15e^6 \cos 4(nt - \omega) \\
& + \{2e^5 - \tfrac{725}{48}e^7 + \tfrac{5}{8}e^3\gamma^4\} \sin 5(nt - \omega) - \tfrac{15}{4}e^6 \sin 6(nt - \omega) \\
& + \tfrac{215}{48}e^7 \sin 7(nt - \omega) + \tfrac{25}{2}e^4\gamma^2 \{\sin 2(\omega - \Omega) - \sin 2(nt - \Omega)\} \\
& + \tfrac{25}{4}e^4\gamma^2 \{\sin(2nt - 4\omega + 2\Omega) + \sin(4nt - 2\omega - 2\Omega)\} \\
& - \tfrac{5}{4}e^4\gamma^2 \{\sin(4nt - 6\omega + 2\Omega) + \sin(6nt - 4\omega - 2\Omega)\} \\
& + \tfrac{25}{8}e^5\gamma^2 \{\sin(5nt - 7\omega + 2\Omega) + \sin(7nt - 5\omega - 2\Omega)\} \\
& + \tfrac{125}{8}e^5\gamma^2 \{\sin(nt - 3\omega + 2\Omega) + \sin(3nt - \omega - 2\Omega)\} \\
& - \tfrac{25}{2}e^5\gamma^2 \{\sin(3nt - 5\omega + 2\Omega) + \sin(5nt - 3\omega - 2\Omega)\} \\
& + \tfrac{25}{8}e^3\gamma^4 \{\sin(3nt + \omega - 4\Omega) - \sin(nt + 3\omega - 4\Omega)\} \\
& - \tfrac{25}{16}e^3\gamma^4 \{\sin(nt - 5\omega + 4\Omega) + \sin(5nt - \omega - 4\Omega)\} \\
& + \tfrac{5}{16}e^3\gamma^4 \{\sin(3nt - 7\omega + 4\Omega) + \sin(7nt - 3\omega - 4\Omega)\}
\end{aligned}\right\}; \qquad (95)$$

$$\left.\begin{aligned}
f(nt)^6 = {} & 20e^6 - \tfrac{45}{2}e^7 \cos(nt - \omega) - 30e^6 \cos 2(nt - \omega) \\
& + \tfrac{81}{2}e^7 \cos 3(nt - \omega) + 12e^6 \cos 4(nt - \omega) - \tfrac{45}{2}e^7 \cos 5(nt - \omega) \\
& - 2e^6 \cos 6(nt - \omega) + \tfrac{9}{2}e^7 \cos 7(nt - \omega) - 30e^5\gamma^2 \cos(nt + \omega - 2\Omega) \\
& + \tfrac{45}{2}e^5\gamma^2 \{\cos(3nt - \omega - 2\Omega) + \cos(nt - 3\omega + 2\Omega)\} \\
& - 9e^5\gamma^2 \{\cos(5nt - 3\omega - 2\Omega) + \cos(3nt - 5\omega + 2\Omega)\} \\
& + \tfrac{3}{2}e^5\gamma^2 \{\cos(7nt - 5\omega - 2\Omega) + \cos(5nt - 7\omega + 2\Omega)\}
\end{aligned}\right\}; \qquad (96)$$

$$\left.\begin{aligned}
f(nt)^7 = {} & 70e^7 \sin(nt - \omega) - 42e^7 \sin 3(nt - \omega) \\
& + 14e^5 \sin 5(nt - \omega) - 2e^7 \sin 7(nt - \omega)
\end{aligned}\right\}. \qquad (97)$$

$$
\left.
\begin{aligned}
\tfrac{1}{24}\cdot\frac{d^3f(nt)^4}{n^3dt^3} = &-\{\tfrac{1}{4}e^5+\tfrac{67}{512}e^7+\tfrac{11}{128}e^3\gamma^4\}\sin(nt-\omega)\\
&-\{\tfrac{8}{3}e^4-\tfrac{37}{48}e^6+\tfrac{1}{2}e^2\gamma^4\}\sin 2(nt-\omega)\\
&+\{\tfrac{81}{8}e^5+\tfrac{513}{512}e^7+\tfrac{891}{256}e^3\gamma^4\}\sin 3(nt-\omega)\\
&+\{\tfrac{16}{3}e^4-\tfrac{52}{3}e^6+e^2\gamma^4\}\sin 4(nt-\omega)\\
&-\{\tfrac{125}{8}e^5-\tfrac{38375}{1536}e^7+\tfrac{3375}{768}e^3\gamma^4\}\sin 5(nt-\omega)\\
&+\tfrac{1305}{48}e^6\sin 6(nt-\omega)-\tfrac{55909}{1536}e^7\sin 7(nt-\omega)\\
&-\{\tfrac{1}{4}e^3\gamma^2-\tfrac{1}{8}e^3\gamma^4+\tfrac{11}{384}e^5\gamma^2+\tfrac{9}{768}e\gamma^6\}\sin(nt+\omega-2\Omega)\\
&+\{\tfrac{9}{2}e^3\gamma^2-\tfrac{9}{4}e^3\gamma^4-\tfrac{1701}{512}e^5\gamma^2+\tfrac{27}{128}e\gamma^6\}\sin(3nt-\omega-2\Omega)\\
&+\{\tfrac{1}{6}e^3\gamma^2-\tfrac{1}{12}e^3\gamma^4-\tfrac{189}{1536}e^5\gamma^2+\tfrac{1}{128}e\gamma^6\}\sin(nt-3\omega+2\Omega)\\
&-\{\tfrac{9}{8}e^3\gamma^2-\tfrac{9}{16}e^3\gamma^4-\tfrac{1539}{256}e^5\gamma^2+\tfrac{27}{512}e\gamma^6\}\sin(3nt-5\omega+2\Omega)\\
&-\{\tfrac{125}{24}e^3\gamma^2-\tfrac{125}{48}e^3\gamma^4-\tfrac{21875}{768}e^5\gamma^2+\tfrac{125}{512}e\gamma^6\}\sin(5nt-3\omega-2\Omega)\\
&+\tfrac{3}{8}e^2\gamma^4\sin(2nt+2\omega-4\Omega)-\tfrac{27}{256}e\gamma^6\sin(3nt+3\omega-6\Omega)\\
&-\tfrac{343}{1536}e\gamma^6\sin(7nt-\omega-6\Omega)-\tfrac{1}{1536}e\gamma^6\sin(nt-7\omega+6\Omega)\\
&-\tfrac{16875}{1536}e^5\gamma^2\sin(5nt-7\omega+2\Omega)-\tfrac{44933}{1536}e^5\gamma^2\sin(7nt-5\omega-2\Omega)\\
&-\tfrac{369}{256}e^3\gamma^4\sin(3nt+\omega-4\Omega)-\tfrac{41}{768}e^3\gamma^4\sin(nt+3\omega-4\Omega)\\
&-\tfrac{369}{512}e^3\gamma^4\sin(3nt-7\omega+4\Omega)-\tfrac{14063}{1536}e^3\gamma^4\sin(7nt-3\omega-4\Omega)\\
&+\tfrac{1}{16}e^2\gamma^4\sin(2nt-6\omega+4\Omega)+\tfrac{27}{16}e^2\gamma^4\sin(6nt-2\omega-4\Omega)\\
&-\tfrac{17}{8}e^4\gamma^2\sin(2nt-4\omega+2\Omega)-17e^4\gamma^2\sin(4nt-2\omega-2\Omega)\\
&+\tfrac{17}{3}e^4\gamma^2\sin(4nt-6\omega+2\Omega)+\tfrac{153}{8}e^4\gamma^2\sin(6nt-4\omega-2\Omega)\\
&+\tfrac{41}{512}e^3\gamma^4\sin(nt-5\omega+4\Omega)+\tfrac{5125}{512}e^3\gamma^4\sin(5nt-\omega-4\Omega)\\
&-2e^2\gamma^4\sin 4(nt-\Omega)+\tfrac{17}{12}e^4\gamma^2\sin 2(nt-\Omega)\\
&+\tfrac{1}{384}e\gamma^6\sin(nt+5\omega-6\Omega)+\tfrac{125}{384}e\gamma^6\sin(5nt+\omega-6\Omega)
\end{aligned}
\right\};\qquad(100)
$$

$$\left.\begin{aligned}
\tfrac{1}{120}\frac{d^4 f(nt)^5}{n^4 dt^4} &= \{\tfrac{1}{6}e^5 + \tfrac{55}{1152}e^7 + \tfrac{5}{96}e^3\gamma^4\}\sin(nt-\omega) - \tfrac{5}{2}e^6 \sin 2(nt-\omega) \\
&- \{\tfrac{27}{4}e^5 - \tfrac{1107}{128}e^7 + \tfrac{135}{64}e^3\gamma^4\}\sin 3(nt-\omega) + 32e^6 \sin 4(nt-\omega) \\
&+ \{\tfrac{125}{12}e^5 - \tfrac{90625}{1152}e^7 + \tfrac{625}{192}e^3\gamma^4\}\sin 5(nt-\omega) - \tfrac{81}{2}e^6 \sin 6(nt-\omega) \\
&+ \tfrac{108248}{1152}e^7 \sin 7(nt-\omega) - \tfrac{5}{3}e^4\gamma^2 \sin 2(nt-\Omega) \\
&+ \tfrac{5}{6}e^4\gamma^2 \sin(2nt-4\omega+2\Omega) + \tfrac{40}{3}e^4\gamma^2 \sin(4nt-2\omega-2\Omega) \\
&- \tfrac{8}{3}e^4\gamma^2 \sin(4nt-6\omega+2\Omega) - \tfrac{27}{2}e^4\gamma^2 \sin(6nt-4\omega-2\Omega) \\
&+ \tfrac{3125}{192}e^5\gamma^2 \sin(5nt-7\omega+2\Omega) + \tfrac{12005}{192}e^5\gamma^2 \sin(7nt-5\omega-2\Omega) \\
&+ \tfrac{25}{192}e^5\gamma^2 \sin(nt-3\omega+2\Omega) + \tfrac{2025}{192}e^5\gamma^2 \sin(3nt-\omega-2\Omega) \\
&- \tfrac{405}{48}e^5\gamma^2 \sin(3nt-5\omega+2\Omega) - \tfrac{3125}{48}e^5\gamma^2 \sin(5nt-3\omega-2\Omega) \\
&+ \tfrac{135}{64}e^3\gamma^4 \sin(3nt+\omega-4\Omega) - \tfrac{5}{192}e^3\gamma^4 \sin(nt+3\omega-4\Omega) \\
&- \tfrac{5}{384}e^3\gamma^4 \sin(nt-5\omega+4\Omega) - \tfrac{3125}{384}e^3\gamma^4 \sin(5nt-\omega-4\Omega) \\
&+ \tfrac{27}{128}e^3\gamma^4 \sin(3nt-7\omega+4\Omega) + \tfrac{2401}{384}e^3\gamma^4 \sin(7nt-3\omega-4\Omega)
\end{aligned}\right\}; \tag{101}$$

$$\left.\begin{aligned}
\tfrac{1}{120}\frac{d^5 f(nt)^6}{n^5 dt^5} &= +\tfrac{1}{32}e^7 \sin(nt-\omega) + \tfrac{4}{3}e^6 \sin 2(nt-\omega) \\
&- \tfrac{2187}{160}e^7 \sin 3(nt-\omega) - \tfrac{256}{15}e^6 \sin 4(nt-\omega) \\
&+ \tfrac{3125}{32}e^7 \sin 5(nt-\omega) + \tfrac{108}{5}e^6 \sin 6(nt-\omega) \\
&+ \tfrac{1}{24}e^5\gamma^2 \sin(nt+\omega-2\Omega) - \tfrac{243}{32}e^5\gamma^2 \sin(3nt-\omega-2\Omega) \\
&- \tfrac{1}{32}e^5\gamma^2 \sin(nt-3\omega+2\Omega) + \tfrac{625}{16}e^5\gamma^2 \sin(5nt-3\omega-2\Omega) \\
&+ \tfrac{243}{80}e^5\gamma^2 \sin(3nt-5\omega+2\Omega) - \tfrac{16807}{480}e^5\gamma^2 \sin(7nt-5\omega-2\Omega) \\
&- \tfrac{625}{96}e^5\gamma^2 \sin(5nt-7\omega+2\Omega) - \tfrac{16807}{160}e^7 \sin 7(nt-\omega)
\end{aligned}\right\}; \tag{102}$$

$$\left.\begin{aligned}
\tfrac{1}{5040}\frac{d^6 f(nt)^7}{n^6 dt^6} &= -\tfrac{1}{72}e^7 \sin(nt-\omega) + \tfrac{243}{40}e^7 \sin 3(nt-\omega) \\
&- \tfrac{3125}{72}e^7 \sin 5(nt-\omega) + \tfrac{16807}{360}e^7 \sin 7(nt-\omega)
\end{aligned}\right\}. \tag{103}$$

If we now substitute equation (91) and equations (98–103) in equation (90), we shall obtain the following value of the true longitude v of the moon in terms of the mean longitude nt.

$$
\left.
\begin{aligned}
\tfrac{1}{24}\cdot\frac{d^3 f(nt)^4}{n^3dt^3} = & -\{\tfrac{1}{4}e^5+\tfrac{67}{512}e^7+\tfrac{11}{128}e^3\gamma^4\}\sin(nt-\omega) \\
& -\{\tfrac{8}{3}e^4-\tfrac{37}{48}e^6+\tfrac{1}{2}e^2\gamma^4\}\sin 2(nt-\omega) \\
& +\{\tfrac{81}{8}e^5+\tfrac{513}{512}e^7+\tfrac{891}{256}e^3\gamma^4\}\sin 3(nt-\omega) \\
& +\{\tfrac{16}{3}e^4-\tfrac{59}{3}e^6+e^2\gamma^4\}\sin 4(nt-\omega) \\
& -\{\tfrac{125}{8}e^5-\tfrac{38375}{1536}e^7+\tfrac{3375}{768}e^3\gamma^4\}\sin 5(nt-\omega) \\
& +\tfrac{1305}{48}e^6\sin 6(nt-\omega)-\tfrac{55909}{1536}e^7\sin 7(nt-\omega) \\
& -\{\tfrac{1}{4}e^3\gamma^2-\tfrac{1}{8}e^3\gamma^4+\tfrac{11}{384}e^5\gamma^2+\tfrac{9}{768}e\gamma^6\}\sin(nt+\omega-2\Omega) \\
& +\{\tfrac{9}{2}e^3\gamma^2-\tfrac{9}{4}e^3\gamma^4-\tfrac{1701}{512}e^5\gamma^2+\tfrac{27}{128}e\gamma^6\}\sin(3nt-\omega-2\Omega) \\
& +\{\tfrac{1}{6}e^3\gamma^2-\tfrac{1}{12}e^3\gamma^4-\tfrac{189}{1536}e^5\gamma^2+\tfrac{1}{128}e\gamma^6\}\sin(nt-3\omega+2\Omega) \\
& -\{\tfrac{9}{8}e^3\gamma^2-\tfrac{9}{16}e^3\gamma^4-\tfrac{1539}{256}e^5\gamma^2+\tfrac{27}{512}e\gamma^6\}\sin(3nt-5\omega+2\Omega) \\
& -\{\tfrac{125}{24}e^3\gamma^2-\tfrac{125}{48}e^3\gamma^4-\tfrac{21875}{768}e^5\gamma^2+\tfrac{125}{512}e\gamma^6\}\sin(5nt-3\omega-2\Omega) \\
& +\tfrac{3}{8}e^2\gamma^4\sin(2nt+2\omega-4\Omega)-\tfrac{27}{256}e\gamma^6\sin(3nt+3\omega-6\Omega) \\
& -\tfrac{343}{1536}e\gamma^6\sin(7nt-\omega-6\Omega)-\tfrac{1}{1536}e\gamma^6\sin(nt-7\omega+6\Omega) \\
& -\tfrac{16875}{1536}e^5\gamma^2\sin(5nt-7\omega+2\Omega)-\tfrac{44933}{1536}e^5\gamma^2\sin(7nt-5\omega-2\Omega) \\
& -\tfrac{369}{256}e^3\gamma^4\sin(3nt+\omega-4\Omega)-\tfrac{41}{768}e^3\gamma^4\sin(nt+3\omega-4\Omega) \\
& -\tfrac{369}{512}e^3\gamma^4\sin(3nt-7\omega+4\Omega)-\tfrac{14063}{1536}e^3\gamma^4\sin(7nt-3\omega-4\Omega) \\
& +\tfrac{1}{16}e^2\gamma^4\sin(2nt-6\omega+4\Omega)+\tfrac{27}{16}e^2\gamma^4\sin(6nt-2\omega-4\Omega) \\
& -\tfrac{17}{8}e^4\gamma^2\sin(2nt-4\omega+2\Omega)-17e^4\gamma^2\sin(4nt-2\omega-2\Omega) \\
& +\tfrac{17}{3}e^4\gamma^2\sin(4nt-6\omega+2\Omega)+\tfrac{153}{8}e^4\gamma^2\sin(6nt-4\omega-2\Omega) \\
& +\tfrac{41}{512}e^3\gamma^4\sin(nt-5\omega+4\Omega)+\tfrac{5125}{512}e^3\gamma^4\sin(5nt-\omega-4\Omega) \\
& -2e^2\gamma^4\sin 4(nt-\Omega)+\tfrac{17}{12}e^4\gamma^2\sin 2(nt-\Omega) \\
& +\tfrac{1}{384}e\gamma^6\sin(nt+5\omega-6\Omega)+\tfrac{125}{384}e\gamma^6\sin(5nt+\omega-6\Omega)
\end{aligned}
\right\}; \qquad (100)
$$

6. Having found the value of v in terms of nt, if we now substitute it in equations (73) and (39), we shall also obtain the values of r and θ in terms of nt. For this purpose we shall first develop equation (73) in a series, carrying the approximation to terms of the fifth order, and we shall find

$$\left.\begin{aligned} \frac{a(1-e^2)}{r} &= 1 - e\{1 - \tfrac{1}{64}\gamma^4\}\cos(v-\omega) - \tfrac{1}{4}e\gamma^2\{1-\tfrac{1}{2}\gamma^2\}\cos(v+\omega-2\Omega) \\ &+ \tfrac{1}{8}e\gamma^2\{1-\tfrac{1}{2}\gamma^2\}\cos\{v-3\omega+2\Omega) + \tfrac{1}{8}e\gamma^2\{1-\tfrac{1}{2}\gamma^2\}\cos(3v-\omega-2\Omega) \\ &+ \tfrac{3}{128}e\gamma^4\cos(v-5\omega+4\Omega) + \tfrac{3}{128}e\gamma^4\cos(5v-\omega-4\Omega) \\ &- \tfrac{3}{128}e\gamma^4\cos(3v+\omega-4\Omega) - \tfrac{3}{128}e\gamma^4\cos(v+3\omega-4\Omega) + \tfrac{1}{64}e\gamma^4\cos 3(v-\omega) \end{aligned}\right\}. \tag{105}$$

In order to substitute the value of v in this equation, we shall observe that if we put all the terms of the second member of equation (104), except the first, equal to β, we shall have

$$v = nt + \beta. \tag{106}$$

This gives
$$mv + \alpha = (mnt + \alpha) + m\beta, \tag{107}$$

m being any positive whole number, and α any angle whatever.

Equation (107) gives

$$\cos(mv+\alpha) = \cos(mnt+\alpha)\cos m\beta - \sin(mnt+\alpha)\sin m\beta. \tag{108}$$

Since β is a small quantity, we may develop $\sin m\beta$ and $\cos m\beta$ in a series, and we shall have, with sufficient accuracy,

$$\sin m\beta = m\beta - \tfrac{1}{6}m^3\beta^3, \qquad \cos m\beta = 1 - \tfrac{1}{2}m^2\beta^2 + \tfrac{1}{24}m^4\beta^4. \tag{109}$$

We shall then find, as far as terms of the fourth order,

$$\left.\begin{aligned} \beta^2 &= 2e^2 + \tfrac{9}{32}e^4 + \tfrac{1}{16}\gamma^4 + \tfrac{5}{2}e^3\cos(nt-\omega) \\ &- \{2e^2 - \tfrac{8}{3}e^4 + \tfrac{1}{16}\gamma^4\}\cos 2(nt-\omega) - \tfrac{5}{2}e^3\cos 3(nt-\omega) \\ &- \tfrac{283}{96}e^4\cos 4(nt-\omega) - \tfrac{37}{16}e^2\gamma^2\cos 2(nt-\Omega) + \tfrac{11}{16}e^2\gamma^2\cos 2(\omega-\Omega) \\ &- \tfrac{1}{32}\gamma^4\cos 4(nt-\Omega) - \tfrac{1}{32}\gamma^4\cos 4(\omega-\Omega) + \tfrac{1}{16}\gamma^4\cos(2nt+2\omega-4\Omega) \\ &- e\gamma^2\cos(nt+\omega-2\Omega) + \tfrac{1}{2}e\gamma^2\cos(3nt-\omega-2\Omega) + \tfrac{1}{2}e\gamma^2\cos(nt-3\omega+2\Omega) \\ &+ \tfrac{21}{16}e^2\gamma^2\cos(4nt-2\omega-2\Omega) + \tfrac{5}{16}e^2\gamma^2\cos(2nt-4\omega+2\Omega) \end{aligned}\right\}. \tag{110}$$

$$\left.\begin{aligned}\beta^3 &= 6e^3\sin(nt-\omega)+\tfrac{15}{2}e^4\sin 2(nt-\omega)-2e^3\sin 3(nt-\omega)\\ &\quad -\tfrac{15}{4}e^4\sin 4(nt-\omega)-\tfrac{9}{4}e^2\gamma^2\sin 2(nt-\Omega)+\tfrac{9}{4}e^2\gamma^2\sin 2(\omega-\Omega)\\ &\quad +\tfrac{3}{4}e^2\gamma^2\sin(4nt-2\omega-2\Omega)+\tfrac{3}{4}e^2\gamma^2\sin(2nt-4\omega+2\Omega)\end{aligned}\right\};\tag{111}$$

$$\beta^4 = 6e^4-8e^4\cos 2(nt-\omega)+2e^4\cos 4(nt-\omega).\tag{112}$$

Therefore equation (108) will become

$$\left.\begin{aligned}&\cos(mv+\alpha)=\cos(mnt+\alpha)\times\\ &\quad\{1-m^2(e^2+\tfrac{9}{64}e^4+\tfrac{1}{32}\gamma^4)+\tfrac{1}{4}m^4e^4-\tfrac{5}{4}m^2e^3\cos(nt-\omega)\\ &\quad+(m^2e^2-\tfrac{4}{3}m^2e^4+\tfrac{1}{32}m^2\gamma^4-\tfrac{1}{3}m^4e^4)\cos 2(nt-\omega)\\ &\quad+\tfrac{5}{4}m^2e^3\cos 3(nt-\omega)+(\tfrac{283}{192}m^2e^4+\tfrac{1}{12}m^4e^4)\cos 4(nt-\omega)\\ &\quad+\tfrac{37}{32}m^2e^2\gamma^2\cos 2(nt-\Omega)-\tfrac{11}{32}m^2e^2\gamma^2\cos 2(\omega-\Omega)\\ &\quad+\tfrac{1}{64}m^2\gamma^4\cos 4(nt-\Omega)+\tfrac{1}{64}m^2\gamma^4\cos 4(\omega-\Omega)\\ &\quad-\tfrac{1}{32}m^2\gamma^4\cos(2nt+2\omega-4\Omega)+\tfrac{1}{2}e\gamma^2m^2\cos(nt+\omega-2\Omega)\\ &\quad-\tfrac{1}{4}m^2e\gamma^2\cos(3nt-\omega-2\Omega)-\tfrac{1}{4}m^2e\gamma^2\cos(nt-3\omega+2\Omega)\\ &\quad-\tfrac{21}{32}m^2e^2\gamma^2\cos(4nt-2\omega-2\Omega)-\tfrac{5}{32}m^2e^2\gamma^2\cos(2nt-4\omega+2\Omega)\}\\ &\quad-\sin(mnt+\alpha)\times\\ &\quad\{(2me-\tfrac{1}{4}me^3-m^3e^3)\sin(nt-\omega)+(\tfrac{5}{4}me^2-\tfrac{11}{24}me^4+\tfrac{1}{16}m\gamma^4-\tfrac{5}{4}m^3e^4)\sin 2(nt-\omega)\\ &\quad+(\tfrac{13}{12}me^3+\tfrac{1}{3}m^3e^3)\sin 3(nt-\omega)+(\tfrac{103}{96}me^4+\tfrac{5}{8}m^3e^4)\sin 4(nt-\omega)\\ &\quad-(\tfrac{1}{4}m\gamma^2-\tfrac{1}{8}m\gamma^4-me^2\gamma^2-\tfrac{3}{8}m^3e^2\gamma^2)\sin 2(nt-\Omega)-\tfrac{1}{2}me\gamma^2\sin(3nt-\omega-2\Omega)\\ &\quad+(\tfrac{1}{4}m\gamma^2-\tfrac{1}{8}m\gamma^4-\tfrac{3}{16}me^2\gamma^2-\tfrac{3}{8}m^3e^2\gamma^2)\sin 2(\omega-\Omega)+\tfrac{1}{2}me\gamma^2\sin(nt+\omega-2\Omega)\\ &\quad+\tfrac{1}{32}m\gamma^4\sin 4(nt-\Omega)+\tfrac{1}{32}m\gamma^4\sin 4(\omega-\Omega)-\tfrac{1}{16}m\gamma^4\sin(2nt+2\omega-4\Omega)\\ &\quad-(\tfrac{13}{16}me^2\gamma^2+\tfrac{1}{8}m^3e^2\gamma^2)\sin(4nt-2\omega-2\Omega)-\tfrac{1}{8}m^3e^2\gamma^2\sin(2nt-4\omega+2\Omega)\}\end{aligned}\right\}.\tag{113}$$

If in this equation we put in succession $m=1$, $\alpha=-\omega$; $m=1$, $\alpha=\omega-2\Omega$; $m=1$, $\alpha=-3\omega+2\Omega$; and $m=3$, $\alpha=-\omega-2\Omega$; we shall get the following equations:

$$\left.\begin{aligned}
\cos(v-\omega) = & -e + \{1 - \tfrac{9}{8}e^2 + \tfrac{25}{192}e^4 - \tfrac{3}{64}\gamma^4\}\cos(nt-\omega) \\
& + \{e - \tfrac{4}{3}e^3\}\cos 2(nt-\omega) + \{\tfrac{9}{8}e^2 - \tfrac{225}{128}e^4 + \tfrac{3}{64}\gamma^4\}\cos 3(nt-\omega) \\
& + \tfrac{4}{3}e^3\cos 4(nt-\omega) + \tfrac{625}{384}e^4\cos 5(nt-\omega) \\
& - \{\tfrac{1}{8}\gamma^2 - \tfrac{1}{16}\gamma^4 - \tfrac{45}{32}e^2\gamma^2\}\cos(3nt-\omega-2\Omega) \\
& + \{\tfrac{1}{4}\gamma^2 - \tfrac{1}{8}\gamma^4 - \tfrac{9}{16}e^2\gamma^2\}\cos(nt+\omega-2\Omega) \\
& - \{\tfrac{1}{8}\gamma^2 - \tfrac{1}{16}\gamma^4 - \tfrac{3}{32}e^2\gamma^2\}\cos(nt-3\omega+2\Omega) \\
& + \tfrac{3}{128}\gamma^4\cos(5nt-\omega-4\Omega) - \tfrac{7}{128}\gamma^4\cos(3nt+\omega-4\Omega) \\
& + \tfrac{5}{128}\gamma^4\cos(nt+3\omega-4\Omega) - \tfrac{1}{128}\gamma^4\cos(nt-5\omega+4\Omega) \\
& + \tfrac{5}{8}e\gamma^2\cos 2(nt-\Omega) - \tfrac{3}{8}e\gamma^2\cos(4nt-2\omega-2\Omega) \\
& - \tfrac{1}{8}e\gamma^2\cos(2nt-4\omega+2\Omega) - \tfrac{1}{8}e\gamma^2\cos 2(\omega-\Omega) \\
& - \tfrac{51}{64}e^2\gamma^2\cos(5nt-3\omega-2\Omega) - \tfrac{9}{64}e^2\gamma^2\cos(3nt-5\omega+2\Omega)
\end{aligned}\right\}; \quad (114)$$

$$\left.\begin{aligned}
\cos(v+\omega-2\Omega) = & (1-e^2)\cos(nt+\omega-2\Omega) + \tfrac{3}{8}e^2\cos(3nt-\omega-2\Omega) \\
& - \tfrac{1}{8}e^2\cos(nt-3\omega+2\Omega) - \tfrac{1}{8}\gamma^2\cos(3nt+\omega-4\Omega) \\
& + \tfrac{1}{8}\gamma^2\cos(nt+3\omega-4\Omega) + e\cos 2(nt-\Omega) - e\cos 2(\omega-\Omega)
\end{aligned}\right\}; \quad (115)$$

$$\left.\begin{aligned}
\cos(v-3\omega+2\Omega) = & (1-e^2)\cos(nt-3\omega+2\Omega) - \tfrac{1}{8}e^2\cos(nt+\omega-2\Omega) \\
& + \tfrac{3}{8}e^2\cos(3nt-5\omega+2\Omega) + \tfrac{1}{8}\gamma^2\cos(nt+3\omega-4\Omega) \\
& - \tfrac{1}{8}\gamma^2\cos(nt-5\omega+4\Omega) + e\cos(2nt-4\omega+2\Omega) \\
& - e\cos 2(\omega-\Omega) + \tfrac{1}{8}\gamma^2\cos(nt-\omega) - \tfrac{1}{8}\gamma^2\cos 3(nt-\omega)
\end{aligned}\right\}; \quad (116)$$

$$\left.\begin{aligned}
\cos(3v-\omega-2\Omega) = & (1-9e^2)\cos(3nt-\omega-2\Omega) + \tfrac{51}{8}e^2\cos(5nt-3\omega-2\Omega) \\
& + \tfrac{21}{8}e^2\cos(nt+\omega-2\Omega) + \tfrac{3}{8}\gamma^2\cos(3nt+\omega-4\Omega) \\
& - \tfrac{3}{8}\gamma^2\cos(5nt-\omega-4\Omega) + 3e\cos(4nt-2\omega-2\Omega) \\
& - 3e\cos 2(nt-\Omega) + \tfrac{3}{8}\gamma^2\cos(nt-\omega) - \tfrac{3}{8}\gamma^2\cos 3(nt-\omega)
\end{aligned}\right\}. \quad (117)$$

If we now substitute these values in equation (105), and at the same time

change v into nt in the terms of the fifth order in that equation, it will become

$$\left.\begin{aligned} \frac{a(1-e^2)}{r} = 1 - e^2 &+ e\{1 - \tfrac{9}{8}e^2 + \tfrac{25}{192}e^4\}\cos(nt-\omega) \\ &+ e^2\{1 - \tfrac{4}{3}e^2\}\cos 2(nt-\omega) + \{\tfrac{9}{8}e^3 - \tfrac{225}{128}e^5\}\cos 3(nt-\omega) \\ &+ \tfrac{4}{3}e^4\cos 4(nt-\omega) + \tfrac{625}{384}e^5\cos 5(nt-\omega) \end{aligned}\right\}. \tag{118}$$

In the substitution in equation (105), the coefficients of all the terms containing Ω become identically equal to nothing, and the quantity γ disappears from the coefficients of all the remaining terms; whence it follows that the radius vector of the orbit is entirely independent of the position of the orbit. We shall therefore have

$$\left.\begin{aligned} \frac{a}{r} = 1 &+ e\{1 - \tfrac{1}{8}e^2 + \tfrac{1}{192}e^4\}\cos(nt-\omega) + e^2\{1 - \tfrac{1}{3}e^2\}\cos 2(nt-\omega) \\ &+ \{\tfrac{9}{8}e^3 - \tfrac{81}{128}e^5\}\cos 3(nt-\omega) + \tfrac{4}{3}e^4\cos 4(nt-\omega) + \tfrac{625}{384}e^5\cos 5(nt-\omega) \end{aligned}\right\}. \tag{119}$$

In order to find the tangent of the latitude it is only necessary to find $\sin(v-\Omega)$, and multiply it by γ. To find $\sin(v-\Omega)$ we shall observe that if we change $\cos(mnt+\alpha)$ into $\sin(mnt+\alpha)$, and $-\sin(mnt+\alpha)$ into $+\cos(mnt+\alpha)$, $\cos(mv+\alpha)$ will change $\sin(mv+\alpha)$. If we then make these changes in equation (113) and put $m=1$, $\alpha=-\Omega$, we shall get, after multiplying by γ, the following value of $\gamma\sin(v-\Omega)$, or $\tan\theta$,

$$\left.\begin{aligned} \tan\theta = \gamma\{1 - e^2 - \tfrac{1}{8}\gamma^2 &+ \tfrac{7}{64}e^4 + \tfrac{1}{32}\gamma^4 + \tfrac{7}{64}e^2\gamma^2\}\sin(nt-\Omega) \\ &- \{\tfrac{1}{8}\gamma^3 - \tfrac{9}{128}\gamma^5 - \tfrac{81}{64}e^2\gamma^3\}\sin 3(nt-\Omega) + \tfrac{3}{128}\gamma^5\sin 5(nt-\Omega) \\ &+ e\gamma\{1 - \tfrac{5}{4}e^2 - \tfrac{1}{8}\gamma^2\}\sin(2nt-\omega-\Omega) - e\gamma\sin(\omega-\Omega) \\ &+ \{\tfrac{9}{8}e^2\gamma - \tfrac{27}{16}e^4\gamma + \tfrac{3}{64}\gamma^5 - \tfrac{9}{64}e^2\gamma^3\}\sin(3nt-2\omega-\Omega) \\ &+ \{\tfrac{1}{8}e^2\gamma - \tfrac{1}{8}\gamma^3 - \tfrac{1}{48}e^4\gamma + \tfrac{5}{64}\gamma^5 + \tfrac{7}{64}e^2\gamma^3\}\sin(nt-2\omega+\Omega) \\ &+ \{\tfrac{1}{8}\gamma^3 - \tfrac{5}{64}\gamma^5 - \tfrac{29}{64}e^2\gamma^3\}\sin(nt+2\omega-3\Omega) - \tfrac{1}{8}e\gamma^3\sin 3(\omega-\Omega) \\ &+ \tfrac{4}{3}e^3\gamma\sin(4nt-3\omega-\Omega) - \tfrac{51}{64}e^2\gamma^3\sin(5nt-2\omega-3\Omega) \\ &+ \{\tfrac{1}{12}e^3\gamma - \tfrac{1}{8}e\gamma^3\}\sin(2nt-3\omega+\Omega) + \tfrac{625}{384}e^4\gamma\sin(5nt-4\omega-\Omega) \\ &+ \{\tfrac{9}{128}e^4\gamma - \tfrac{9}{64}e^2\gamma^3\}\sin(3nt-4\omega+\Omega) + \tfrac{3}{128}\gamma^5\sin(nt+4\omega-5\Omega) \\ &+ \{\tfrac{1}{64}e^2\gamma^3 - \tfrac{1}{128}\gamma^5\}\sin(nt-4\omega+3\Omega) - \tfrac{3}{8}e\gamma^3\sin(4nt-\omega-3\Omega) \\ &- \tfrac{3}{64}\gamma^5\sin(3nt+2\omega-5\Omega) + \tfrac{1}{2}e\gamma^3\sin(2nt+\omega-3\Omega) \end{aligned}\right\}. \tag{120}$$

This equation will give

$$\left.\begin{aligned}\tfrac{1}{3}\tan^3\theta = {} & \tfrac{1}{4}\gamma^3\{1-e^2-\tfrac{1}{4}\gamma^2\}\sin(nt-\Omega)-\tfrac{1}{12}\gamma^3\{1-9e^2+\tfrac{3}{8}\gamma^2\}\sin 3(nt-\Omega)\\ & +\tfrac{1}{32}\gamma^5\sin 5(nt-\Omega)-\tfrac{1}{4}e\gamma^3\sin(\omega-\Omega)+\tfrac{1}{4}e\gamma^3\sin(2nt+\omega-3\Omega)\\ & +\tfrac{1}{4}e\gamma^3\sin(2nt-\omega-\Omega)-\tfrac{1}{4}e\gamma^3\sin(4nt-\omega-3\Omega)-\tfrac{1}{32}\gamma^5\sin(3nt+2\omega-5\Omega)\\ & +\tfrac{1}{32}\gamma^3\{\gamma^2+9e^2\}\sin(3nt-2\omega-\Omega)-\tfrac{17}{32}e^2\gamma^3\sin(5nt-2\omega-3\Omega)\\ & +\tfrac{1}{32}\gamma^3\{e^2-\gamma^2\}\sin(nt-2\omega+\Omega)+\tfrac{1}{32}\gamma^3\{\gamma^2-7e^2\}\sin(nt+2\omega-3\Omega)\end{aligned}\right\};\tag{121}$$

$$\tfrac{1}{5}\tan^5\theta=\tfrac{1}{8}\gamma^5\sin(nt-\Omega)-\tfrac{1}{16}\gamma^5\sin 3(nt-\Omega)+\tfrac{1}{80}\gamma^5\sin 5(nt-\Omega).\tag{122}$$

Now we have $$\theta=\tan\theta-\tfrac{1}{3}\tan^3\theta+\tfrac{1}{5}\tan^5\theta-,\text{ etc.}\tag{123}$$

If we now substitute these values of $\tan\theta$ and its powers in equation (123) we shall get the following expression for the latitude of the moon, which is correct to terms of the fifth order:

$$\left.\begin{aligned}\theta = {} & \gamma\{1-e^2-\tfrac{3}{8}\gamma^2+\tfrac{7}{64}e^4+\tfrac{7}{32}\gamma^4+\tfrac{23}{64}e^2\gamma^2\}\sin(nt-\Omega)\\ & -\{\tfrac{1}{24}\gamma^3-\tfrac{5}{128}\gamma^5-\tfrac{33}{64}e^2\gamma^3\}\sin 3(nt-\Omega)+\tfrac{3}{640}\gamma^5\sin 5(nt-\Omega)\\ & +e\gamma\{1-\tfrac{5}{4}e^2-\tfrac{3}{8}\gamma^2\}\sin(2nt-\omega-\Omega)-e\gamma\{1-\tfrac{1}{4}\gamma^2\}\sin(\omega-\Omega)\\ & +\{\tfrac{9}{8}e^2\gamma-\tfrac{27}{16}e^4\gamma+\tfrac{1}{64}\gamma^5-\tfrac{27}{64}e^2\gamma^3\}\sin(3nt-2\omega-\Omega)-\tfrac{1}{8}e\gamma^3\sin 3(\omega-\Omega)\\ & +\{\tfrac{1}{8}e^2\gamma-\tfrac{1}{8}\gamma^3-\tfrac{1}{48}e^4\gamma+\tfrac{7}{64}\gamma^5+\tfrac{5}{64}e^2\gamma^3\}\sin(nt-2\omega+\Omega)\\ & +\{\tfrac{1}{8}\gamma^3-\tfrac{7}{64}\gamma^5-\tfrac{15}{64}e^2\gamma^3\}\sin(nt+2\omega-3\Omega)+\tfrac{4}{3}e^3\gamma\sin(4nt-3\omega-\Omega)\\ & -\tfrac{1}{8}e\gamma^3\sin(4nt-\omega-3\Omega)+\tfrac{1}{4}e\gamma^3\sin(2nt+\omega-3\Omega)\\ & +\{\tfrac{1}{12}e^3\gamma-\tfrac{1}{8}e\gamma^3\}\sin(2nt-3\omega+\Omega)+\tfrac{625}{384}e^4\gamma\sin(5nt-4\omega-\Omega)\\ & +\{\tfrac{9}{128}e^4\gamma-\tfrac{9}{64}e^2\gamma^3\}\sin(3nt-4\omega+\Omega)+\tfrac{3}{128}\gamma^5\sin(nt+4\omega-5\Omega)\\ & +\{\tfrac{1}{64}e^2\gamma^3-\tfrac{1}{128}\gamma^5\}\sin(nt-4\omega+3\Omega)-\tfrac{1}{64}\gamma^5\sin(3nt+2\omega-5\Omega)\\ & -\tfrac{17}{64}e^2\gamma^3\sin(5nt-2\omega-3\Omega) \quad -\end{aligned}\right\}.\tag{124}$$

7. We have thus found the values of the three co-ordinates r, v and θ in terms of the time. We may however find θ directly from the differential equation (28), which will serve as a verification of all the developments which have thus far been made in the determination of t, v, r and θ. For this purpose we shall sub-

stitute the values of c, c' and c'' in equation (28), by which means it will become

$$\frac{d\theta}{dt} = \frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}} \cdot \frac{\gamma}{r^2} \cos(v - \Omega). \tag{125}$$

Now equation (119) will give

$$\left.\begin{aligned} \frac{a^2}{r^2} = 1 &+ \tfrac{1}{2}e^2 + \tfrac{3}{8}e^4 + \{2e + \tfrac{3}{4}e^3 + \tfrac{65}{96}e^5\} \cos(nt - \omega) \\ &+ \{\tfrac{5}{2}e^2 + \tfrac{1}{3}e^4\} \cos 2(nt - \omega) + \tfrac{103}{24}e^4 \cos 4(nt - \omega) \\ &+ \{\tfrac{13}{4}e^3 - \tfrac{25}{64}e^5\} \cos 3(nt - \omega) + \tfrac{1097}{192}e^5 \cos 5(nt - \omega) \end{aligned}\right\}; \tag{126}$$

and equation (113) gives, by putting $m = 1$, and $a = -\Omega$,

$$\left.\begin{aligned} \cos(v - \Omega) = {} & \{1 - e^2 + \tfrac{1}{8}\gamma^2 + \tfrac{7}{64}e^4 - \tfrac{3}{32}\gamma^4 - \tfrac{7}{64}e^2\gamma^2\} \cos(nt - \Omega) \\ & - e\cos(\omega - \Omega) + \{e - \tfrac{5}{4}e^3 + \tfrac{1}{8}e\gamma^2\} \cos(2nt - \omega - \Omega) \\ & + \{\tfrac{9}{8}e^2 - \tfrac{27}{16}e^4 + \tfrac{3}{64}\gamma^4 + \tfrac{9}{64}e^2\gamma^2\} \cos(3nt - 2\omega - \Omega) \\ & - \{\tfrac{1}{8}\gamma^2 - \tfrac{7}{128}\gamma^4 - \tfrac{81}{64}e^2\gamma^2\} \cos 3(nt - \Omega) \\ & + \{\tfrac{1}{8}\gamma^2 - \tfrac{3}{64}\gamma^4 - \tfrac{29}{64}e^2\gamma^2\} \cos(nt + 2\omega - 3\Omega) \\ & - \{\tfrac{1}{8}e^2 + \tfrac{1}{8}\gamma^2 - \tfrac{1}{48}e^4 - \tfrac{3}{64}\gamma^4 - \tfrac{7}{64}e^2\gamma^2\} \cos(nt - 2\omega + \Omega) \\ & + \tfrac{4}{3}e^3 \cos(4nt - 3\omega - \Omega) - \{\tfrac{1}{12}e^3 + \tfrac{1}{8}e\gamma^2\} \cos(2nt - 3\omega + \Omega) \\ & - \tfrac{1}{8}e\gamma^2 \cos 3(\omega - \Omega) + \tfrac{1}{2}e\gamma^2 \cos(2nt + \omega - 3\Omega) \\ & - \tfrac{3}{8}e\gamma^2 \cos(4nt - \omega - 3\Omega) + \tfrac{625}{384}e^4 \cos(5nt - 4\omega - \Omega) \\ & - \{\tfrac{9}{128}e^4 + \tfrac{9}{64}e^2\gamma^2\} \cos(3nt - 4\omega + \Omega) + \tfrac{3}{128}\gamma^4 \cos 5(nt - \Omega) \\ & + \tfrac{3}{128}\gamma^4 \cos(nt + 4\omega - 5\Omega) - \{\tfrac{1}{128}\gamma^4 + \tfrac{1}{64}e^2\gamma^2\} \cos(nt - 4\omega + 3\Omega) \\ & - \tfrac{3}{64}\gamma^4 \cos(3nt + 2\omega - 5\Omega) - \tfrac{51}{64}e^2\gamma^2 \cos(5nt - 2\omega - 3\Omega) \end{aligned}\right\}. \tag{127}$$

If we now multiply equations (126) and (127) together, and the product by γ, we shall obtain

$$
\left.
\begin{aligned}
\frac{\gamma\cos(v-\Omega)}{r^2} &= \frac{1}{a^2}\{\gamma-\tfrac{1}{2}e^2\gamma+\tfrac{1}{8}\gamma^3-\tfrac{3}{32}\gamma^5-\tfrac{1}{64}e^4\gamma-\tfrac{5}{64}e^2\gamma^3\}\cos(nt-\Omega)\\
&-\{\tfrac{1}{8}\gamma^3-\tfrac{7}{128}\gamma^5-\tfrac{85}{64}e^2\gamma^3\}\cos 3(nt-\Omega)+\tfrac{3}{128}\gamma^5\cos 5(nt-\Omega)\\
&+\{2e\gamma-\tfrac{3}{2}e^3\gamma+\tfrac{1}{4}e\gamma^3\}\cos(2nt-\omega-\Omega)+\tfrac{16}{3}e^3\gamma\cos(4nt-3\omega-\Omega)\\
&+\{\tfrac{27}{8}e^2\gamma-\tfrac{27}{8}e^4\gamma+\tfrac{27}{64}e^2\gamma^3+\tfrac{3}{64}\gamma^5\}\cos(3nt-2\omega-\Omega)\\
&+\{\tfrac{1}{8}\gamma^3-\tfrac{3}{64}\gamma^5-\tfrac{11}{64}e^2\gamma^3\}\cos(nt+2\omega-3\Omega)\\
&+\{\tfrac{1}{8}e^2\gamma-\tfrac{1}{8}\gamma^3+\tfrac{1}{24}e^4\gamma+\tfrac{3}{64}\gamma^5+\tfrac{5}{64}e^2\gamma^3\}\cos(nt-2\omega+\Omega)\\
&-\{\tfrac{1}{4}e\gamma^3-\tfrac{1}{6}e^3\gamma\}\cos(2nt-3\omega+\Omega)+\tfrac{1}{2}e\gamma^3\cos(2nt+\omega-3\Omega)\\
&+\{\tfrac{27}{128}e^4\gamma-\tfrac{27}{64}e^2\gamma^3\}\cos(3nt-4\omega+\Omega)-\tfrac{1}{2}e\gamma^3\cos(4nt-\omega-3\Omega)\\
&+\{\tfrac{1}{64}e^2\gamma^3-\tfrac{1}{128}\gamma^5\}\cos(nt-4\omega+3\Omega)+\tfrac{3125}{384}e^4\gamma\cos(5nt-4\omega-\Omega)\\
&+\tfrac{3}{128}\gamma^5\cos(nt+4\omega-5\Omega)-\tfrac{3}{64}\gamma^5\cos(3nt+2\omega-5\Omega)\\
&-\tfrac{85}{64}e^2\gamma^3\cos(5nt-2\omega-3\Omega)
\end{aligned}
\right\}. \quad (128)
$$

If we multiply this equation by

$$\sqrt{a\mu}\,\frac{\sqrt{1-e^2}}{\sqrt{1+\gamma^2}}=\sqrt{a\mu}\,\{1-\tfrac{1}{2}e^2-\tfrac{1}{2}\gamma^2-\tfrac{1}{8}e^4+\tfrac{3}{8}\gamma^4+\tfrac{1}{4}e^2\gamma^2\},$$

and put $\dfrac{\sqrt{\mu}}{a^{\frac{3}{2}}}=n$, we shall obtain

$$
\left.
\begin{aligned}
\frac{d\theta}{ndt} &= \gamma\{1-e^2-\tfrac{3}{8}\gamma^2+\tfrac{7}{32}\gamma^4+\tfrac{7}{64}e^4+\tfrac{23}{64}e^2\gamma^2\}\cos(nt-\Omega)\\
&-\tfrac{1}{8}\gamma^3\{1-\tfrac{15}{16}\gamma^2-\tfrac{22}{8}e^2\}\cos 3(nt-\Omega)+\tfrac{3}{128}\gamma^5\cos 5(nt-\Omega)\\
&+2e\gamma\{1-\tfrac{5}{4}e^2-\tfrac{3}{8}\gamma^2\}\cos(2nt-\omega-\Omega)+\tfrac{16}{3}e^3\gamma\cos(4nt-3\omega-\Omega)\\
&+\{\tfrac{27}{8}e^2\gamma-\tfrac{81}{16}e^4\gamma-\tfrac{81}{64}e^2\gamma^3+\tfrac{3}{64}\gamma^5\}\cos(3nt-2\omega-\Omega)\\
&+\{\tfrac{1}{8}\gamma^3-\tfrac{7}{64}\gamma^5-\tfrac{15}{64}e^2\gamma^3\}\cos(nt+2\omega-3\Omega)-\tfrac{85}{64}e^2\gamma^3\cos(5nt-2\omega-3\Omega)\\
&+\{\tfrac{1}{8}e^2\gamma-\tfrac{1}{8}\gamma^3-\tfrac{1}{48}e^4\gamma+\tfrac{7}{64}\gamma^5+\tfrac{5}{64}e^2\gamma^3\}\cos(nt-2\omega+\Omega)\\
&-\{\tfrac{1}{4}e\gamma^3-\tfrac{1}{6}e^3\gamma\}\cos(2nt-3\omega+\Omega)+\tfrac{1}{2}e\gamma^3\cos(2nt+\omega-3\Omega)\\
&+\{\tfrac{27}{128}e^4\gamma-\tfrac{27}{64}e^2\gamma^3\}\cos(3nt-4\omega+\Omega)-\tfrac{1}{2}e\gamma^3\cos(4nt-\omega-3\Omega)\\
&+\{\tfrac{1}{64}e^2\gamma^3-\tfrac{1}{128}\gamma^5\}\cos(nt-4\omega+3\Omega)+\tfrac{3125}{384}e^4\gamma\cos(5nt-4\omega-\Omega)\\
&+\tfrac{3}{128}\gamma^5\cos(nt+4\omega-5\Omega)-\tfrac{3}{64}\gamma^5\cos(3nt+2\omega-5\Omega).
\end{aligned}
\right\}. \quad (129)
$$

Equation (129) gives, by integration,

$$\left.\begin{aligned}
\theta = (\theta) &+ \gamma\{1 - e^2 - \tfrac{3}{8}\gamma^2 + \tfrac{7}{64}e^4 + \tfrac{7}{32}\gamma^4 + \tfrac{23}{64}e^2\gamma^2\}\sin(nt - \Omega) \\
&- \{\tfrac{1}{24}\gamma^3 - \tfrac{5}{128}\gamma^5 - \tfrac{33}{64}e^2\gamma^3\}\sin 3(nt - \Omega) + \tfrac{3}{640}\gamma^5 \sin 5(nt - \Omega) \\
&+ e\gamma\{1 - \tfrac{5}{4}e^2 - \tfrac{3}{8}\gamma^2\}\sin(2nt - \omega - \Omega) - \tfrac{17}{64}e^2\gamma^3 \sin(5nt - 2\omega - 3\Omega) \\
&+ \{\tfrac{9}{8}e^2\gamma - \tfrac{27}{16}e^4\gamma + \tfrac{1}{64}\gamma^5 - \tfrac{27}{64}e^2\gamma^3\}\sin(3nt - 2\omega - \Omega) \\
&+ \{\tfrac{1}{8}e^2\gamma - \tfrac{1}{8}\gamma^3 - \tfrac{1}{48}e^4\gamma + \tfrac{7}{64}\gamma^5 + \tfrac{5}{64}e^2\gamma^3\}\sin(nt - 2\omega + \Omega) \\
&+ \{\tfrac{1}{8}\gamma^3 - \tfrac{7}{64}\gamma^5 - \tfrac{15}{64}e^2\gamma^3\}\sin(nt + 2\omega - 3\Omega) + \tfrac{4}{3}e^3\gamma \sin(4nt - 3\omega - \Omega) \\
&- \tfrac{1}{8}e\gamma^3 \sin(4nt - \omega - 3\Omega) + \tfrac{1}{4}e\gamma^3 \sin(2nt + \omega - 3\Omega) \\
&+ \{\tfrac{1}{12}e^3\gamma - \tfrac{1}{8}e\gamma^3\}\sin(2nt - 3\omega + \Omega) + \tfrac{625}{384}e^4\gamma \sin(5nt - 4\omega - \Omega) \\
&+ \{\tfrac{9}{128}e^4\gamma - \tfrac{9}{64}e^2\gamma^3\}\sin(3nt - 4\omega + \Omega) + \tfrac{3}{128}\gamma^5 \sin(nt + 4\omega - 5\Omega) \\
&+ \{\tfrac{1}{64}e^2\gamma^3 - \tfrac{1}{128}\gamma^5\}\sin(nt - 4\omega + 3\Omega) - \tfrac{1}{64}\gamma^5 \sin(3nt + 2\omega - 5\Omega)
\end{aligned}\right\}; \tag{130}$$

(θ) being the constant quantity to complete the integral. It is evident that the two expressions for the value of θ given in equations (124) and (130) will be identical if we make the constant

$$(\theta) = -e\gamma\{1 - \tfrac{1}{4}\gamma^2\}\sin(\omega - \Omega) - \tfrac{1}{8}e\gamma^3 \sin 3(\omega - \Omega), \tag{131}$$

which makes equation (130) satisfy the condition that the latitude of the moon shall be equal to the latitude of the perigee of the orbit when $nt = \omega$, or when the moon is at the extremities of the transverse axis of the orbit. The perfect agreement of these two determinations of the value of θ proves conclusively that all the preceding analytical developments have been correctly made.

8. In order to show, by a few numerical examples, that the values of v, r and θ, given by equations (104), (119) and (124), are correct, and at the same time show that the development of these quantities in series to terms of the fifth order is sufficient, in the lunar theory, we shall now reduce these equations to numbers by using the values of e and γ, corresponding to the elements of the moon's orbit which were employed by DELAUNAY in reducing his equations of the lunar theory to numbers. These values are, $e = 0.05489930$, and $\gamma = 0.09004560$. And instead of giving the value of r in terms of the moon's mean distance as the unit, we have multiplied equation (119) by the constant term of the moon's parallax, supposing it to be equal to 3422.″3.

We shall therefore find

$$\left.\begin{aligned}
v = nt &+ 22638.''97 \sin(nt - \omega) + 777.''074 \sin 2(nt - \omega) \\
&+ 36.''997 \sin 3(nt - \omega) + 2.''010 \sin 4(nt - \omega) + 0.''118 \sin 5(nt - \omega) \\
&- 411.''374 \sin 2(nt - \Omega) + 415.''469 \sin 2(\omega - \Omega) \\
&- 45.''255 \sin(3nt - \omega - 2\Omega) + 45.''691 \sin(nt + \omega - 2\Omega) \\
&- 4.''096 \sin(4nt - 2\omega - 2\Omega) + 0.''093 \sin(nt + 3\omega - 4\Omega) \\
&- 0.''186 \sin(3nt + \omega - 4\Omega) + 0.''093 \sin(5nt - \omega - 4\Omega) \\
&- 0.''340 \sin(5nt - 3\omega - 2\Omega) - 0.''848 \sin(2nt + 2\omega - 4\Omega) \\
&+ 0.''424 \sin 4(nt - \Omega) + 0.''424 \sin 4(\omega - \Omega)
\end{aligned}\right\}; \quad (132)$$

$$\left.\begin{aligned}
\theta = &\ 18461.''23 \sin(nt - \Omega) + 1012.''716 \sin(2nt - \omega - \Omega) \\
&- 1017.''590 \sin(\omega - \Omega) + 62.''519 \sin(3nt - 2\omega - \Omega) \\
&+ 18.''584 \sin(nt + 2\omega - 3\Omega) - 11.''662 \sin(nt - 2\omega + \Omega) \\
&- 5.''993 \sin 3(nt - \Omega) + 2.''067 \sin(2nt + \omega - 3\Omega) \\
&- 1.''0335 \sin(4nt - \omega - 3\Omega) - 1.''0335 \sin 3(\omega - \Omega) \\
&+ 4.''098 \sin(4nt - 3\omega - \Omega) + 0.''2746 \sin(5nt - 4\omega - \Omega) \\
&+ 0.''0286 \sin(nt + 4\omega - 5\Omega) - 0.''1206 \sin(5nt - 2\omega - 3\Omega) \\
&- 0.''0191 \sin(3nt + 2\omega - 5\Omega) - 0.''0024 \sin(nt - 4\omega + 3\Omega) \\
&+ 0.''0057 \sin 5(nt - \Omega) - 0.''7773 \sin(2nt - 3\omega + \Omega) \\
&- 0.''0520 \sin(3nt - 4\omega + \Omega)
\end{aligned}\right\}; \quad (133)$$

$$\left.\begin{aligned}
\pi = &\ 3422.''300 + 187.''811 \cos(nt - \omega) + 10.''304 \cos 2(nt - \omega) \\
&+ 0.''636 \sin 3(nt - \omega) + 0.''0414 \cos 4(nt - \omega) \\
&+ 0.''0028 \cos 5(nt - \omega)
\end{aligned}\right\}; \quad (134)$$

π denoting the moon's parallax.

We shall now compare these approximate values of v, θ and π with the values derived by an exact calculation from rigorous formulæ for the same co-ordinates. For this purpose we shall observe that if we denote by u the eccentric anomaly, and by $nt-\omega'$ the mean anomaly measured on the plane of the orbit, the relation between u and $nt-\omega'$ will be given by the equation

$$nt-\omega'=u-e\sin u. \tag{135}$$

Then we shall have the following equations for the determination of the true anomaly $v'-\omega'$, and radius vector r,

$$\left.\begin{aligned}\tan\tfrac{1}{2}(v'-\omega')&=\frac{\sqrt{1+e}}{\sqrt{1-e}}.\tan\tfrac{1}{2}u\\ r&=a(1-e\cos u)\end{aligned}\right\}. \tag{136}$$

Equations (135) and (136) are entirely rigorous; and if, for a given value of the mean anomaly $nt-\omega'$, we compute the corresponding value of u with accuracy by means of equation (135), we can then compute the corresponding values of $v'-\omega'$ and r by means of equations (136). Now we also have the value of $\omega'-\Omega$ by means of the equation

$$\tan(\omega'-\Omega)=\tan(\omega-\Omega)\sec i; \tag{137}$$

then adding the value of $v'-\omega'$ to $\omega'-\Omega$, we shall have the true distance, $v'-\Omega$, of the moon from the node measured on the orbit.

Then the spherical triangle $vv'\Omega$ will give

$$\left.\begin{aligned}\tan(v-\Omega)&=\tan(v'-\Omega)\cos i\\ \text{and}\qquad \sin\theta&=\sin(v'-\Omega)\sin i\end{aligned}\right\}. \tag{138}$$

The formulæ (135–138) will give the rigorous values of r, v and θ corresponding to any value of the mean anomaly $nt-\omega'$.

If we suppose that $i=5°\ 8'\ 43.''28$, and $\omega-\Omega=30°$, we shall find that $\omega'-\Omega=30°\ 6'\ 0.''99$. Then if we designate the values of v and θ, derived from equation (138), as "observed values," and the values of the same quantities derived from equations (132) and (133) as "calculated values," we shall obtain the following table showing the values of the co-ordinates v and θ for the values of the mean anomaly given in the first column of the table.

Mean Anomaly.	Observed.		Calculated.		Calculated.—Observed.	
	$v - \Omega$	θ	$v - \Omega$	θ	Δv	$\Delta\theta$
30°	63° 20′ 58″.23	+4° 36′ 4″.30	63° 20′ 58″.06	+4° 36′ 4″.30	−0″.17	0″.00
40	74 18 13.61	4 57 16.05	74 18 13.72	4 57 16.06	+0.11	+0.01
60	95 45 20.32	5 7 10.39	95 45 20.51	5 7 10.34	+0.19	−0.05
80	116 27 0.21	4 36 33.14	116 27 0.22	4 36 33.10	+0.01	−0.04
100	136 19 36.09	3 33 29.25	136 19 35.97	3 33 29.18	−0.12	−0.07
130	164 46 9.63	+1 21 18.38	164 46 9.80	+1 21 18.40	+0.17	+0.02
160	192 4 24.48	−1 4 44.42	192 4 24.44	−1 4 44.40	+0.04	+0.02

The columns of residuals show that the development of the longitude and latitude in series as far as terms of the fifth order can in no case be in error to a greater extent than about 0.″2 in longitude, and 0.″1 in latitude; and it would therefore seem that the development of the lunar theory to that degree of approximation would be amply sufficient for all the purposes of astronomy, except for those terms of perturbations producing the inequalities of long period. We would here also observe that the value of the parallax given by equation (134) can in no case differ from the rigorous value of that co-ordinate to a greater extent than 0.″1; and a more perfect agreement between approximative and rigorous formulæ could hardly be expected or desired.

9. The whole of the mathematical theory of the moon's motion is contained in the preceding articles (1–8), when we neglect the consideration of the effects of the disturbing forces. The expressions for the three co-ordinates v, θ and r, given in equations (104), (124) and (119), are perfectly homogeneous in their development, every circular function or argument of the different terms of the equations being measured on the fixed plane of projection instead of the plane of the orbit. There are therefore no terms depending on the mean distance of the moon from the perigee measured on the plane of the orbit, usually called the mean anomalies; but the equivalent terms depending on the difference of the mean longitudes of the moon and of the perigee measured on the plane of projection are given instead. For this reason it is unnecessary to make any reference to the distance of the perigee from the node of the orbit measured on the plane of the orbit, but simply to designate the longitude of the perigee by the distance of its projection from the origin of longitudes and measured on the fixed plane. And therefore the expressions for these three co-ordinates which we have given are simpler than the equivalent expressions given by Delaunay, who reckons longitudes on the fixed plane, and the other arguments of his equations upon the plane of the orbit, referring them to the fixed plane in terms of certain angular functions depending on the distance between the node and perigee, and also to the mean anomalies which are measured on the plane of the orbit. Delaunay's

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$$\frac{dv}{dt}=\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}\; r^2\cos^2\theta}\left\{1-\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\cdot\int\left(\frac{dR}{dv}\right)dt\right\}; \tag{C}$$

$$\left.\begin{aligned}\frac{d\theta}{dt}=&\frac{\sqrt{a\mu(1-e^2)}\,\gamma}{\sqrt{1+\gamma^2}\; r^2}\cos(v-\Omega)\\ &-\frac{\cos v}{r^2}\int\left\{\tan\theta\sin v\left(\frac{dR}{dv}\right)+\cos v\left(\frac{dR}{d\theta}\right)\right\}dt\\ &+\frac{\sin v}{r^2}\int\left\{\tan\theta\cos v\left(\frac{dR}{dv}\right)-\sin v\left(\frac{dR}{d\theta}\right)\right\}dt\end{aligned}\right\}; \tag{D}$$

$$\frac{d\Omega}{dt}=\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\left\{\sin(v-\Omega)\cos(v-\Omega)\left(\frac{dR}{dv}\right)-\frac{\sin(v-\Omega)}{\gamma}\left(\frac{dR}{d\theta}\right)\right\}; \tag{E}$$

$$\frac{d\gamma}{dt}=\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\left\{\gamma\cos^2(v-\Omega)\left(\frac{dR}{dv}\right)-\cos(v-\Omega)\left(\frac{dR}{d\theta}\right)\right\}; \tag{F}$$

$$\left.\begin{aligned}\mu e\cos\theta_0\frac{d\omega}{dt}=&\,r^2\left\{\cos\theta\cos(v-\omega)\frac{dv}{dt}-\sin\theta\sin(v-\omega)\frac{d\theta}{dt}\right\}\left(\frac{dR}{dr}\right)\\ &+\left\{\frac{\cos(v-\omega)}{\cos\theta}\frac{dr}{dt}-2r\cos\theta\sin(v-\omega)\frac{dv}{dt}\right\}\left(\frac{dR}{dv}\right)\\ &-\left\{\sin\theta\frac{dr}{dt}+2r\cos\theta\frac{d\theta}{dt}\right\}\sin(v-\omega)\left(\frac{dR}{d\theta}\right)\end{aligned}\right\}; \tag{G}$$

$$\left.\begin{aligned}\mu\frac{de}{dt}=&\,r^2\left\{\{\sin\theta_0\cos\theta-\cos\theta_0\sin\theta\cos(v-\omega)\}\frac{d\theta}{dt}-\cos\theta_0\cos\theta\sin(v-\omega)\frac{dv}{dt}\right\}\left(\frac{dR}{dr}\right)\\ &-\left\{\frac{\cos\theta_0}{\cos\theta}\sin(v-\omega)\frac{dr}{dt}+2r\{\cos\theta_0\cos\theta\cos(v-\omega)+\sin\theta_0\sin\theta\}\frac{dv}{dt}\right\}\left(\frac{dR}{dv}\right)\\ &+\left\{\begin{aligned}&\{\sin\theta_0\cos\theta-\cos\theta_0\sin\theta\cos(v-\omega)\}\frac{dr}{dt}\\ &-2r\{\cos\theta_0\cos\theta\cos(v-\omega)+\sin\theta_0\sin\theta\}\frac{d\theta}{dt}\end{aligned}\right\}\left(\frac{dR}{d\theta}\right)\end{aligned}\right\}. \tag{H}$$

And lastly, equation (17) will give

$$d\frac{\mu}{a}=2\left\{\left(\frac{dR}{dr}\right)dr+\left(\frac{dR}{dv}\right)dv+\left(\frac{dR}{d\theta}\right)d\theta\right\}=2\mathrm{d}R. \qquad \text{(I)}$$

We have already integrated equations (B), (C), and (D), when the function R is equal to nothing, which has conducted us to the equations of the elliptical motion; and we shall now proceed to determine the effect of the first power of the disturbing force upon the values of r, v, and θ already calculated. Then by substituting the values of the co-ordinates r, v, and θ in equations (E–I), we shall obtain by integration the variation of the elements of the elliptical motion.

CHAPTER II.

PERTURBATIONS OF THE MOON'S MOTIONS.

11. Having given the differential equations of the moon's motion, together with their integrals when the disturbing force is neglected, which has conducted us to the theory of the elliptical motion, we shall now determine the integrals of the same equations when increased by the terms arising from the disturbing forces. In this investigation we shall notice in the first approximation only the terms arising from the first power of the sun's disturbing force, but shall carry on the approximation to terms of the fourth order depending on the eccentricities and inclinations of the orbits. These terms will all be multiplied by a factor of the second order depending on the sun's mass and distance; and, consequently, the investigation will include all the terms to the sixth order.

For this purpose, we must first develop the function R. If we denote the sun's mass by m', and mark with one accent, for the sun, the quantities x, y, z, r, v, θ, a, n, e, γ, ω, ☊, which refer to the moon, we shall have the elliptical value of the sun's co-ordinates. The value of R will then be given by means of the equation,

$$R = \frac{m'\ \{xx' + yy' + zz'\}}{r'^3} - \frac{m'}{\sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}}. \tag{145}$$

If in this equation we substitute the values of the co-ordinates x, y, z; x', y', z'; given by the equations

$$\left.\begin{array}{l} x = r\cos\theta\cos v,\ y = r\cos\theta\sin v,\ z = r\sin\theta, \\ x' = r'\cos\theta'\cos v',\ y' = r'\cos\theta'\sin v',\ z' = r'\sin\theta' \end{array}\right\}, \tag{146}$$

it will become

$$\left.\begin{array}{l} R = \dfrac{m'\,rr'\ \{\cos\theta\cos\theta'\cos(v-v') + \sin\theta\sin\theta'\}}{r'^3} \\ \quad - \dfrac{m'}{\sqrt{r'^2 - 2rr'\,[\cos\theta\cos\theta'\cos(v-v') + \sin\theta\sin\theta'] + r^2}} \end{array}\right\}. \tag{147}$$

We may simplify the calculations by taking the ecliptic for the plane of projection, in which case the quantities z' and θ' will disappear from the value of R. It is true that the plane of the ecliptic is not fixed, and we shall, in the course of this work, determine the effect of its secular motion on the co-ordinates of the moon.

If we put $\theta' = 0$ in equation (147), it will become

$$R = \frac{m'\,rr'\cos\theta\cos(v-v')}{r'^3} - \frac{m'}{\{r'^2 - 2rr'\cos\theta\cos(v-v') + r^2\}^{\frac{1}{2}}}. \tag{148}$$

If we take the partial differentials of this equation with respect to r, v and θ, we shall find

$$\left(\frac{dR}{dr}\right)=\frac{m'r'\cos\theta\cos(v-v')}{r'^3}+\frac{m'\{r-r'\cos\theta\cos(v-v')\}}{\{r'^2-2rr'\cos\theta\cos(v-v')+r^2\}^{\frac{3}{2}}}; \tag{149}$$

$$\left(\frac{dR}{dv}\right)=-\frac{m'rr'\cos\theta\sin(v-v')}{r'^3}+\frac{m'rr'\cos\theta\sin(v-v')}{\{r'^2-2rr'\cos\theta\cos(v-v')+r^2\}^{\frac{3}{2}}}; \tag{150}$$

$$\left(\frac{dR}{d\theta}\right)=-\frac{m'rr'\sin\theta\cos(v-v')}{r'^3}+\frac{m'rr'\sin\theta\cos(v-v')}{\{r'^2-2rr'\cos\theta\cos(v-v')+r^2\}^{\frac{3}{2}}}. \tag{151}$$

By development in series, we shall find

$$\left.\begin{aligned}&\{r'^2-2rr'\cos\theta\cos(v-v')+r^2\}^{-\frac{3}{2}}=\frac{1}{r'^3}\Big\{1+3\frac{r}{r'}\cos\theta\cos(v-v')\\&-\tfrac{3}{2}\frac{r^2}{r'^2}+\tfrac{15}{8}\left[\frac{r^2}{r'^2}-2\frac{r}{r'}\cos\theta\cos(v-v')\right]^2-\tfrac{35}{16}\left[\frac{r^2}{r'^2}-2\frac{r}{r'}\cos\theta\cos(v-v')\right]^3\Big\}\end{aligned}\right\}. \tag{152}$$

If we substitute this value in equations (149–151), they will become

$$\left(\frac{dR}{dr}\right)=\left.\begin{aligned}&\frac{m'r}{r'^3}-\tfrac{3}{2}\frac{m'r}{r'^3}\cos^2\theta+\tfrac{9}{2}\frac{m'r^2}{r'^4}\cos\theta\cos(v-v')\\&-\tfrac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\cos(v-v')-\tfrac{3}{2}\frac{m'r}{r'^3}\cos^2\theta\cos2(v-v')\\&-\tfrac{15}{8}\frac{m'r^2}{r'^4}\cos^3\theta\cos3(v-v')\end{aligned}\right\}; \tag{153}$$

$$\left(\frac{dR}{dv}\right)=\left.\begin{aligned}&-\tfrac{3}{2}\frac{m'r^3}{r'^4}\cos\theta\sin(v-v')+\tfrac{15}{8}\frac{m'r^3}{r'^4}\cos^3\theta\sin(v-v')\\&+\tfrac{3}{2}\frac{m'r^2}{r'^3}\cos\theta\sin2(v-v')+\tfrac{15}{8}\frac{m'r^3}{r'^4}\cos^3\theta\sin3(v-v')\end{aligned}\right\}; \tag{154}$$

$$\left(\frac{dR}{d\theta}\right)=\left.\begin{aligned}&\tfrac{3}{2}\frac{m'r^2}{r'^3}\sin\theta\cos\theta+\tfrac{3}{2}\frac{m'r^3}{r'^4}\left\{\tfrac{15}{4}\cos^2\theta-1\right\}\sin\theta\cos(v-v')\\&+\tfrac{3}{2}\frac{m'r^2}{r'^3}\sin\theta\cos\theta\cos2(v-v')\\&+\tfrac{15}{8}\frac{m'r^3}{r'^4}\sin\theta\cos^2\theta\cos3(v-v')\end{aligned}\right\}. \tag{155}$$

In these equations r, r', v, v' and θ denote the true values of these co-ordinates. But the true values of the co-ordinates are equal to their elliptical values increased by the terms produced by the disturbing forces. We shall therefore put

$$r=r_1+\delta r,\ r'=r_1'+\delta r',\ v=v_1+\delta v,\ v'=v_1'+\delta v' \text{ and } \theta=\theta_1+\delta\theta, \tag{156}$$

in which r_1, r_1', v_1, v_1', and θ_1, denote the elliptical values, and δr, $\delta r'$, etc., denote the terms arising from the disturbing forces.

If we substitute the elliptical values of r, v, and θ in equations (B), (C), and (D), and also in equations (153–155), we shall obtain the values of δr, δv, and $\delta\theta$, which will be correct to terms of the first power of the disturbing force. Equations (E–I) will also give the variations of the elements of the elliptical motion correct to terms of the same order. After having found the values of δr, δv, and $\delta\theta$, we may find the influence of these terms in the different functions and forces which enter into the several differential equations of the co-ordinates and elements in the following manner.

If we suppose that u is any function of r, v, and θ, the variation of u corresponding to finite variations of r, v, and θ will be given by the equation

$$\left.\begin{aligned}
\delta u = {} & \left(\frac{du}{dr}\right)\delta r + \left(\frac{du}{dv}\right)\delta v + \left(\frac{du}{d\theta}\right)\delta\theta \\
& + \Big\{ \tfrac{1}{2}\left(\frac{d^2u}{dr^2}\right)\delta r^2 + \tfrac{1}{2}\left(\frac{d^2u}{dv^2}\right)\delta v^2 + \tfrac{1}{2}\left(\frac{d^2u}{d\theta^2}\right)\delta\theta^2 + \left(\frac{d^2u}{dr\,dv}\right)\delta r\delta v \\
& \qquad + \left(\frac{d^2u}{dr\,d\theta}\right)\delta r\delta\theta + \left(\frac{d^2u}{dv\,d\theta}\right)\delta v\delta\theta \Big\} \\
& + \Big\{ \tfrac{1}{6}\left(\frac{d^3u}{dr^3}\right)\delta r^3 + \tfrac{1}{6}\left(\frac{d^3u}{dv^3}\right)\delta v^3 + \tfrac{1}{6}\left(\frac{d^3u}{d\theta^3}\right)\delta\theta^3 + \tfrac{1}{2}\left(\frac{d^3u}{dr^2dv}\right)\delta r^2\delta v \\
& \qquad + \tfrac{1}{2}\left(\frac{d^3u}{dr^2d\theta}\right)\delta r^2\delta\theta + \tfrac{1}{2}\left(\frac{d^3u}{dr\,dv^2}\right)\delta r\delta v^2 + \tfrac{1}{2}\left(\frac{d^3u}{dr\,d\theta^2}\right)\delta r\delta\theta^2 \\
& \qquad + \tfrac{1}{2}\left(\frac{d^3u}{dv^2d\theta}\right)\delta v^2\delta\theta + \tfrac{1}{2}\left(\frac{d^3u}{dv\,d\theta^2}\right)\delta v\delta\theta^2 + \left(\frac{d^3u}{dr\,dv\,d\theta}\right)\delta r\delta v\delta\theta \Big\} \\
& + \Big\{ \tfrac{1}{24}\left(\frac{d^4u}{dr^4}\right)\delta r^4 + \tfrac{1}{24}\left(\frac{d^4u}{dv^4}\right)\delta v^4 + \tfrac{1}{24}\left(\frac{d^4u}{d\theta^4}\right)\delta\theta^4 + \tfrac{1}{6}\left(\frac{d^4u}{dr^3dv}\right)\delta r^3\delta v \\
& \qquad + \tfrac{1}{6}\left(\frac{d^4u}{dr^3\partial\theta}\right)\delta r^3\delta\theta + \tfrac{1}{6}\left(\frac{d^4u}{dv^3d\theta}\right)\delta v^3 d\theta + \tfrac{1}{6}\left(\frac{d^4u}{dv\,d\theta^3}\right)\delta v\delta\theta^3 \\
& \qquad + \tfrac{1}{6}\left(\frac{d^4u}{dr\,d\theta^3}\right)\delta r\delta\theta^3 + \tfrac{1}{6}\left(\frac{d^4u}{dr\,dv^3}\right)\delta r\delta v^3 + \tfrac{1}{4}\left(\frac{d^4u}{dr^2dv^2}\right)\delta r^2\delta v^2 \\
& \qquad + \tfrac{1}{4}\left(\frac{d^4u}{dr^2d\theta^2}\right)\delta r^2\delta\theta^2 + \tfrac{1}{4}\left(\frac{d^4u}{dv^2d\theta^2}\right)\delta v^2\delta\theta^2 + \tfrac{1}{2}\left(\frac{d^4u}{dr^2dv\,d\theta}\right)\delta r^2\delta v\delta\theta \\
& \qquad + \tfrac{1}{2}\left(\frac{d^4u}{dr\,dv^2d\theta}\right)\delta r\delta v^2 d\theta + \tfrac{1}{2}\left(\frac{d^4u}{dr\,dv\,d\theta^2}\right)\delta r\delta v\delta\theta^2 \Big\} \\
& + \Big\{ \tfrac{1}{120}\left(\frac{d^5u}{dr^5}\right)\delta r^5 + \tfrac{1}{120}\left(\frac{d^5u}{dv^5}\right)\delta v^5 + \tfrac{1}{120}\left(\frac{d^5u}{d\theta^5}\right)\delta\theta^5 + \tfrac{1}{24}\left(\frac{d^5u}{dr^4dv}\right)\delta r^4\delta v
\end{aligned}\right\} ; \quad (157)$$

(Continued on the next page.)

$$\left.\begin{array}{l}
+\frac{1}{24}\left(\frac{d^5u}{dr^4d\theta}\right)\delta r^4\delta\theta+\frac{1}{24}\left(\frac{d^5u}{drd\theta^4}\right)\delta r\delta\theta^4+\frac{1}{24}\left(\frac{d^5u}{dv^4d\theta}\right)\delta v^4\delta\theta \\
+\frac{1}{24}\left(\frac{d^5u}{drdv^4}\right)\delta r\delta v^4+\frac{1}{24}\left(\frac{d^5u}{dvd\theta^4}\right)\delta v\delta\theta^4+\frac{1}{12}\left(\frac{d^5u}{dr^3dv^2}\right)\delta r^3\delta v^2 \\
+\frac{1}{12}\left(\frac{d^5u}{dr^3d\theta^2}\right)\delta r^3\delta\theta^2+\frac{1}{12}\left(\frac{d^5u}{dr^2dv^3}\right)\delta r^2\delta v^3+\frac{1}{12}\left(\frac{d^5u}{dr^2d\theta^3}\right)\delta r^2\delta\theta^3 \\
+\frac{1}{12}\left(\frac{d^5u}{dv^3d\theta^2}\right)\delta v^3\delta\theta^2+\frac{1}{12}\left(\frac{d^5u}{dv^2d\theta^3}\right)\delta v^2\delta\theta^3+\frac{1}{6}\left(\frac{d^5u}{dr^3dvd\theta}\right)\delta r^3\delta v\delta\theta \\
+\frac{1}{6}\left(\frac{d^5u}{drdv^3d\theta}\right)\delta r\delta v^3\delta\theta+\frac{1}{6}\left(\frac{d^5u}{drdvd\theta^3}\right)\delta r\delta v\delta\theta^3+\frac{1}{4}\left(\frac{d^5u}{dr^2dv^2d\theta}\right)\delta r^2\delta v^2\delta\theta \\
+\frac{1}{4}\left(\frac{d^5u}{drdv^2d\theta^2}\right)\delta r\delta v^2\delta\theta^2+\frac{1}{4}\left(\frac{d^5u}{dr^2dvd\theta^2}\right)\delta r^2\delta v\delta\theta^2\Big\} \\
\text{etc.}
\end{array}\right\}; \quad (157)$$

As an example of the use of the preceding equation, we may suppose that u represents the value of $\left(\frac{dR}{dr}\right)$ given by equation (153). If we take the partial differential coefficients of this function with respect to r, v, and θ, and neglect the terms divided by r'^4 in the second differential coefficients, we shall obtain the following values:

$$\left.\begin{array}{l}
\left(\frac{du}{dr}\right)=\frac{m'}{r'^3}-\frac{3}{2}\frac{m'}{r'^3}\cos^2\theta+9\frac{m'r}{r'^4}\cos\theta\cos(v-v')-\frac{45}{4}\frac{m'r}{r'^4}\cos^3\theta\cos(v-v') \\
\qquad\qquad -\frac{3}{2}\frac{m'}{r'^3}\cos^2\theta\cos 2(v-v')-\frac{15}{4}\frac{m'r}{r'^4}\cos^3\theta\cos 3(v-v')
\end{array}\right\}; \quad (158)$$

$$\left.\begin{array}{l}
\left(\frac{du}{dv}\right)=-\frac{9}{2}\frac{m'r^2}{r'^4}\cos\theta\sin(v-v')+\frac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\sin(v-v') \\
\qquad\qquad +3\frac{m'r}{r'^3}\cos^2\theta\sin 2(v-v')+\frac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\sin 3(v-v')
\end{array}\right\}; \quad (159)$$

$$\left.\begin{array}{l}
\left(\frac{du}{d\theta}\right)=3\frac{m'r}{r'^3}\sin\theta\cos\theta-\frac{9}{2}\frac{m'r^2}{r'^4}\sin\theta\cos(v-v') \\
\quad +\frac{135}{8}\frac{m'r^2}{r'^4}\cos^2\theta\sin\theta\cos(v-v')+3\frac{m'r}{r'^3}\sin\theta\cos\theta\cos 2(v-v') \\
\quad +\frac{45}{8}\frac{m'r^2}{r'^4}\sin\theta\cos^2\theta\cos 3(v-v')
\end{array}\right\}; \quad (160)$$

$$\left(\frac{d^2u}{dr^2}\right)=0; \quad (161)$$

$$\left(\frac{d^2u}{dv^2}\right)=6\frac{m'r}{r'^3}\cos^2\theta\cos 2(v-v'); \quad (162)$$

$$\left(\frac{d^2u}{d\theta^2}\right)=3\frac{m'r}{r'^3}\{\cos^2\theta-\sin^2\theta\}+3\frac{m'r}{r'^3}\{\cos^2\theta-\sin^2\theta\}\cos 2(v-v'); \quad (163)$$

$$\left(\frac{d^2u}{drdv}\right) = 3\,\frac{m'r}{r'^3}\cos^2\theta \sin 2\,(v - v'); \tag{164}$$

$$\left(\frac{d^2u}{drd\theta}\right) = 3\,\frac{m'}{r'^3}\sin\theta\cos\theta + 3\,\frac{m'}{r'^3}\sin\theta\cos\theta\cos 2\,(v - v') \tag{165}$$

$$\left(\frac{d^2u}{dvd\theta}\right) = -\,6\,\frac{m'r}{r'^3}\sin\theta\cos\theta\sin 2\,(v - v'). \tag{166}$$

If we substitute these values in equation (157), we shall obtain the value of $\delta\left(\dfrac{dR}{dr}\right)$ correct to terms of the order of the cube of the disturbing force, as follows:

$$\left.\begin{aligned}
\delta\left(\frac{dR}{dr}\right) = &\left\{\frac{m'}{r'^3} - \tfrac{3}{2}\frac{m'}{r'^3}\cos^2\theta + 9\,\frac{m'r}{r'^4}\cos\theta\cos(v - v')\right.\\
&\quad - \tfrac{45}{4}\frac{m'r}{r'^4}\cos^3\theta\cos(v - v') - \tfrac{3}{2}\frac{m'}{r'^3}\cos^2\theta\cos 2\,(v - v')\\
&\quad \left. - \tfrac{15}{4}\frac{m'r}{r'^4}\cos^3\theta\cos 3\,(v - v')\right\}\delta r\\
&+\left\{-\tfrac{9}{2}\frac{m'r^2}{r'^4}\cos\theta\sin(v - v') + \tfrac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\sin(v - v')\right.\\
&\quad \left. + 3\,\frac{m'}{r'^3}\cos^2\theta\sin 2\,(v - v') + \tfrac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\sin 3\,(v - v')\right\}\delta v\\
&+\left\{3\,\frac{m'r}{r'^3}\sin\theta\cos\theta - \tfrac{9}{2}\frac{m'r^2}{r'^4}\sin\theta\cos(v - v') + \tfrac{135}{8}\frac{m'r^2}{r'^4}\cos^2\sin\theta\cos(v - v')\right.\\
&\quad \left. + 3\,\frac{m'r}{r'^3}\sin\theta\cos\theta\cos 2\,(v - v') + \tfrac{45}{8}\frac{m'r^2}{r'^4}\sin\theta\cos^2\theta\cos 3\,(v - v')\right\}\delta\theta\\
&+ 3\,\frac{m'r}{r'^3}\cos^2\theta\cos 2\,(v - v')\,\delta v^2 + 3\,\frac{m'}{r'^3}\cos^2\theta\sin 2\,(v - v')\,\delta r\delta v\\
&+ \tfrac{3}{2}\frac{m'r}{r'^3}\{\cos^2\theta - \sin^2\theta\}\{1 + \cos 2\,(v - v')\}\delta\theta^2\\
&+ 3\,\frac{m'}{r'^3}\sin\theta\cos\theta\{1 + \cos 2\,(v - v')\}\delta r\delta\theta\\
&- 6\,\frac{m'r}{r'^3}\sin\theta\cos\theta\sin 2\,(v - v')\,\delta v\delta\theta
\end{aligned}\right\}; \tag{167}$$

When δr, δv, and $\delta\theta$ have been determined, to terms of the order of the first power of the disturbing force, their substitution in the first three terms of equation (167) will give the variation of the force $\left(\dfrac{dR}{dr}\right)$ correct to terms of the order of the square of the disturbing force. If we then compute in the same manner the variations of the forces $\left(\dfrac{dR}{dv}\right)$ and $\left(\dfrac{dR}{d\theta}\right)$; and also of the coefficients of these functions which enter into the differential expressions of the

co-ordinates r, v, and θ, given by equations (B), (C), and (D), we shall easily obtain the values of δr, δv, and $\delta\theta$ correct to terms of the order of the square of the disturbing forces. We may then, by repeating the process just described, obtain by means of equation (157), the variations of the various functions and forces correct to terms of the order of the cube of the disturbing force; with which we may find the variations of the co-ordinates to terms of the same order. In the same manner we can obtain the various terms arising from the higher powers of the disturbing force.

Since the determination of the co-ordinates r, v, and θ is the principal object of the analysis, it is important to observe that we may obtain these quantities by means of a different method from that just indicated. We have given the elliptical values of r, v, and θ, in equations (104), (119), and (124); from which it will be seen that these quantities are functions of the elements of the orbit a, e, ω, γ, Ω, and also of the mean longitude of the moon at a given epoch. By taking the partial differential coefficients of the co-ordinates with respect to the elements, we may obtain the value of the variation of the co-ordinates by means of an equation similar to (157). For example, the variation of v would be given by the equation,

$$\delta v = \left(\frac{dv}{da}\right)\delta a + \left(\frac{dv}{de}\right)\delta e + \left(\frac{dv}{d\omega}\right)\delta\omega + \left(\frac{dv}{d\gamma}\right)\delta\gamma + \left(\frac{dv}{d\Omega}\right)\delta\Omega + \frac{dv}{d(nt)}\delta(nt) + \text{etc.}, \tag{168}$$

the terms depending on the squares and higher powers of the variations of the elements being indicated by + etc.

Since the values of δa, δe, etc., are given by means of equations (E–I), we shall obtain in a very simple manner the corresponding variation of δv by means of equation (168).

If we change successively v into r, and θ, in equation (168) we shall obtain the values of δr and $\delta\theta$.

It is important to observe that the values of δa, δe, etc., which enter into equation (168) are explicit functions of the forces, and not of the time t; therefore the parts of the variations of δa, δe, etc., which are proportional to the time must be neglected in the application of equation (168), since the terms would increase indefinitely in the same direction.

Both the methods which we have explained have been successfully applied to the perturbations of the heavenly bodies. The facility with which they may be severally applied depends upon the circumstances of each particular case. In orbits of small eccentricity and inclination the perturbations of the elements are in general much greater than the perturbations of the co-ordinates; consequently the infinite series which determines the perturbations by means of the variation of elements would be less converging than the direct method. We shall see hereafter that the perturbations of the moon's co-ordinates never exceed two and one-half degrees, while the perturbations of some of the elements amount to more than twelve degrees. For this reason we have not hesitated to adopt the direct

method of computation, and have used the method of the variation of elements simply as a means of verification in particular cases.

12. If we change v into v_1, in equation (104), and put the second member equal to $nt+\beta$, we shall have the elliptical value of the longitude v_1 equal to

$$v_1 = nt + \beta\,; \tag{170}$$

and if we neglect terms of a higher order than the fourth depending on e and γ, the value of β will be given by the equation

$$\left.\begin{aligned}\beta = {} & 2e\,\{1-\tfrac{1}{8}e^2\}\sin(nt-\omega)+\tfrac{5}{4}e^2\,\{1-\tfrac{11}{30}e^2\}\sin 2\,(nt-\omega)+\tfrac{13}{12}e^3\sin 3\,(nt-\omega)\\ & +\tfrac{103}{96}e^4\sin 4\,(nt-\omega)-\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-4e^2\}\sin 2\,(nt-\Omega)\\ & +\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-\tfrac{3}{4}e^2\}\sin 2\,(\omega-\Omega)+\tfrac{1}{32}\gamma^4\sin 4\,(nt-\Omega)\\ & +\tfrac{1}{32}\gamma^4\sin 4\,(\omega-\Omega)+\tfrac{1}{2}e\gamma^2\sin(nt+\omega-2\Omega)-\tfrac{1}{2}e\gamma^2\sin(3nt-\omega-2\Omega)\\ & -\tfrac{13}{16}e^2\gamma^2\sin(4nt-2\omega-2\Omega)-\tfrac{1}{16}\gamma^4\sin(2nt+2\omega-4\Omega)\end{aligned}\right\}. \tag{171}$$

If we mark with one accent for the sun, the quantities which refer to the moon, we shall have

$$v_1' = n't + \beta'\,; \tag{172}$$

and putting $\gamma=0$ in equation (171) we shall get

$$\left.\begin{aligned}\beta' = {} & 2e'\,\{1-\tfrac{1}{8}e'^2\}\sin(n't-\omega')+\tfrac{5}{4}e'^2\,\{1-\tfrac{11}{30}e'^2\}\sin 2\,(n't-\omega')\\ & +\tfrac{13}{12}e'^3\sin 3\,(n't-\omega')+\tfrac{103}{96}e'^4\sin 4\,(n't-\omega')\end{aligned}\right\}. \tag{173}$$

We shall therefore have

$$(v_1-v_1') = (nt-n't) + (\beta-\beta')\,; \tag{174}$$

whence we get

$$\left.\begin{aligned}\sin m\,(v_1-v_1') &= \sin m\,(nt-n't)\cos m\,(\beta-\beta')+\cos m\,(nt-n't)\sin m\,(\beta-\beta')\\ \cos m\,(v_1-v_1') &= \cos m\,(nt-n't)\cos m\,(\beta-\beta')-\sin m\,(nt-n't)\sin m\,(\beta-\beta')\end{aligned}\right\}. \tag{175}$$

Developing $\sin m\,(\beta-\beta')$ and $\cos m\,(\beta-\beta')$ in series, we shall have, with sufficient accuracy,

$$\left.\begin{aligned}\cos m\,(\beta-\beta') &= 1-\tfrac{1}{2}m^2(\beta-\beta')^2+\tfrac{1}{24}m^4(\beta-\beta')^4\\ \sin m\,(\beta-\beta') &= m\,(\beta-\beta')-\tfrac{1}{6}m^3(\beta-\beta')^3\end{aligned}\right\}. \tag{176}$$

Now we have

$$\left.\begin{aligned}(\beta-\beta')^2 &= \beta^2-2\beta\beta'+\beta'^2\\ (\beta-\beta')^3 &= \beta^3-3\beta^2\beta'+3\beta\beta'^2-\beta'^3\\ (\beta-\beta')^4 &= \beta^4-4\beta^3\beta'+6\beta^2\beta'^2-4\beta\beta'^3+\beta'^4\end{aligned}\right\}. \tag{177}$$

Substituting the values of β, β' in these equations, we shall obtain

$$\left.\begin{aligned}
(\beta-\beta')^2 = {} & \{2e^2+2e'^2+\tfrac{9}{32}e^4+\tfrac{9}{32}e'^4+\tfrac{1}{16}\gamma^4\}+\tfrac{5}{2}e^3\cos(nt-\omega) \\
& -2e^2\{1-\tfrac{4}{3}e^2+\tfrac{1}{32}\tfrac{\gamma^4}{e^2}\}\cos 2(nt-\omega)-\tfrac{5}{2}e^3\cos 3(nt-\omega) \\
& -\tfrac{283}{96}e^4\cos 4(nt-\omega)-\tfrac{27}{16}e^2\gamma^2\cos 2(nt-\Omega)+\tfrac{11}{16}e^2\gamma^2\cos 2(\omega-\Omega) \\
& -\tfrac{1}{32}\gamma^4\cos 4(nt-\Omega)-\tfrac{1}{32}\gamma^4\cos 4(\omega-\Omega)+\tfrac{1}{16}\gamma^4\cos(2nt+2\omega-4\Omega) \\
& -e\gamma^2\cos(nt+\omega-2\Omega)+\tfrac{1}{2}e\gamma^2\cos(3nt-\omega-2\Omega) \\
& +\tfrac{1}{2}e\gamma^2\cos(nt-3\omega+2\Omega)+\tfrac{21}{16}e^2\gamma^2\cos(4nt-2\omega-2\Omega) \\
& +\tfrac{5}{16}e^2\gamma^2\cos(2nt-4\omega+2\Omega)+\tfrac{5}{2}e'^3\cos(n't-\omega') \\
& -2e'^2\{1-\tfrac{4}{3}e'^2\}\cos 2(n't-\omega')-\tfrac{5}{2}e'^3\cos 3(n't-\omega') \\
& -\tfrac{283}{96}e'^4\cos 4(n't-\omega') \\
& -4ee'\{1-\tfrac{1}{8}e^2-\tfrac{1}{8}e'^2\}\{\cos(nt-n't-\omega+\omega')-\cos(nt+n't-\omega-\omega')\} \\
& +\tfrac{5}{2}e^2e'\{\cos(2nt+n't-2\omega-\omega')-\cos(2nt-n't-2\omega+\omega')\} \\
& +\tfrac{13}{6}e^3e'\{\cos(3nt+n't-3\omega-\omega')-\cos(3nt-n't-3\omega+\omega')\} \\
& +\tfrac{13}{6}ee'^3\{\cos(nt+3n't-\omega-3\omega')-\cos(nt-3n't-\omega+3\omega')\} \\
& +\tfrac{1}{2}e'\gamma^2\{\cos(2nt-n't+\omega-2\Omega)-\cos(2nt+n't-\omega'-2\Omega)\} \\
& +\tfrac{1}{2}e'\gamma^2\{\cos(n't+2\omega-\omega'-2\Omega)-\cos(n't-2\omega-\omega'+2\Omega)\} \\
& -ee'\gamma^2\cos(nt-n't+\omega+\omega'-2\Omega)+ee'\gamma^2\cos(nt+n't+\omega-\omega'-2\Omega) \\
& +ee'\gamma^2\cos(3nt-n't-\omega+\omega'-2\Omega)-ee'\gamma^2\cos(3nt+n't-\omega-\omega'-2\Omega) \\
& +\tfrac{5}{2}ee'^2\cos(nt+2n't-\omega-2\omega')-\tfrac{5}{2}ee'^2\cos(nt-2n't-\omega+2\omega') \\
& +\tfrac{25}{16}e^2e'^2\cos(2nt+2n't-2\omega-2\omega')-\tfrac{25}{16}e^2e'^2\cos(2nt-2n't-2\omega+2\omega') \\
& +\tfrac{5}{16}e'^2\gamma^2\cos(2nt-2n't+2\omega'-2\Omega)-\tfrac{5}{16}e'^2\gamma^2\cos(2nt+2n't-2\omega'-2\Omega) \\
& +\tfrac{5}{16}e'^2\gamma^2\cos(2n't+2\omega-2\omega'-2\Omega)-\tfrac{5}{16}e'^2\gamma^2\cos(2n't-2\omega-2\omega'+2\Omega)
\end{aligned}\right\}; \quad (178)$$

$$\left.\begin{aligned}
\tfrac{1}{6}(\beta-\beta')^3 &= e\{e^2+2e'^2\}\sin(nt-\omega)-e'\{2e^2+e'^2\}\sin(n't-\omega') \\
&+\tfrac{5}{4}e^2\{e^2+e'^2\}\sin 2(nt-\omega)-\tfrac{5}{4}e'^2\{e^2+e'^2\}\sin 2(n't-\omega') \\
&-\tfrac{1}{3}e^3\sin 3(nt-\omega)+\tfrac{1}{3}e'^3\sin 3(n't-\omega')-\tfrac{5}{8}e^4\sin 4(nt-\omega) \\
&+\tfrac{5}{8}e'^4\sin 4(n't-\omega')-\tfrac{1}{4}\gamma^2\{\tfrac{3}{2}e^2+e'^2\}\sin 2(nt-\Omega) \\
&+\tfrac{1}{4}\gamma^2\{\tfrac{3}{2}e^2+e'^2\}\sin 2(\omega-\Omega)+\tfrac{1}{8}e^2\gamma^2\sin(4nt-2\omega-2\Omega) \\
&+\tfrac{1}{8}e^2\gamma^2\sin(2nt-4\omega+2\Omega)+\tfrac{5}{4}\{ee'^3-e^3e'\}\sin(nt+n't-\omega-\omega') \\
&+\tfrac{5}{4}\{ee'^3+e^3e'\}\sin(nt-n't-\omega+\omega')-ee'^2\sin(nt+2n't-\omega-2\omega') \\
&-ee'^2\sin(nt-2n't-\omega+2\omega')+e^2e'\sin(2nt+n't-2\omega-\omega') \\
&-e^2e'\sin(2nt-n't-2\omega+\omega')-\tfrac{5}{4}ee'^3\sin(nt+3n't-\omega-3\omega') \\
&-\tfrac{5}{4}ee'^3\sin(nt-3n't-\omega+3\omega')+\tfrac{5}{4}e^3e'\sin(3nt+n't-3\omega-\omega') \\
&-\tfrac{5}{4}e^3e'\sin(3nt-n't-3\omega+\omega')+\tfrac{1}{4}ee'\gamma^2\sin(3nt-n't-\omega+\omega'-2\Omega) \\
&-\tfrac{1}{2}ee'\gamma^2\sin(nt-n't+\omega+\omega'-2\Omega)-\tfrac{3}{4}ee'\gamma^2\sin(3nt+n't-\omega-\omega'-2\Omega) \\
&+\tfrac{1}{2}ee'\gamma^2\sin(nt+n't+\omega-\omega'-2\Omega)+\tfrac{1}{4}ee'\gamma^2\sin(nt-n't-3\omega+\omega'+2\Omega) \\
&-\tfrac{1}{4}ee'\gamma^2\sin(nt+n't-3\omega-\omega'+2\Omega)-\tfrac{5}{4}e^2e'^2\sin(2nt-2n't-2\omega+2\omega') \\
&+\tfrac{1}{8}e'^2\gamma^2\sin(2nt-2n't+2\omega'-2\Omega)+\tfrac{1}{8}e'^2\gamma^2\sin(2nt+2n't-2\omega'-2\Omega) \\
&+\tfrac{1}{8}e'^2\gamma^2\sin(2n't-2\omega-2\omega'+2\Omega)-\tfrac{1}{8}e'^2\gamma^2\sin(2n't+2\omega-2\omega'-2\Omega)
\end{aligned}\right\};\quad(179)$$

$$\left.\begin{aligned}
\tfrac{1}{24}(\beta-\beta')^4 &= \tfrac{1}{4}\{e^4+4e^2e'^2+e'^4\}-\tfrac{1}{3}e^2\{e^2+3e'^2\}\cos 2(nt-\omega) \\
&-\tfrac{1}{3}e'^2\{3e^2+e'^2\}\cos 2(n't-\omega')+\tfrac{1}{12}e^4\cos 4(nt-\omega) \\
&+\tfrac{1}{12}e'^4\cos 4(n't-\omega')-\{e^3e'+ee'^3\}\cos(nt-n't-\omega+\omega') \\
&+\{e^3e'+ee'^3\}\cos(nt+n't-\omega-\omega')+\tfrac{1}{3}ee'^3\cos(nt-3n't-\omega+3\omega') \\
&-\tfrac{1}{3}ee'^3\cos(nt+3n't-\omega-3\omega')-\tfrac{1}{3}e^3e'\cos(3nt+n't-3\omega-\omega') \\
&+\tfrac{1}{3}e^3e'\cos(3nt-n't-3\omega+\omega')+\tfrac{1}{2}e^2e'^2\cos(2nt+2n't-2\omega-2\omega') \\
&+\tfrac{1}{2}e^2e'^2\cos(2nt-2n't-2\omega+2\omega')
\end{aligned}\right\}.\quad(180)$$

If we substitute these values in equations (176), and the resulting values of $\sin m\,(\beta-\beta)$ and $\cos m\,(\beta-\beta')$ in the first of equations (175), we shall get,

$$
\left.\begin{aligned}
\sin m\,(v_1-v_1') = \{1 &- m^2(e^2+e'^2+\tfrac{9}{64}e^4+\tfrac{9}{64}e'^4+\tfrac{1}{32}\gamma^4) \\
&+ \tfrac{1}{4}m^4(e^4+4e^2e'^2+e'^4)\}\sin m\,(nt-n't) \\
&+ m^2 \sin m\,(nt-n't)\times \\
&\Big\{-\tfrac{5}{4}e^3\cos(nt-\omega)+e^2\{1-\tfrac{4}{3}e^2+\tfrac{1}{32}\frac{\gamma^4}{e^2}-\tfrac{1}{3}m^2(e^2+3e'^2)\}\cos 2\,(nt-\omega) \\
&+\tfrac{5}{4}e^3\cos 3\,(nt-\omega)+e^4\{\tfrac{283}{192}+\tfrac{1}{12}m^2\}\cos 4\,(nt-\omega)-\tfrac{5}{4}e'^3\cos(n't-\omega') \\
&+e'^2\{1-\tfrac{4}{3}e'^2-\tfrac{1}{3}m^2(3e^2+e'^2)\}\cos 2\,(n't-\omega')+\tfrac{5}{4}e'^3\cos 3\,(n't-\omega') \\
&+e'^4\{\tfrac{283}{192}+\tfrac{1}{12}m^2\}\cos 4\,(n't-\omega')+\tfrac{87}{32}e^2\gamma^2\cos 2\,(nt-\Omega) \\
&-\tfrac{11}{32}e^2\gamma^2\cos 2\,(\omega-\Omega)+\tfrac{1}{64}\gamma^4\cos 4\,(nt-\Omega)+\tfrac{1}{64}\gamma^4\cos 4\,(\omega-\Omega) \\
&-\tfrac{1}{32}\gamma^4\cos(2nt+2\omega-4\Omega)+\tfrac{1}{2}e\gamma^2\cos(nt+\omega-2\Omega) \\
&-\tfrac{1}{4}e\gamma^2\cos(3nt-\omega-2\Omega)-\tfrac{1}{4}e\gamma^2\cos(nt-3\omega+2\Omega) \\
&-\tfrac{21}{32}e^2\gamma^2\cos(4nt-2\omega-2\Omega)-\tfrac{5}{32}e^2\gamma^2\cos(2nt-4\omega+2\Omega) \\
&+2ee'\{1-\tfrac{1}{8}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}m^2(e^2+e'^2)\}\{\cos(nt-n't-\omega+\omega') \\
&-\cos(nt+n't-\omega-\omega')\}-\tfrac{5}{4}e^2e'\cos(2nt+n't-2\omega-\omega') \\
&+\tfrac{5}{4}e^2e'\cos(2nt-n't-2\omega+\omega')+e^3e'\{\tfrac{13}{12}+\tfrac{1}{3}m^2\}\cos(3nt-n't-3\omega+\omega') \\
&-e^3e'\{\tfrac{13}{12}+\tfrac{1}{3}m^2\}\cos(3nt+n't-3\omega-\omega') \\
&+ee'^3\{\tfrac{13}{12}+\tfrac{1}{3}m^2\}\cos(nt-3n't-\omega+3\omega') \\
&-ee'^3\{\tfrac{13}{12}+\tfrac{1}{3}m^2\}\cos(nt+3n't-\omega-3\omega') \\
&+e^2e'^2\{\tfrac{25}{32}+\tfrac{1}{2}m^2\}\cos(2nt-2n't-2\omega+2\omega') \\
&+e^2e'^2\{\tfrac{1}{2}m^2-\tfrac{25}{32}\}\cos(2nt-2n't-2\omega-2\omega') \\
&+\tfrac{1}{4}e'\gamma^2\cos(2nt+n't-\omega'-2\Omega)-\tfrac{1}{4}e'\gamma^2\cos(2nt-n't+\omega'-2\Omega) \\
&+\tfrac{1}{4}e'\gamma^2\cos(n't-2\omega-\omega'+2\Omega)-\tfrac{1}{4}e'\gamma^2\cos(n't+2\omega-\omega'-2\Omega) \\
&+\tfrac{1}{2}ee'\gamma^2\cos(nt-n't+\omega+\omega'-2\Omega)-\tfrac{1}{2}ee'\gamma^2\cos(nt+n't+\omega-\omega'-2\Omega) \\
&+\tfrac{1}{2}ee'\gamma^2\cos(3nt+n't-\omega-\omega'-2\Omega)-\tfrac{1}{2}ee'\gamma^2\cos(3nt-n't-\omega+\omega'-2\Omega) \\
&+\tfrac{5}{4}ee'^2\cos(nt-2n't-\omega+2\omega')-\tfrac{5}{4}ee'^2\cos(nt+2n't-\omega-2\omega') \\
&+\tfrac{5}{32}e'^2\gamma^2\cos(2nt+2n't-2\omega'-2\Omega)-\tfrac{5}{32}e'^2\gamma^2\cos(2nt-2n't+2\omega'-2\Omega) \\
&+\tfrac{5}{32}e'^2\gamma^2\cos(2n't-2\omega-2\omega'+2\Omega)-\tfrac{5}{32}e'^2\gamma^2\cos(2n't+2\omega-2\omega'-2\Omega)\Big\}
\end{aligned}\right\};\quad (181)
$$

(This equation is continued on the next page.)

$$\left.\begin{aligned}
&+ m\cos m\,(nt-n't)\\
&\quad \Big\{ \{2e-\tfrac{1}{4}e^3-m^2e\,(e^2+2e'^2)\}\sin(nt-\omega)\\
&\quad - \{2e'-\tfrac{1}{4}e'^3-m^2e'\,(2e^2+e'^2)\}\sin(n't-\omega')\\
&\quad + \{\tfrac{5}{4}e^2-\tfrac{11}{24}e^4-\tfrac{5}{4}m^2e^2\,(e^2+e'^2)\}\sin 2\,(nt-\omega)\\
&\quad - \{\tfrac{5}{4}e'^2-\tfrac{11}{24}e'^4-\tfrac{5}{4}m^2e'^2\,(e^2+e'^2)\}\sin 2\,(n't-\omega')\\
&\quad + \{\tfrac{13}{12}+\tfrac{1}{8}m^2\}\,e^3\sin 3\,(nt-\omega)-\{\tfrac{13}{12}+\tfrac{1}{8}m^2\}\,e'^3\sin 3\,(n't-\omega')\\
&\quad + \{\tfrac{103}{96}+\tfrac{5}{8}m^2\}\,e^4\cos 4\,(nt-\omega)-\{\tfrac{103}{96}+\tfrac{5}{8}m^2\}\,e'^4\sin 4\,(n't-\omega')\\
&\quad - \tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-4e^2-m^2\,(\tfrac{3}{2}e^2+e'^2)\}\sin 2\,(nt-\Omega)\\
&\quad + \tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-12e^2-m^2\,(\tfrac{3}{2}e^2+e'^2)\}\sin 2\,(\omega-\Omega)\\
&\quad + \tfrac{1}{32}\gamma^4\sin 4\,(nt-\Omega)+\tfrac{1}{32}\gamma^4\sin 4\,(\omega-\Omega)+\tfrac{1}{2}e\gamma^2\sin(nt+\omega-2\Omega)\\
&\quad - \tfrac{1}{2}e\gamma^2\sin(3nt-\omega-2\Omega)-e^2\gamma^2\,\{\tfrac{13}{16}+\tfrac{1}{8}m^2\}\sin(4nt-2\omega-2\Omega)\\
&\quad - \tfrac{1}{16}\gamma^4\sin(2nt+2\omega-4\Omega)-\tfrac{1}{8}m^2e^2\gamma^2\sin(2nt-4\omega+2\Omega)\\
&\quad + \tfrac{5}{4}m^2\,\{e^3e'-ee'^3\}\sin(nt+n't-\omega-\omega')\\
&\quad - \tfrac{5}{4}m^2\,\{e^3e'+ee'^3\}\sin(nt-n't-\omega+\omega')\\
&\quad + m^2ee'^2\sin(nt+2n't-\omega-2\omega')+m^2ee'^2\sin(nt-2n't-\omega+2\omega')\\
&\quad - m^2e^2e'\sin(2nt+n't-2\omega-\omega')+m^2e^2e'\sin(2nt-n't-2\omega+\omega')\\
&\quad + \tfrac{5}{4}m^2ee'^3\sin(nt+3n't-\omega-3\omega')+\tfrac{5}{4}m^2ee'^3\sin(nt-3n't-\omega+3\omega')\\
&\quad - \tfrac{5}{4}m^2e^3e'\sin(3nt+n't-3\omega-\omega')+\tfrac{5}{4}m^2e^3e'\sin(3nt-n't-3\omega+\omega')\\
&\quad - \tfrac{1}{4}m^2ee'\gamma^2\sin(3nt-n't-\omega+\omega'-2\Omega)\\
&\quad + \tfrac{1}{2}m^2ee'\gamma^2\sin(nt-n't+\omega+\omega'-2\Omega)\\
&\quad + \tfrac{3}{4}m^2ee'\gamma^2\sin(3nt+n't-\omega-\omega'-2\Omega)\\
&\quad - \tfrac{1}{2}m^2ee'\gamma^2\sin(nt+n't+\omega-\omega'-2\Omega)\\
&\quad - \tfrac{1}{4}m^2ee'\gamma^2\sin(nt-n't-3\omega+\omega'+2\Omega)\\
&\quad + \tfrac{1}{4}m^2ee'\gamma^2\sin(nt+n't-3\omega-\omega'+2\Omega)\\
&\quad + \tfrac{5}{4}m^2e^2e'^2\sin(2nt-2n't-2\omega+2\omega')-\tfrac{1}{8}m^2e'^2\gamma^2\sin(2nt-2n't+2\omega'-2\Omega)\\
&\quad - \tfrac{1}{8}m^2e'^2\gamma^2\sin(2nt+2n't-2\omega'-2\Omega)-\tfrac{1}{8}m^2e'^2\gamma^2\sin(2n't-2\omega-2\omega'+2\Omega)\\
&\quad + \tfrac{1}{8}m^2e'^2\gamma^2\sin(2n't+2\omega-2\omega'-2\Omega)\Big\}
\end{aligned}\right\}. \quad (181)$$

This equation will give the value of *cos* $m\,(v_1-v_1')$ by changing *sin* $m\,(nt-n't)$ into *cos* $m\,(nt-n't)$ and *cos* $m\,(nt-n't)$ into $-$*sin* $m\,(nt-n't)$.

If in the preceding equation we put $m=2$, we shall obtain

$$
\left.\begin{aligned}
&\sin 2\,(v_1-v_1') = \\
&\quad \{1-4e^2-4e'^2+\tfrac{55}{16}e^4+\tfrac{55}{16}e'^4+16e^2e'^2-\tfrac{1}{8}\gamma^4\}\sin 2\,(nt-n't) \\
&\quad +2e\,\{1-\tfrac{27}{8}e^2-4e'^2\}\sin(3nt-2n't-\omega) \\
&\quad -2e\,\{1-\tfrac{7}{8}e^2-4e'^2\}\sin(nt-2n't+\omega) \\
&\quad -2e'\,\{1-\tfrac{7}{8}e'^2-4e^2\}\sin(2nt-n't-\omega') \\
&\quad +2e'\,\{1-\tfrac{27}{8}e'^2-4e^2\}\sin(2nt-3n't+\omega') \\
&\quad +\{\tfrac{13}{4}e^2-\tfrac{259}{24}e^4-13e^2e'^2+\tfrac{1}{16}\gamma^4\}\sin(4nt-2n't-2\omega) \\
&\quad \mp\{\tfrac{3}{4}e^2+\tfrac{1}{8}e^4-3e^2e'^2+\tfrac{1}{16}\gamma^4\}\sin 2\,(n't-\omega) \\
&\quad +\tfrac{3}{4}e'^2\,\{1-4e^2+\tfrac{1}{6}e'^2\}\sin 2\,(nt-\omega') \\
&\quad +e'^2\,\{\tfrac{13}{4}-\tfrac{259}{24}e'^2-13e^2\}\sin(2nt-4n't+2\omega') \\
&\quad +\tfrac{59}{12}e^3\sin(5nt-2n't-3\omega)\mp\tfrac{1}{12}e^3\sin(nt+2n't-3\omega) \\
&\quad +\tfrac{1}{12}e'^3\sin(2nt+n't-3\omega')+\tfrac{59}{12}e'^3\sin(2nt-5n't+3\omega') \\
&\quad -\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-\tfrac{77}{4}e^2-4e'^2\}\sin(4nt-2n't-2\Omega) \\
&\quad \mp\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-\tfrac{3}{4}e^2-4e'^2\}\sin 2\,(n't-\Omega) \\
&\quad +\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-\tfrac{83}{4}e^2-4e'^2\}\sin(2nt-2n't+2\omega-2\Omega) \\
&\quad -\tfrac{1}{4}\gamma^2\,\{1-\tfrac{1}{2}\gamma^2-\tfrac{61}{4}e^2-4e'^2\}\sin(2nt-2n't-2\omega+2\Omega) \\
&\quad +\tfrac{3}{2}e\gamma^2\sin(3nt-2n't+\omega-2\Omega)+\tfrac{1}{2}e\gamma^2\sin(nt-2n't-\omega+2\Omega) \\
&\quad -e\gamma^2\sin(5nt-2n't-\omega-2\Omega)-\tfrac{1}{2}e\gamma^2\sin(3nt-2n't-3\omega+2\Omega) \\
&\quad -\tfrac{1}{2}e\gamma^2\sin(nt-2n't+3\omega-2\Omega)+\tfrac{1}{2}e'\gamma^2\sin(4nt-n't-\omega'-2\Omega) \\
&\quad \mp e'\gamma^2\sin(3n't-\omega'-2\Omega)-\tfrac{1}{2}e'\gamma^2\sin(4nt-3n't+\omega'-2\Omega) \\
&\quad \pm\tfrac{1}{2}e'\gamma^2\sin(n't+\omega'-2\Omega)+\tfrac{1}{2}e'\gamma^2\sin(2nt-n't-2\omega-\omega'+2\Omega) \\
&\quad +\tfrac{1}{2}e'\gamma^2\sin(2nt-3n't+2\omega+\omega'-2\Omega) \\
&\quad -\tfrac{1}{2}e'\gamma^2\sin(2nt-n't+2\omega-\omega'-2\Omega) \\
&\quad -\tfrac{1}{2}e'\gamma^2\sin(2nt-3n't-2\omega+\omega'+2\Omega) \\
&\quad -4ee'\,\{1-\tfrac{27}{8}e^2-\tfrac{7}{8}e'^2\}\sin(3nt-n't-\omega-\omega') \\
&\quad -4ee'\,\{1-\tfrac{7}{8}e^2-\tfrac{27}{8}e'^2\}\sin(nt-3n't+\omega+\omega') \\
&\quad +4ee'\,\{1-\tfrac{27}{8}e^2-\tfrac{27}{8}e'^2\}\sin(3nt-3n't-\omega+\omega')
\end{aligned}\right\}\quad(182)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+4ee'\left\{1-\tfrac{7}{8}e^2-\tfrac{7}{8}e'^2\right\}\sin(nt-n't+\omega-\omega')\\
&+\tfrac{3}{2}ee'^2\sin(3nt-\omega-2\omega')-\tfrac{3}{2}ee'^2\sin(nt+\omega-2\omega')\\
&-\tfrac{13}{2}ee'^2\sin(nt-4n't+\omega+2\omega')+\tfrac{13}{2}ee'^2\sin(3nt-4n't-\omega+2\omega')\\
&-\tfrac{13}{2}e^2e'\sin(4nt-n't-2\omega-\omega')\mp\tfrac{3}{2}e^2e'\sin(3n't-2\omega-\omega')\\
&+\tfrac{13}{2}e^2e'\sin(4nt-3n't-2\omega+\omega')\pm\tfrac{3}{2}e^2e'\sin(n't-2\omega+\omega')\\
&+\tfrac{115}{16}e^4\sin(6nt-2n't-4\omega)\mp\tfrac{1}{24}e^4\sin(2nt+2n't-4\omega)\\
&+\tfrac{1}{24}e'^4\sin(2nt+2n't-4\omega')+\tfrac{115}{16}e'^4\sin(2nt-6n't+4\omega')\\
&+\tfrac{1}{16}\gamma^4\sin(6nt-2n't-4\Omega)+\tfrac{1}{16}\gamma^4\sin(2nt-2n't+4\omega-4\Omega)\\
&-\tfrac{1}{8}\gamma^4\sin(4nt-2n't+2\omega-4\Omega)-\tfrac{13}{16}e^2\gamma^2\sin(4nt-2n't-4\omega+2\Omega)\\
&\mp e^2\gamma^2\sin(2n't-4\omega+2\Omega)-\tfrac{21}{8}e^2\gamma^2\sin(6nt-2n't-2\omega-2\Omega)\\
&+\tfrac{1}{6}ee'^3\sin(3nt+n't-\omega-3\omega')-\tfrac{1}{6}ee'^3\sin(nt+n't+\omega-3\omega')\\
&-\tfrac{59}{6}ee'^3\sin(nt-5n't+\omega+3\omega')+\tfrac{59}{6}ee'^3\sin(3nt-5n't-\omega+3\omega')\\
&\mp\tfrac{1}{6}e^3e'\sin(nt+3n't-3\omega-\omega')\pm\tfrac{1}{6}e^3e'\sin(nt+n't-3\omega+\omega')\\
&-\tfrac{59}{6}e^3e'\sin(5nt-n't-3\omega-\omega')+\tfrac{59}{6}e^3e'\sin(5nt-3n't-3\omega+\omega')\\
&-2ee'\gamma^2\sin(5nt-3n't-\omega+\omega'-2\Omega)\\
&+3ee'\gamma^2\sin(3nt-3n't+\omega+\omega'-2\Omega)\\
&-ee'\gamma^2\sin(nt-n't-\omega-\omega'+2\Omega)+4ee'\gamma^2\sin(5nt-n't-\omega-\omega'-2\Omega)\\
&\pm2ee'\gamma^2\sin(nt+3n't-\omega-\omega'-2\Omega)\\
&-3ee'\gamma^2\sin(3nt-n't+\omega-\omega'-2\Omega)\\
&+ee'\gamma^2\sin(nt-3n't-\omega+\omega'+2\Omega)\\
&-ee'\gamma^2\sin(3nt-3n't-3\omega+\omega'+2\Omega)\\
&+ee'\gamma^2\sin(nt-n't+3\omega-\omega'-2\Omega)+ee'\gamma^2\sin(3nt-n't-3\omega-\omega'+2\Omega)\\
&-ee'\gamma^2\sin(nt-3n't+3\omega+\omega'-2\Omega)\\
&+\tfrac{169}{16}e^2e'^2\sin(4nt-4n't-2\omega+2\omega')\\
&+\tfrac{9}{16}e^2e'^2\sin2(\omega-\omega')-\tfrac{13}{16}e'^2\gamma^2\sin(4nt-4n't+2\omega'-2\Omega)\\
&\mp\tfrac{3}{16}e'^2\gamma^2\sin2(\omega'-\Omega)-\tfrac{3}{16}e'^2\gamma^2\sin(4nt-2\omega'-2\Omega)\\
&\mp\tfrac{13}{16}e'^2\gamma^2\sin(4n't-2\omega'-2\Omega)-\tfrac{3}{16}e'^2\gamma^2\sin(2nt-2\omega-2\omega'+2\Omega)\\
&+\tfrac{13}{16}e'^2\gamma^2\sin(2nt-4n't+2\omega+2\omega'-2\Omega)\\
&+\tfrac{3}{16}e'^2\gamma^2\sin(2nt+2\omega-2\omega'-2\Omega)\\
&-\tfrac{13}{16}e'^2\gamma^2\sin(2nt-4n't-2\omega+2\omega'+2\Omega)\\
&+\tfrac{33}{16}e^2e'^2\sin(4nt-2\omega-2\omega')\mp\tfrac{33}{16}e^2e'^2\sin(4n't-2\omega-2\omega')
\end{aligned}\right\}\quad.\ (182)
$$

Since the terms of equations (167–169), which depend on the angles $v_1 - v_1'$, and $3(v_1 - v_1')$, have r'^4 for a divisor, it will be unnecessary to develop the sines and cosines of these angles to terms of a higher order than the second depending on e and γ. We shall therefore obtain, by putting $m = 1$, and $m = 3$, in equation (181),

$$\sin(v_1 - v_1') = \left.\begin{array}{l} \{1 - e^2 - e'^2\}\sin(nt - n't) + e\sin(2nt - n't - \omega) \\ \pm e\sin(n't - \omega) - e'\sin(nt - \omega') + e'\sin(nt - 2n't + \omega') \\ - ee'\sin(2nt - \omega - \omega') \pm ee'\sin(2n't - \omega - \omega') + ee'\sin(\omega - \omega') \\ + ee'\sin(2nt - 2n't - \omega + \omega') + \frac{9}{8}e^2\sin(3nt - n't - 2\omega) \\ \pm \frac{1}{8}e^2\sin(nt + n't - 2\omega) - \frac{1}{8}e'^2\sin(nt + n't - 2\omega') \\ + \frac{9}{8}e'^2\sin(nt - 3n't + 2\omega') \mp \frac{1}{8}\gamma^2\sin(nt + n't - 2\Omega) \\ - \frac{1}{8}\gamma^2\sin(nt - n't - 2\omega + 2\Omega) - \frac{1}{8}\gamma^2\sin(3nt - n't - 2\Omega) \\ + \frac{1}{8}\gamma^2\sin(nt - n't + 2\omega - 2\Omega) \end{array}\right\}; \quad (183)$$

$$\sin 3(v_1 - v_1') = \left.\begin{array}{l} \{1 - 9e^2 - 9e'^2\}\sin 3(nt - n't) + 3e\sin(4nt - 3n't - \omega) \\ - 3e\sin(2nt - 3n't + \omega) + 3e'\sin(3nt - 4n't + \omega') \\ - 3e'\sin(3nt - 2n't - \omega') + 9ee'\sin(4nt - 4n't - \omega + \omega') \\ + 9ee'\sin(2nt - 2n't + \omega - \omega') - 9ee'\sin(4nt - 2n't - \omega - \omega') \\ - 9ee'\sin(2nt - 4n't + \omega + \omega') + \frac{51}{8}e^2\sin(5nt - 3n't - 2\omega) \\ + \frac{21}{8}e^2\sin(nt - 3n't + 2\omega) + \frac{21}{8}e'^2\sin(3nt - n't - 2\omega') \\ + \frac{51}{8}e'^2\sin(3nt - 5n't + 2\omega') - \frac{9}{8}\gamma^2\sin(5nt - 3n't - 2\Omega) \\ + \frac{9}{8}\gamma^2\sin(nt - 3n't + 2\Omega) + \frac{9}{8}\gamma^2\sin(3nt - 3n't + 2\omega - 2\Omega) \\ - \frac{9}{8}\gamma^2\sin(3nt - 3n't - 2\omega + 2\Omega) \end{array}\right\}. \quad (184)$$

If, in equations (182–184), we change *sin* to *cos*, and use the lower sign where two are given, we shall obtain the values of the *cosines* of the same angles.

And here it may be well to explain some useful theorems in regard to the derivation of the expression for the *cosine* of an angle from that of its *sine*, and the reverse.

If we have expressions of the form

$$v = nt + e\sin(int + f\omega + h\Omega)$$
$$v' = n't + e'\sin(i'n't + f'\omega' + h'\Omega')$$

in which i, i', f, f', h, h', are integral numbers, positive or negative, and satisfying the equations $i + f + h = 0$, $i' + f' + h' = 0$, then

Theorem I.—The development of $\sin(v - v')$ will have the term $\sin(nt - n't)$ plus a series of terms of the form $\sin(int - i'n't + f\omega - f'\omega' + h\Omega - h'\Omega')$,

in which $i+f+h$ and $i'+f'+h'$ *must* be either *positive* or *negative*, and numerically equal to each other, but cannot vanish, so that we shall have $i-i'+f-f'+h-h'=0$; then

Theorem II.—The co-efficient of the *cosine* of any angle will be equal to the co-efficient of the *sine* of the same angle; and

Theorem III.—The sign of the co-efficient of the *cosine* of any angle will be the *same* as that of its *sine* if $i+f+h$ be positive, but *different* if $i+f+h$ be *negative.*

We may illustrate these theorems by means of the two following terms which occur in equation (182),

$$+4ee'\gamma^2 \sin(5nt-n't-\omega-\omega'-2\Omega) \pm 2ee'\gamma^2 \sin(nt+3n't-\omega-\omega'-2\Omega).$$

In the first we have $i+f+h=5-1-2=+2$, consequently, the sign of the cosine will be the same as that of its sine, while in the second we have $i+f+h=1-1-2=-2$, and therefore the sign of the cosine will be different from that of its sine. Theorem III. is very useful as a check to the accuracy of independent developments of the sine and cosine of functions of the above forms.

13. Equation (119) will give

$$\left.\begin{array}{l}\dfrac{r_1}{a}=1+\frac{1}{2}e^2-e\{1-\frac{3}{8}e^2\}\cos(nt-\omega)-\frac{1}{2}e^2\{1-\frac{2}{3}e^2\}\cos 2(nt-\omega) \\ \qquad -\frac{3}{8}e^3\cos 3(nt-\omega)-\frac{1}{3}e^4\cos 4(nt-\omega)\end{array}\right\}; \tag{185}$$

$$\left.\begin{array}{l}\dfrac{r_1^2}{a^2}=1+\frac{3}{2}e^2-2e\{1-\frac{1}{8}e^2\}\cos(nt-\omega)-\frac{1}{2}e^2\{1-\frac{1}{3}e^2\}\cos 2(nt-\omega) \\ \qquad -\frac{1}{4}e^3\cos 3(nt-\omega)-\frac{1}{6}e^4\cos 4(nt-\omega)\end{array}\right\}; \tag{186}$$

$$\frac{r_1^3}{a^3}=1+3e^2-3e\cos(nt-\omega). \tag{187}$$

Marking with one accent for the sun the quantities in equation (119), it will give

$$\left.\begin{array}{l}\dfrac{a'^3}{r_1'^3}=1+\frac{3}{2}e'^2+\frac{15}{8}e'^4+3e'\{1+\frac{9}{8}e'^2\}\cos(n't-\omega') \\ \qquad +\frac{9}{2}e'^2\{1+\frac{7}{9}e'^2\}\cos 2(n't-\omega')+\frac{53}{8}e'^3\cos 3(n't-\omega') \\ \qquad +\frac{77}{8}e'^4\cos 4(n't-\omega')\end{array}\right\}; \tag{188}$$

$$\frac{a'^4}{r'^4}=1+3e'^2+4e'\cos(n't-\omega')+7e'^2\cos 2(n't-\omega'). \tag{189}$$

These equations will give

$$
\left.
\begin{aligned}
\frac{r_1}{r_1'^3} = \frac{a}{a'^3}\Big\{ & \{1 + \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 + \tfrac{3}{4}e^2e'^2 + \tfrac{15}{8}e'^4\} - e\,\{1 - \tfrac{3}{8}e^2 + \tfrac{3}{2}e'^2\} \cos(nt - \omega) \\
& + 3e'\,\{1 + \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2\} \cos(n't - \omega') - \tfrac{1}{2}e^2\,\{1 - \tfrac{4}{3}e^2 + \tfrac{3}{2}e'^2\} \cos 2(nt - \omega) \\
& + \tfrac{9}{2}e'^2\,\{1 + \tfrac{1}{2}e^2 + \tfrac{7}{9}e'^2\} \cos 2(n't - \omega') \\
& - \tfrac{3}{2}ee'\,\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2\} \cos(nt + n't - \omega - \omega') \\
& - \tfrac{3}{2}ee'\,\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2\} \cos(nt - n't - \omega + \omega') \\
& - \tfrac{3}{8}e^3 \cos 3(nt - \omega) + \tfrac{53}{8}e'^3 \cos 3(n't - \omega') \\
& - \tfrac{9}{4}ee'^2 \cos(nt + 2n't - \omega - 2\omega') - \tfrac{9}{4}ee'^2 \cos(nt - 2n't - \omega + 2\omega') \\
& - \tfrac{3}{4}e^2e' \cos(2nt + n't - 2\omega - \omega') - \tfrac{3}{4}e^2e' \cos(2nt - n't - 2\omega + \omega') \\
& - \tfrac{53}{16}ee'^3 \cos(nt + 3n't - \omega - 3\omega') - \tfrac{53}{16}ee'^3 \cos(nt - 3n't - \omega + 3\omega') \\
& - \tfrac{9}{16}e^3e' \cos(3nt + n't - 3\omega - \omega') - \tfrac{9}{16}e^3e' \cos(3nt - n't - 3\omega + \omega') \\
& - \tfrac{9}{8}e^2e'^2 \cos(2nt + 2n't - 2\omega - 2\omega') - \tfrac{9}{8}e^2e'^2 \cos(2nt - 2n't - 2\omega + 2\omega') \\
& - \tfrac{1}{3}e^4 \cos 4(nt - \omega) + \tfrac{77}{8}e'^4 \cos 4(n't - \omega')\Big\}
\end{aligned}
\right\} ; \quad (190)
$$

$$
\left.
\begin{aligned}
\frac{r_1^2}{r_1'^3} = \frac{a^2}{a'^3}\Big\{ & \{1 + \tfrac{3}{2}e^2 + \tfrac{3}{2}e'^2 + \tfrac{9}{4}e^2e'^2 + \tfrac{15}{8}e'^4\} - 2e\,\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2\} \cos(nt - \omega) \\
& + 3e'\,\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2\} \cos(n't - \omega') - \tfrac{1}{2}e^2\,\{1 - \tfrac{1}{3}e^2 + \tfrac{3}{2}e'^2\} \cos 2(nt - \omega) \\
& + \tfrac{9}{2}e'^2\,\{1 + \tfrac{3}{2}e^2 + \tfrac{7}{9}e'^2\} \cos 2(n't - \omega') - \tfrac{1}{4}e^3 \cos 3(nt - \omega) \\
& - 3ee'\,\{1 - \tfrac{1}{8}e^2 + \tfrac{9}{8}e'^2\} \cos(nt + n't - \omega - \omega') \\
& - 3ee'\,\{1 - \tfrac{1}{8}e^2 + \tfrac{9}{8}e'^2\} \cos(nt - n't - \omega + \omega') \\
& - \tfrac{9}{2}ee'^2 \cos(nt + 2n't - \omega - 2\omega') - \tfrac{9}{2}ee'^2 \cos(nt - 2n't - \omega + 2\omega') \\
& + \tfrac{53}{8}e'^3 \cos 3(n't - \omega') - \tfrac{3}{4}e^2e' \cos(2nt + n't - 2\omega - \omega') \\
& - \tfrac{3}{4}e^2e' \cos(2nt - n't - 2\omega + \omega') - \tfrac{53}{8}ee'^3 \cos(nt + 3n't - \omega - 3\omega') \\
& - \tfrac{53}{8}ee'^3 \cos(nt - 3n't - \omega + 3\omega') - \tfrac{3}{8}e^3e' \cos(3nt + n't - 3\omega - \omega') \\
& - \tfrac{3}{8}e^3e' \cos(3nt - n't - 3\omega + \omega') - \tfrac{9}{8}e^2e'^2 \cos(2nt + 2n't - 2\omega - 2\omega') \\
& - \tfrac{9}{8}e^2e'^2 \cos(2nt - 2n't - 2\omega + 2\omega') - \tfrac{1}{6}e^4 \cos 4(nt - \omega) \\
& + \tfrac{77}{8}e'^4 \cos 4(n't - \omega')
\end{aligned}
\right\} ; \quad (191)
$$

$$
\left.
\begin{aligned}
\frac{r_1^2}{r_1'^4} = \frac{a^2}{a'^4}\Big\{ & 1 + \tfrac{3}{2}e^2 + 3e'^2 - 2e \cos(nt - \omega) - \tfrac{1}{2}e^2 \cos 2(nt - \omega) \\
& + 4e' \cos(n't - \omega') + 7e'^2 \cos 2(n't - \omega') - 4ee' \cos(nt + n't - \omega - \omega') \\
& - 4ee' \cos(nt - n't - \omega + \omega')
\end{aligned}
\right\} ; \quad (192)
$$

$$\left.\begin{aligned}\frac{r_1^{3}}{r_1'^{4}}=\frac{a^3}{a'^4}\Big\{&1+3e^2+3e'^2-3e\cos(nt-\omega)+4e'\cos(n't-\omega')\\&+7e'^2\cos 2(n't-\omega')-6ee'\cos(nt+n't-\omega-\omega')\\&-6ee'\cos(nt-n't-\omega+\omega')\end{aligned}\right\}. \tag{193}$$

Now equation (124) will give

$$\left.\begin{aligned}\sin\theta_1=\gamma\{1-e^2-\tfrac{1}{2}\gamma^2\}&\sin(nt-\Omega)+e\gamma\{1-\tfrac{5}{4}e^2-\tfrac{1}{2}\gamma^2\}\sin(2nt-\omega-\Omega)\\&-e\gamma\{1-\tfrac{3}{8}\gamma^2\}\sin(\omega-\Omega)+\tfrac{9}{8}e^2\gamma\sin(3nt-2\omega-\Omega)\\&+\tfrac{1}{8}\gamma\{e^2-\gamma^2\}\sin(nt-2\omega+\Omega)+\tfrac{1}{8}\gamma^3\sin(nt+2\omega-3\Omega)\\&-\tfrac{1}{8}e\gamma^3\sin 3(\omega-\Omega)+\tfrac{4}{3}e^3\gamma\sin(4nt-3\omega-\Omega)\\&+\tfrac{1}{8}e\gamma^3\sin(2nt+\omega-3\Omega)+\tfrac{1}{12}e\gamma\{e^2-\tfrac{3}{2}\gamma^2\}\sin(2nt-3\omega+\Omega)\end{aligned}\right\}; \tag{194}$$

$$\left.\begin{aligned}\cos\theta_1=1-\tfrac{1}{4}\gamma^2&+\tfrac{13}{64}\gamma^4+\tfrac{1}{4}\gamma^2\{1-4e^2-\tfrac{3}{4}\gamma^2\}\cos 2(nt-\Omega)\\&+\tfrac{1}{2}e\gamma^2\cos(3nt-\omega-2\Omega)-\tfrac{1}{2}e\gamma^2\cos(nt+\omega-2\Omega)\\&+\tfrac{13}{16}e^2\gamma^2\cos(4nt-2\omega-2\Omega)+\tfrac{3}{16}e^2\gamma^2\cos 2(\omega-\Omega)\\&+\tfrac{1}{16}\gamma^4\cos(2nt+2\omega-4\Omega)-\tfrac{1}{16}\gamma^4\cos 2(nt-\omega)-\tfrac{1}{64}\gamma^4\cos 4(nt-\Omega)\end{aligned}\right\}. \tag{195}$$

These equations give

$$\left.\begin{aligned}\cos^2\theta_1=1-\tfrac{1}{2}\gamma^2&+\tfrac{1}{4}\gamma^4+\tfrac{1}{2}\gamma^2\{1-4e^2-\gamma^2\}\cos 2(nt-\Omega)\\&+e\gamma^2\cos(3nt-\omega-2\Omega)-e\gamma^2\cos(nt+\omega-2\Omega)\\&+\tfrac{13}{8}e^2\gamma^2\cos(4nt-2\omega-2\Omega)+\tfrac{3}{8}e^2\gamma^2\cos 2(\omega-\Omega)\\&+\tfrac{1}{8}\gamma^4\cos(2nt+2\omega-4\Omega)-\tfrac{1}{8}\gamma^4\cos 2(nt-\omega)\end{aligned}\right\}; \tag{196}$$

$$\left.\begin{aligned}\cos^3\theta_1=1-\tfrac{3}{4}\gamma^2&+\tfrac{57}{64}\gamma^4+\tfrac{3}{4}\gamma^2\{1-4e^2-\tfrac{5}{4}\gamma^2\}\cos 2(nt-\Omega)\\&+\tfrac{3}{2}e\gamma^2\cos(3nt-\omega-2\Omega)-\tfrac{3}{2}e\gamma^2\cos(nt+\omega-2\Omega)\\&+\tfrac{39}{16}e^2\gamma^2\cos(4nt-2\omega-2\Omega)+\tfrac{9}{16}e^2\gamma^2\cos 2(\omega-\Omega)\\&+\tfrac{3}{16}\gamma^4\cos(2nt+2\omega-4\Omega)-\tfrac{3}{16}\gamma^4\cos 2(nt-\omega)+\tfrac{3}{64}\gamma^4\cos 4(nt-\Omega)\end{aligned}\right\}; \tag{197}$$

$$\sin\theta_1\cos\theta_1=$$

$$\left.\begin{aligned}&\gamma\{1-e^2-\tfrac{7}{8}\gamma^2\}\sin(nt-\Omega)\\&+e\gamma\{1-\tfrac{5}{4}e^2-\tfrac{7}{8}\gamma^2\}\sin(2nt-\omega-\Omega)-e\gamma\{1-\tfrac{3}{4}\gamma^2\}\sin(\omega-\Omega)\\&+\tfrac{9}{8}e^2\gamma\sin(3nt-2\omega-\Omega)+\tfrac{1}{8}\gamma\{e^2-\gamma^2\}\sin(nt-2\omega+\Omega)\\&+\tfrac{1}{8}\gamma^3\sin(nt+2\omega-3\Omega)+\tfrac{1}{8}\gamma^3\sin 3(nt-\Omega)-\tfrac{1}{8}e\gamma^3\sin 3(\omega-\Omega)\\&-\tfrac{1}{4}e\gamma^3\sin(2nt+\omega-3\Omega)+\tfrac{3}{8}e\gamma^3\sin(4nt-\omega-3\Omega)\\&+\tfrac{4}{3}e^3\gamma\sin(4nt-3\omega-\Omega)+\tfrac{1}{12}e\gamma\{e^2-\tfrac{3}{2}\gamma^2\}\sin(2nt-3\omega+\Omega)\end{aligned}\right\}. \tag{198}$$

If we multiply equations (189) and (195) together, we obtain

$$
\begin{aligned}
&\frac{r_1}{{r_1}'^3}\cos^2\theta_1 = \\
&\left.\begin{aligned}
&\frac{a}{a'^3}\Big\{ \{1 + \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{2}\gamma^2 + \tfrac{15}{8}e'^4 + \tfrac{3}{4}e^2e'^2 - \tfrac{3}{4}e'^2\gamma^2 - \tfrac{1}{4}e^2\gamma^2 + \tfrac{1}{2}\gamma^4\} \\
&- e\,\{1 - \tfrac{3}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{2}\gamma^2\} \cos (nt - \omega) \\
&+ 3e'\,\{1 + \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{2}\gamma^2\} \cos (n't - \omega') \\
&- \tfrac{1}{2}e^2\,\{1 - \tfrac{4}{3}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{2}\gamma^2 + \tfrac{1}{4}\tfrac{\gamma^4}{e^2}\} \cos 2\,(nt - \omega) \\
&+ \tfrac{9}{2}e'^2\,\{1 + \tfrac{1}{2}e^2 + \tfrac{7}{9}e'^2 - \tfrac{1}{2}\gamma^2\} \cos 2\,(n't - \omega') \\
&- \tfrac{3}{2}ee'\,\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{2}\gamma^2\} \cos (nt + n't - \omega - \omega') \\
&- \tfrac{3}{2}ee'\,\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{2}\gamma^2\} \cos (nt - n't - \omega + \omega') - \tfrac{3}{8}e^3 \cos 3\,(nt - \omega) \\
&+ \tfrac{53}{8}e'^3 \cos 3\,(n't - \omega') + \tfrac{1}{2}\gamma^2\,\{1 - \tfrac{7}{2}e^2 + \tfrac{3}{2}e'^2 - \gamma^2\} \cos 2\,(nt - \Omega) \\
&+ \tfrac{3}{4}e\gamma^2 \cos (3nt - \omega - 2\Omega) - \tfrac{5}{4}e\gamma^2 \cos (nt + \omega - 2\Omega) \\
&+ \tfrac{3}{4}e'\gamma^2 \cos (2nt + n't - \omega' - 2\Omega) + \tfrac{3}{4}e'\gamma^2 \cos (2nt - n't + \omega' - 2\Omega) \\
&- \tfrac{9}{4}ee'^2 \cos (nt + 2n't - \omega - 2\omega') - \tfrac{9}{4}ee'^2 \cos (nt - 2n't - \omega + 2\omega') \\
&- \tfrac{3}{4}e^2e' \cos (2nt + n't - 2\omega - \omega') - \tfrac{3}{4}e^2e' \cos (2nt - n't - 2\omega + \omega') \\
&- \tfrac{53}{16}ee'^3 \cos (nt + 3n't - \omega - 3\omega') - \tfrac{53}{16}ee'^3 \cos (nt - 3n't - \omega + 3\omega') \\
&- \tfrac{9}{16}e^3e' \cos (3nt + n't - 3\omega - \omega') - \tfrac{9}{16}e^3e' \cos (3nt - n't - 3\omega + \omega') \\
&- \tfrac{9}{8}e^2e'^2 \cos (2nt + 2n't - 2\omega - 2\omega') - \tfrac{9}{8}e^2e'^2 \cos (2nt - 2n't - 2\omega + 2\omega') \\
&+ \tfrac{9}{8}e'^2\gamma^2 \cos (2nt + 2n't - 2\omega' - 2\Omega) + \tfrac{9}{8}e'^2\gamma^2 \cos (2nt - 2n't + 2\omega - 2\Omega) \\
&+ \tfrac{9}{8}ee'\gamma^2 \cos (3nt + n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{9}{8}ee'\gamma^2 \cos (3nt - n't - \omega + \omega' - 2\Omega) \\
&- \tfrac{15}{8}ee'\gamma^2 \cos (nt + n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{15}{8}ee'\gamma^2 \cos (nt - n't + \omega + \omega' - 2\Omega) - \tfrac{1}{3}e^4 \cos 4\,(nt - \omega) \\
&+ \tfrac{77}{8}e'^4 \cos 4\,(n't - \omega') + e^2\gamma^2 \cos (4nt - 2\omega - 2\Omega) \\
&+ \tfrac{3}{4}e^2\gamma^2 \cos 2\,(\omega - \Omega) + \tfrac{1}{8}\gamma^4 \cos (2nt + 2\omega - 4\Omega)\Big\}
\end{aligned}\right\} . \quad (199)
\end{aligned}
$$

In like manner equations (191) and (195) will give

$$
\frac{r_1^{\,2}}{r_1^{\prime 3}}\cos\theta_1 =
$$

$$
\left.
\begin{aligned}
&\frac{a^2}{a'^3}\Big\{ \{1 + \tfrac{3}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{4}\gamma^2 + \tfrac{15}{8}e'^4 + \tfrac{9}{4}e^2e'^2 - \tfrac{3}{8}e^2\gamma^2 - \tfrac{3}{8}e'^2\gamma^2 + \tfrac{13}{64}\gamma^4\} \\
&\quad - 2e\,\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{4}\gamma^2\}\cos(nt - \omega) \\
&\quad + 3e'\,\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{4}\gamma^2\}\cos(n't - \omega') \\
&\quad - \tfrac{1}{2}e^2\,\{1 - \tfrac{1}{3}e^2 + \tfrac{3}{2}e'^2 - \tfrac{1}{4}\gamma^2 + \tfrac{1}{8}\tfrac{\gamma^4}{e^2}\}\cos 2(nt - \omega) \\
&\quad + \tfrac{9}{2}e'^2\,\{1 + \tfrac{3}{2}e^2 + \tfrac{7}{9}e'^2 - \tfrac{1}{4}\gamma^2\}\cos 2(n't - \omega') \\
&\quad - 3ee'\,\{1 - \tfrac{1}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{4}\gamma^2\}\cos(nt + n't - \omega - \omega') \\
&\quad - 3ee'\,\{1 - \tfrac{1}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{4}\gamma^2\}\cos(nt - n't - \omega + \omega') \\
&\quad + \tfrac{1}{4}\gamma^2\,\{1 - \tfrac{5}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\}\cos 2(nt - \Omega) - \tfrac{1}{4}e^3\cos 3(nt - \omega) \\
&\quad + \tfrac{53}{8}e'^3\cos 3(n't - \omega') + \tfrac{1}{4}e\gamma^2\cos(3nt - \omega - 2\Omega) \\
&\quad - \tfrac{3}{4}e\gamma^2\cos(nt + \omega - 2\Omega) + \tfrac{3}{8}e'\gamma^2\cos(2nt + n't - \omega' - 2\Omega) \\
&\quad + \tfrac{3}{8}e'\gamma^2\cos(2nt - n't + \omega' - 2\Omega) - \tfrac{9}{2}ee'^2\cos(nt + 2n't - \omega - 2\omega') \\
&\quad - \tfrac{9}{2}ee'^2\cos(nt - 2n't - \omega + 2\omega') - \tfrac{3}{4}e^2e'\cos(2nt + n't - 2\omega - \omega') \\
&\quad - \tfrac{3}{4}e^2e'\cos(2nt - n't - 2\omega + \omega') - \tfrac{53}{8}ee'^3\cos(nt + 3n't - \omega - 3\omega') \\
&\quad - \tfrac{53}{8}ee'^3\cos(nt - 3n't - \omega + 3\omega') - \tfrac{3}{8}e^3e'\cos(3nt + n't - 3\omega - \omega') \\
&\quad - \tfrac{3}{8}e^3e'\cos(3nt - n't - 3\omega + \omega') - \tfrac{9}{8}e^2e'^2\cos(2nt + 2n't - 2\omega - 2\omega') \\
&\quad - \tfrac{9}{8}e^2e'^2\cos(2nt - 2n't - 2\omega + 2\omega') + \tfrac{9}{16}e'^2\gamma^2\cos(2nt + 2n't - 2\omega' - 2\Omega) \\
&\quad + \tfrac{9}{16}e'^2\gamma^2\cos(2nt - 2n't + 2\omega' - 2\Omega) \\
&\quad + \tfrac{3}{8}ee'\gamma^2\cos(3nt + n't - \omega - \omega' - 2\Omega) \\
&\quad + \tfrac{3}{8}ee'\gamma^2\cos(3nt - n't - \omega + \omega' - 2\Omega) \\
&\quad - \tfrac{9}{8}ee'\gamma^2\cos(nt + n't + \omega - \omega' - 2\Omega) \\
&\quad - \tfrac{9}{8}ee'\gamma^2\cos(nt - n't + \omega + \omega' - 2\Omega) - \tfrac{1}{6}e^4\cos 4(nt - \omega) \\
&\quad + \tfrac{77}{8}e'^4\cos 4(n't - \omega') + \tfrac{1}{4}e^2\gamma^2\cos(4nt - 2\omega - 2\Omega) \\
&\quad + \tfrac{5}{8}e^2\gamma^2\cos 2(\omega - \Omega) + \tfrac{1}{16}\gamma^4\cos(2nt + 2\omega - 4\Omega) - \tfrac{1}{64}\gamma^4\cos 4(nt - \Omega)\Big\}
\end{aligned}
\right\} ; \quad (200)
$$

If we now multiply equation (191) by (198), we shall find

$$
\begin{aligned}
&\frac{r_1^{\,2}}{r_1'^{\,3}} \sin\theta_1 \cos\theta_1 = \\
&\left.\begin{aligned}
&\frac{a^2}{a'^3}\Big\{\gamma\,\{1 + \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{8}\gamma^2\}\sin(nt - \Omega) \\
&\quad - 2e\gamma\,\{1 + \tfrac{1}{4}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\}\sin(\omega - \Omega) \\
&\quad + \tfrac{3}{2}e'\gamma\,\{1 + \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{8}\gamma^2\}\sin(nt + n't - \omega' - \Omega) \\
&\quad + \tfrac{3}{2}e'\gamma\,\{1 + \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{8}\gamma^2\}\sin(nt - n't + \omega' - \Omega) \\
&\quad - \tfrac{1}{8}e^2\gamma\sin(3nt - 2\omega - \Omega) + \tfrac{1}{8}\gamma^3\sin(nt + 2\omega - 3\Omega) + \tfrac{1}{8}\gamma^3\sin 3(nt - \Omega) \\
&\quad + \tfrac{9}{4}e'^2\gamma\sin(nt + 2n't - 2\omega' - \Omega) + \tfrac{9}{4}e'^2\gamma\sin(nt - 2n't + 2\omega' - \Omega) \\
&\quad + 3ee'\gamma\sin(n't - \omega - \omega' + \Omega) - 3ee'\gamma\sin(n't + \omega - \omega' - \Omega) \\
&\quad - \tfrac{1}{8}\gamma\,\{5e^2 + \gamma^2\}\sin(nt - 2\omega + \Omega) + \tfrac{1}{2}e^3\gamma\sin(2nt - \omega - \Omega) \\
&\quad - \tfrac{1}{6}e^3\gamma\sin(4nt - 3\omega - \Omega) - \tfrac{1}{6}e^3\gamma\sin(2nt - 3\omega + \Omega) \\
&\quad + \tfrac{53}{16}e'^3\gamma\sin(nt + 3n't - 3\omega' - \Omega) + \tfrac{53}{16}e'^3\gamma\sin(nt - 3n't + 3\omega' - \Omega) \\
&\quad + \tfrac{9}{2}ee'^2\gamma\sin(2n't - \omega - 2\omega' + \Omega) - \tfrac{9}{2}ee'^2\gamma\sin(2n't + \omega - 2\omega' - \Omega) \\
&\quad - \tfrac{3}{16}e^2e'\gamma\sin(3nt + n't - 2\omega - \omega' - \Omega) \\
&\quad - \tfrac{3}{16}e^2e'\gamma\sin(3nt - n't - 2\omega + \omega' - \Omega) + \tfrac{1}{4}e\gamma^3\sin(4nt - \omega - 3\Omega) \\
&\quad - \tfrac{1}{4}e\gamma^3\sin 3(\omega - \Omega) + \tfrac{3}{16}e'\gamma^3\sin(3nt + n't - \omega' - 3\Omega) \\
&\quad + \tfrac{3}{16}e'\gamma^3\sin(3nt - n't + \omega' - 3\Omega) + \tfrac{3}{16}e'\gamma^3\sin(nt + n't + 2\omega - \omega' - 3\Omega) \\
&\quad + \tfrac{3}{16}e'\gamma^3\sin(nt - n't + 2\omega + \omega' - 3\Omega) - \tfrac{1}{2}e\gamma^3\sin(2nt + \omega - 3\Omega) \\
&\quad - \tfrac{3}{16}e'\gamma\,\{5e^2 + \gamma^2\}\sin(nt + n't - 2\omega - \omega' + \Omega) \\
&\quad - \tfrac{3}{16}e'\gamma\,\{5e^2 + \gamma^2\}\sin(nt - n't - 2\omega + \omega' + \Omega)\Big\}
\end{aligned}\right\} . \quad (201)
\end{aligned}
$$

From equations (192), (195) and (197) we get

$$
\begin{aligned}
&\frac{r_1^{\,2}}{r_1'^{\,4}}\{\tfrac{9}{2}\cos\theta_1 - \tfrac{45}{8}\cos^3\theta_1\} = \\
&\left.\begin{aligned}
&-\tfrac{9}{8}\frac{a^2}{a'^4}\Big\{1 + \tfrac{3}{2}e^2 + 3e'^2 - \tfrac{11}{4}\gamma^2 - 2e\cos(nt - \omega) + 4e'\cos(n't - \omega') \\
&\qquad - \tfrac{1}{2}e^2\cos 2(nt - \omega) + 7e'^2\cos 2(n't - \omega') - 4ee'\cos(nt + n't - \omega - \omega') \\
&\qquad - 4ee'\cos(nt - n't - \omega + \omega') + \tfrac{11}{4}\gamma^2\cos 2(nt - \Omega)\Big\}
\end{aligned}\right\} . \quad (202)
\end{aligned}
$$

Equations (192) and (197) give

$$\left.\begin{aligned}\frac{r_1^{\,2}}{r_1^{\prime 4}}\cos^3\theta_1=\frac{a^2}{a'^4}\Big\{&1+\tfrac{3}{2}e^2+3e'^2-\tfrac{3}{4}\gamma^2-2e\cos(nt-\omega)+4e'\cos(n't-\omega')\\&-\tfrac{1}{2}e^2\cos 2(nt-\omega)+7e'^2\cos 2(n't-\omega')\\&-4ee'\cos(nt+n't-\omega-\omega')-4ee'\cos(nt-n't-\omega+\omega')\\&+\tfrac{3}{4}\gamma^2\cos 2(nt-\Omega)\Big\}\end{aligned}\right\}. \qquad (203)$$

Equations (193), (195) and (197) will give

$$\left.\begin{aligned}&\frac{r_1^{\,3}}{r_1^{\prime 4}}\{\tfrac{15}{8}\cos^3\theta_1-\tfrac{3}{2}\cos\theta_1\}=\\&\tfrac{3}{8}\frac{a^3}{a'^4}\Big\{1+3e^2+3e'^2-\tfrac{15}{2}\gamma^2-3e\cos(nt-\omega)+4e'\cos(n't-\omega')\\&\quad+7e'^2\cos 2(n't-\omega')-6ee'\cos(nt+n't-\omega-\omega')\\&\quad-6ee'\cos(nt-n't-\omega+\omega')+\tfrac{15}{2}\gamma^2\cos 2(nt-\Omega)\Big\}\end{aligned}\right\}. \qquad (204)$$

Equations (193) and (197) will give

$$\left.\begin{aligned}\frac{r_1^{\,3}}{r_1^{\prime 4}}=\frac{a^3}{a'^4}\Big\{&1+3e^2+3e'^2-\tfrac{3}{4}\gamma^2-3e\cos(nt-\omega)+4e'\cos(n't-\omega')\\&+7e'^2\cos 2(n't-\omega')-6ee'\cos(nt+n't-\omega-\omega')\\&-6ee'\cos(nt-n't-\omega+\omega')+\tfrac{3}{4}\gamma^2\cos 2(nt-\Omega)\Big\}\end{aligned}\right\}. \qquad (205)$$

Equations (193), (194) and (196) will give the two following,

$$\left.\begin{aligned}&\frac{r_1^{\,3}}{r_1^{\prime 4}}\{\tfrac{15}{4}\cos^2\theta_1-1\}\sin\theta_1=\\&\tfrac{11}{4}\frac{a^3}{a'^4}\Big\{\gamma\sin(nt-\Omega)-\tfrac{1}{2}e\gamma\sin(2nt-\omega-\Omega)-\tfrac{5}{2}e\gamma\sin(\omega-\Omega)\\&\quad+2e'\gamma\sin(nt+n't-\omega'-\Omega)+2e'\gamma\sin(nt-n't+\omega'-\Omega)\Big\}\end{aligned}\right\}. \qquad (206)$$

$$\left.\begin{aligned}\frac{r_1^{\,3}}{r_1^{\prime 4}}\sin\theta_1\cos^2\theta_1=\frac{a^3}{a'^4}\Big\{&\gamma\sin(nt-\Omega)-\tfrac{1}{2}e\gamma\sin(2nt-\omega-\Omega)\\&-\tfrac{5}{2}e\gamma\sin(\omega-\Omega)+2e'\gamma\sin(nt+n't-\omega'-\Omega)\\&+2e'\gamma\sin(nt-n't+\omega'-\Omega)\Big\}\end{aligned}\right\} \qquad (207)$$

Equations (182) and (199) will give

$$\frac{r_1}{r_1'^3}\cos^2\theta_1\cos 2\,(v_1-v_1')=$$

$$\left.\begin{aligned}
&\frac{a}{a'^3}\left\{\begin{matrix}1-\tfrac{7}{2}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{47}{16}e^4+\tfrac{13}{16}e'^4\\ +\tfrac{35}{4}e^2e'^2+\tfrac{7}{4}e^2\gamma^2+\tfrac{5}{4}e'^2\gamma^2+\tfrac{3}{8}\gamma^4\end{matrix}\right\}\cos 2\,(nt-n't)\\
&+\tfrac{3}{2}e\,\{1-\tfrac{25}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-2n't-\omega)\\
&-\tfrac{5}{2}e\,\{1-\tfrac{33}{40}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-2n't+\omega)\\
&-\tfrac{1}{2}e'\,\{1-\tfrac{1}{8}e'^2-\tfrac{7}{2}e^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-n't-\omega')\\
&+\tfrac{7}{2}e'\,\{1-\tfrac{5}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-3n't+\omega')\\
&+2e^2\{1-\tfrac{19}{6}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2-\tfrac{1}{32}\tfrac{\gamma^4}{e^2}\}\cos(4nt-2n't-2\omega)\\
&+\tfrac{3}{2}e^2\{1-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{1}{24}\tfrac{\gamma^4}{e^2}\}\cos 2\,(n't-\omega)\\
&+\tfrac{17}{2}e'^2\,\{1-\tfrac{115}{51}e'^2-\tfrac{119}{34}e^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-4n't+2\omega')\\
&+\tfrac{1}{2}\gamma^2\,\{1-\gamma^2-\tfrac{1}{4}e^2-\tfrac{5}{2}e'^2\}\cos 2\,(n't-\Omega)\\
&+\tfrac{1}{4}\gamma^2\,\{1-\gamma^2-\tfrac{59}{4}e^2-\tfrac{5}{2}e'^2\}\cos(2nt-2n't+2\omega-2\Omega)\\
&-\tfrac{1}{4}\gamma^2\,\{1-\gamma^2-\tfrac{57}{4}e^2-\tfrac{5}{2}e'^2\}\cos(2nt-2n't-2\omega+2\Omega)\\
&+\tfrac{125}{48}e^3\cos(5nt-2n't-3\omega)+\tfrac{1}{48}e^3\cos(nt+2n't-3\omega)\\
&+\tfrac{1}{48}e'^3\cos(2nt+n't-3\omega')+\tfrac{845}{48}e'^3\cos(2nt-5n't+3\omega')\\
&+\tfrac{3}{8}e\gamma^2\cos(3nt-2n't+\omega-2\Omega)+\tfrac{3}{8}e\gamma^2\cos(nt-2n't-\omega+2\Omega)\\
&-\tfrac{3}{8}e\gamma^2\cos(3nt-2n't-3\omega+2\Omega)-\tfrac{5}{8}e\gamma^2\cos(nt-2n't+3\omega-2\Omega)\\
&+\tfrac{7}{4}e'\gamma^2\cos(3n't-\omega'-2\Omega)-\tfrac{1}{4}e'\gamma^2\cos(n't+\omega'-2\Omega)\\
&+\tfrac{1}{8}e'\gamma^2\cos(2nt-n't-2\omega-\omega'+2\Omega)\\
&+\tfrac{7}{8}e'\gamma^2\cos(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-\tfrac{1}{8}e'\gamma^2\cos(2nt-n't+2\omega-\omega'-2\Omega)\\
&-\tfrac{7}{8}e'\gamma^2\cos(2nt-3n't-2\omega+\omega'+2\Omega)\\
&-\tfrac{3}{4}ee'\,\{1-\tfrac{25}{8}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-n't-\omega-\omega')\\
&-\tfrac{35}{4}ee'\,\{1-\tfrac{231}{280}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-3n't+\omega+\omega')
\end{aligned}\right\}\quad(208)$$

(*Continued on the next page.*)

$$
\left.
\begin{aligned}
&+\tfrac{21}{4}ee'\{1-\tfrac{175}{56}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-3n't-\omega+\omega')\\
&+\tfrac{5}{4}ee'\{1-\tfrac{83}{40}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-n't+\omega-\omega')\\
&-\tfrac{85}{4}ee'^2\cos(nt-4n't+\omega+2\omega')+\tfrac{51}{4}ee'^2\cos(3nt-4n't-\omega+2\omega')\\
&-e^2e'\cos(4nt-n't-2\omega-\omega')+\tfrac{21}{4}e^2e'\cos(3n't-2\omega-\omega')\\
&+7e^2e'\cos(4nt-3n't-2\omega+\omega')-\tfrac{3}{4}e^2e'\cos(n't-2\omega+\omega')\\
&-\tfrac{1}{4}e\gamma^2\cos(nt+2n't-\omega-2\Omega)+\tfrac{27}{8}e^4\cos(6nt-2n't-4\omega)\\
&+\tfrac{1}{48}e^4\cos(2nt+2n't-4\omega)+\tfrac{1}{24}e'^4\cos(2nt+2n't-4\omega')\\
&+\tfrac{583}{16}e'^4\cos(2nt-6n't+4\omega')+\tfrac{1}{16}\gamma^4\cos(2nt-2n't+4\omega-4\Omega)\\
&+\tfrac{1}{16}\gamma^4\cos(2nt+2n't-4\Omega)-\tfrac{1}{2}e^2\gamma^2\cos(4nt-2n't-4\omega+2\Omega)\\
&+\tfrac{3}{8}e^2\gamma^2\cos(2n't-4\omega+2\Omega)+\tfrac{1}{2}e^2\gamma^2\cos(4nt-2n't-2\Omega)\\
&-\tfrac{1}{8}e^2\gamma^2\cos(2nt+2n't-2\omega-2\Omega)+\tfrac{1}{32}ee'^3\cos(3nt+n't-\omega-3\omega')\\
&-\tfrac{5}{96}ee'^3\cos(nt+n't+\omega-3\omega')-\tfrac{4225}{96}ee'^3\cos(nt-5n't+\omega+3\omega')\\
&+\tfrac{845}{32}ee'^3\cos(3nt-5n't-\omega+3\omega')+\tfrac{7}{96}e^3e'\cos(nt+3n't-3\omega-\omega')\\
&-\tfrac{1}{96}e^3e'\cos(nt+n't-3\omega+\omega')-\tfrac{125}{96}e^3e'\cos(5nt-n't-3\omega-\omega')\\
&+\tfrac{875}{96}e^3e'\cos(5nt-3n't-3\omega+\omega')+17e^2e'^2\cos(4nt-4n't-2\omega+2\omega')\\
&+\tfrac{51}{4}e^2e'^2\cos(4n't-2\omega-2\omega')+\tfrac{17}{8}e'^2\gamma^2\cos(2nt-4n't+2\omega+2\omega'-2\Omega)\\
&-\tfrac{17}{8}e'^2\gamma^2\cos(2nt-4n't-2\omega+2\omega'+2\Omega)+\tfrac{17}{4}e'^2\gamma^2\cos(4n't-2\omega'-2\Omega)\\
&+\tfrac{21}{16}ee'\gamma^2\cos(3nt-3n't+\omega+\omega'-2\Omega)\\
&-\tfrac{3}{16}ee'\gamma^2\cos(nt-n't-\omega-\omega'+2\Omega)\\
&+2ee'\gamma^2\cos(5nt-n't-\omega-\omega'-2\Omega)\\
&+\tfrac{21}{16}ee'\gamma^2\cos(nt-3n't-\omega+\omega'+2\Omega)\\
&-\tfrac{21}{16}ee'\gamma^2\cos(3nt-3n't-3\omega+\omega'+2\Omega)\\
&+\tfrac{5}{16}ee'\gamma^2\cos(nt-n't+3\omega-\omega'-2\Omega)\\
&+\tfrac{3}{16}ee'\gamma^2\cos(3nt-n't-3\omega-\omega'+2\Omega)\\
&-\tfrac{35}{16}ee'\gamma^2\cos(nt-3n't+3\omega+\omega'-2\Omega)\\
&+\tfrac{1}{8}ee'\gamma^2\cos(nt+n't-\omega+\omega'-2\Omega)\\
&-\tfrac{23}{8}ee'\gamma^2\cos(nt+3n't-\omega-\omega'-2\Omega)\\
&-\tfrac{3}{16}ee'\gamma^2\cos(3nt-n't+\omega-\omega'-2\Omega)\Big\}
\end{aligned}
\right\}\quad . \qquad (208)
$$

If we now multiply equation (191) by (198), we shall find

$$
\left.
\begin{aligned}
&\frac{r_1^{\,2}}{r_1^{\prime 3}}\sin\theta_1\cos\theta_1 = \\
&\frac{a^2}{a'^3}\Big\{\gamma\,\{1+\tfrac{1}{2}e^2+\tfrac{3}{2}e'^2-\tfrac{7}{8}\gamma^2\}\sin(nt-\Omega) \\
&\quad -2e\gamma\,\{1+\tfrac{1}{4}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(\omega-\Omega) \\
&\quad +\tfrac{3}{2}e'\gamma\,\{1+\tfrac{1}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{7}{8}\gamma^2\}\sin(nt+n't-\omega'-\Omega) \\
&\quad +\tfrac{3}{2}e'\gamma\,\{1+\tfrac{1}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{7}{8}\gamma^2\}\sin(nt-n't+\omega'-\Omega) \\
&\quad -\tfrac{1}{8}e^2\gamma\sin(3nt-2\omega-\Omega)+\tfrac{1}{8}\gamma^3\sin(nt+2\omega-3\Omega)+\tfrac{1}{8}\gamma^3\sin 3(nt-\Omega) \\
&\quad +\tfrac{9}{4}e'^2\gamma\sin(nt+2n't-2\omega'-\Omega)+\tfrac{9}{4}e'^2\gamma\sin(nt-2n't+2\omega'-\Omega) \\
&\quad +3ee'\gamma\sin(n't-\omega-\omega'+\Omega)-3ee'\gamma\sin(n't+\omega-\omega'-\Omega) \\
&\quad -\tfrac{1}{8}\gamma\,\{5e^2+\gamma^2\}\sin(nt-2\omega+\Omega)+\tfrac{1}{2}e^3\gamma\sin(2nt-\omega-\Omega) \\
&\quad -\tfrac{1}{6}e^3\gamma\sin(4nt-3\omega-\Omega)-\tfrac{1}{6}e^3\gamma\sin(2nt-3\omega+\Omega) \\
&\quad +\tfrac{53}{16}e'^3\gamma\sin(nt+3n't-3\omega'-\Omega)+\tfrac{53}{16}e'^3\gamma\sin(nt-3n't+3\omega'-\Omega) \\
&\quad +\tfrac{9}{2}ee'^2\gamma\sin(2n't-\omega-2\omega'+\Omega)-\tfrac{9}{2}ee'^2\gamma\sin(2n't+\omega-2\omega'-\Omega) \\
&\quad -\tfrac{3}{16}e^2e'\gamma\sin(3nt+n't-2\omega-\omega'-\Omega) \\
&\quad -\tfrac{3}{16}e^2e'\gamma\sin(3nt-n't-2\omega+\omega'-\Omega)+\tfrac{1}{4}e\gamma^3\sin(4nt-\omega-3\Omega) \\
&\quad -\tfrac{1}{4}e\gamma^3\sin 3(\omega-\Omega)+\tfrac{3}{16}e'\gamma^3\sin(3nt+n't-\omega'-3\Omega) \\
&\quad +\tfrac{3}{16}e'\gamma^3\sin(3nt-n't+\omega'-3\Omega)+\tfrac{3}{16}e'\gamma^3\sin(nt+n't+2\omega-\omega'-3\Omega) \\
&\quad +\tfrac{3}{16}e'\gamma^3\sin(nt-n't+2\omega+\omega'-3\Omega)-\tfrac{1}{2}e\gamma^3\sin(2nt+\omega-3\Omega) \\
&\quad -\tfrac{3}{16}e'\gamma\,\{5e^2+\gamma^2\}\sin(nt+n't-2\omega-\omega'+\Omega) \\
&\quad -\tfrac{3}{16}e'\gamma\,\{5e^2+\gamma^2\}\sin(nt-n't-2\omega+\omega'+\Omega)\Big\}
\end{aligned}
\right\}. \quad (201)
$$

From equations (192), (195) and (197) we get

$$
\left.
\begin{aligned}
&\frac{r_1^{\,2}}{r_1^{\prime 4}}\{\tfrac{3}{2}\cos\theta_1-\tfrac{45}{8}\cos^3\theta_1\} = \\
&-\tfrac{3}{8}\frac{a^2}{a'^4}\Big\{1+\tfrac{3}{2}e^2+3e'^2-\tfrac{11}{4}\gamma^2-2e\cos(nt-\omega)+4e'\cos(n't-\omega') \\
&\qquad -\tfrac{1}{2}e^2\cos 2(nt-\omega)+7e'^2\cos 2(n't-\omega')-4ee'\cos(nt+n't-\omega-\omega') \\
&\qquad -4ee'\cos(nt-n't-\omega+\omega')+\tfrac{11}{4}\gamma^2\cos 2(nt-\Omega)\Big\}
\end{aligned}
\right\}. \quad (202)
$$

$$\left.\begin{aligned}
&+\tfrac{5}{4}e^2e'\sin(n't-2\omega+\omega')+\tfrac{25}{24}e^3\sin(5nt-2n't-3\omega)\\
&+\tfrac{7}{24}e^3\sin(nt+2n't-3\omega)+\tfrac{1}{48}e'^3\sin(2nt+n't-3\omega')\\
&+\tfrac{845}{48}e'^3\sin(2nt-5n't+3\omega')+\tfrac{9}{8}e^4\sin(6nt-2n't-4\omega)\\
&+\tfrac{1}{16}e^4\sin(2nt+2n't-4\omega)+\tfrac{1}{24}e'^4\sin(2nt+2n't-4\omega')\\
&+\tfrac{533}{16}e'^4\sin(2nt-6n't+4\omega')+\tfrac{3}{128}\gamma^4\sin(6nt-2n't-4\Omega)\\
&+\tfrac{1}{16}\gamma^4\sin(2nt-2n't+4\omega-4\Omega)-\tfrac{1}{16}\gamma^4\sin(4nt-2n't+2\omega-4\Omega)\\
&-\tfrac{3}{128}\gamma^4\sin(2nt+2n't-4\Omega)-\tfrac{1}{4}e^2\gamma^2\sin(4nt-2n't-4\omega+2\Omega)\\
&-\tfrac{5}{8}e^2\gamma^2\sin(2n't-4\omega+2\Omega)-\tfrac{25}{32}e^2\gamma^2\sin(6nt-2n't-2\omega-2\Omega)\\
&+\tfrac{3}{32}e^2\gamma^2\sin(2nt+2n't-2\omega-2\Omega)-\tfrac{51}{16}e'^2\gamma^2\sin(4n't-2\omega'-2\Omega)\\
&+\tfrac{17}{2}e^2e'^2\sin(4nt-4n't-2\omega+2\omega')\\
&-\tfrac{17}{16}e'^2\gamma^2\sin(4nt-4n't+2\omega'-2\Omega)\\
&+\tfrac{17}{8}e'^2\gamma^2\sin(2nt-4n't+2\omega+2\omega'-2\Omega)\\
&-\tfrac{17}{8}e'^2\gamma^2\sin(2nt-4n't-2\omega+2\omega'+2\Omega)\\
&-\tfrac{85}{4}e^2e'^2\sin(4n't-2\omega-2\omega')+\tfrac{1}{48}ee'^3\sin(3nt+n't-\omega-3\omega')\\
&-\tfrac{1}{16}ee'^3\sin(nt+n't+\omega-3\omega')-\tfrac{845}{16}ee'^3\sin(nt-5n't+\omega+3\omega')\\
&+\tfrac{845}{48}ee'^3\sin(3nt-5n't-\omega+3\omega')+\tfrac{49}{48}e^3e'\sin(nt+3n't-3\omega-\omega')\\
&-\tfrac{7}{48}e^3e'\sin(nt+n't-3\omega+\omega')-\tfrac{25}{48}e^3e'\sin(5nt-n't-3\omega-\omega')\\
&+\tfrac{175}{48}e^3e'\sin(5nt-3n't-3\omega+\omega')\\
&-\tfrac{21}{16}ee'\gamma^2\sin(5nt-3n't-\omega+\omega'-2\Omega)\\
&+\tfrac{49}{16}ee'\gamma^2\sin(3nt-3n't+\omega+\omega'-2\Omega)\\
&-\tfrac{3}{16}ee'\gamma^2\sin(nt-n't-\omega-\omega'+2\Omega)\\
&+\tfrac{35}{16}ee'\gamma^2\sin(5nt-n't-\omega-\omega'-2\Omega)\\
&+\tfrac{53}{16}ee'\gamma^2\sin(nt+3n't-\omega-\omega'-2\Omega)\\
&-\tfrac{7}{16}ee'\gamma^2\sin(3nt-n't+\omega-\omega'-2\Omega)\\
&+\tfrac{3}{2}ee'\gamma^2\sin(nt-3n't-\omega+\omega'+2\Omega)\\
&-\tfrac{7}{8}ee'\gamma^2\sin(3nt-3n't-3\omega+\omega'+2\Omega)\\
&+\tfrac{3}{8}ee'\gamma^2\sin(nt-n't+3\omega-\omega'-2\Omega)\\
&+\tfrac{1}{8}ee'\gamma^2\sin(3nt-n't-3\omega-\omega'+2\Omega)\\
&-\tfrac{45}{16}ee'\gamma^2\sin(nt-3n't+3\omega+\omega'-2\Omega)\\
&-\tfrac{3}{16}ee'\gamma^2\sin(nt+n't-\omega+\omega'-2\Omega)\Big\}
\end{aligned}\right\}\ . \quad (209)$$

Equations (182) and (199) will give

$$\frac{r_1}{r_1'^3}\cos^2\theta_1\cos 2\,(v_1-v_1')=$$

$$\left.\begin{aligned}
&\frac{a}{a'^3}\left\{\begin{aligned}&1-\tfrac{7}{2}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{47}{16}e^4+\tfrac{13}{16}e'^4\\ &+\tfrac{35}{4}e^2e'^2+\tfrac{7}{4}e^2\gamma^2+\tfrac{5}{4}e'^2\gamma^2+\tfrac{3}{8}\gamma^4\end{aligned}\right\}\cos 2\,(nt-n't)\\
&+\tfrac{3}{2}e\,\{1-\tfrac{25}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-2n't-\omega)\\
&-\tfrac{5}{2}e\,\{1-\tfrac{33}{40}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-2n't+\omega)\\
&-\tfrac{1}{2}e'\,\{1-\tfrac{1}{8}e'^2-\tfrac{7}{2}e^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-n't-\omega')\\
&+\tfrac{7}{2}e'\,\{1-\tfrac{5}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-3n't+\omega')\\
&+2e^2\{1-\tfrac{19}{6}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2-\tfrac{1}{32}\tfrac{\gamma^4}{e^2}\}\cos(4nt-2n't-2\omega)\\
&+\tfrac{3}{2}e^2\{1-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{1}{24}\tfrac{\gamma^4}{e^2}\}\cos 2\,(n't-\omega)\\
&+\tfrac{17}{2}e'^2\,\{1-\tfrac{115}{51}e'^2-\tfrac{119}{34}e^2-\tfrac{1}{2}\gamma^2\}\cos(2nt-4n't+2\omega')\\
&+\tfrac{1}{2}\gamma^2\,\{1-\gamma^2-\tfrac{1}{4}e^2-\tfrac{5}{2}e'^2\}\cos 2\,(n't-\Omega)\\
&+\tfrac{1}{4}\gamma^2\,\{1-\gamma^2-\tfrac{59}{4}e^2-\tfrac{5}{2}e'^2\}\cos(2nt-2n't+2\omega-2\Omega)\\
&-\tfrac{1}{4}\gamma^2\,\{1-\gamma^2-\tfrac{57}{4}e^2-\tfrac{5}{2}e'^2\}\cos(2nt-2n't-2\omega+2\Omega)\\
&+\tfrac{125}{48}e^3\cos(5nt-2n't-3\omega)+\tfrac{1}{48}e^3\cos(nt+2n't-3\omega)\\
&+\tfrac{1}{48}e'^3\cos(2nt+n't-3\omega')+\tfrac{845}{48}e'^3\cos(2nt-5n't+3\omega')\\
&+\tfrac{3}{8}e\gamma^2\cos(3nt-2n't+\omega-2\Omega)+\tfrac{3}{8}e\gamma^2\cos(nt-2n't-\omega+2\Omega)\\
&-\tfrac{3}{8}e\gamma^2\cos(3nt-2n't-3\omega+2\Omega)-\tfrac{5}{8}e\gamma^2\cos(nt-2n't+3\omega-2\Omega)\\
&+\tfrac{7}{4}e'\gamma^2\cos(3n't-\omega'-2\Omega)-\tfrac{1}{4}e'\gamma^2\cos(n't+\omega'-2\Omega)\\
&+\tfrac{1}{8}e'\gamma^2\cos(2nt-n't-2\omega-\omega'+2\Omega)\\
&+\tfrac{7}{8}e'\gamma^2\cos(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-\tfrac{1}{8}e'\gamma^2\cos(2nt-n't+2\omega-\omega'-2\Omega)\\
&-\tfrac{7}{8}e'\gamma^2\cos(2nt-3n't-2\omega+\omega'+2\Omega)\\
&-\tfrac{3}{4}ee'\,\{1-\tfrac{25}{8}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-n't-\omega-\omega')\\
&-\tfrac{35}{4}ee'\,\{1-\tfrac{231}{280}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-3n't+\omega+\omega')
\end{aligned}\right\}\quad(208)$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&-\tfrac{7}{32}e'\gamma^3\sin(5nt-3n't+\omega'-3\Omega)-\tfrac{3}{32}e'\gamma^3\sin(nt+n't+\omega'-3\Omega)\\
&-\tfrac{1}{32}e'\gamma^3\sin(nt-n't-2\omega-\omega'+3\Omega)\\
&+\tfrac{21}{32}e'\gamma^3\sin(3nt-3n't+2\omega+\omega'-3\Omega)\\
&-\tfrac{8}{32}e'\gamma^3\sin(3nt-n't+2\omega-\omega'-3\Omega)\\
&+\tfrac{7}{32}e'\gamma^3\sin(nt-3n't-2\omega+\omega'+3\Omega)\\
&+\tfrac{1}{32}e'\gamma\,\{3\gamma^2-e^2\}\sin(3nt-n't-2\omega-\omega'+\Omega)\\
&+\tfrac{7}{32}e'\gamma\,\{43e^2-\gamma^2\}\sin(nt-3n't+2\omega+\omega'-\Omega)\\
&-\tfrac{1}{32}e'\gamma\,\{43e^2-\gamma^2\}\sin(nt-n't+2\omega-\omega'-\Omega)\\
&+\tfrac{7}{32}e'\gamma\,\{e^2-3\gamma^2\}\sin(3nt-3n't-2\omega+\omega'+\Omega)\\
&-17ee'^2\gamma\sin(2nt-4n't+\omega+2\omega'-\Omega)\\
&-\tfrac{17}{2}ee'^2\gamma\sin(4n't-\omega-2\omega'-\Omega)+\tfrac{17}{2}ee'^2\gamma\sin(4nt-4n't-\omega+2\omega'-\Omega)\\
&-\tfrac{25}{32}e^2e'\gamma\sin(5nt-n't-2\omega-\omega'-\Omega)\\
&+\tfrac{25}{32}e^2e'\gamma\sin(nt+3n't-2\omega-\omega'-\Omega)\\
&+\tfrac{175}{32}e^2e'\gamma\sin(5nt-3n't-2\omega+\omega'-\Omega)\\
&-\tfrac{5}{32}e^2e'\gamma\sin(nt+n't-2\omega+\omega'-\Omega)\Big\}
\end{aligned}\right\} \quad . \ (210)
$$

Equations (183) and (202) will give

$$
\frac{r_1^{\,2}}{r_1^{\prime 4}}\cos\theta_1\,\{\tfrac{9}{2}-\tfrac{45}{8}\cos^2\theta_1\}\cos(v_1-v_1')=
$$

$$
\left.\begin{aligned}
-\tfrac{9}{8}\frac{a^2}{a'^4}\Big\{&\{1+\tfrac{1}{2}e^2+2e'^2-\tfrac{11}{4}\gamma^2\}\cos(nt-n't)-2e\cos(n't-\omega)\\
&+e'\cos(nt-\omega')+3e'\cos(nt-2n't+\omega')-2ee'\cos(\omega-\omega')\\
&-6ee'\cos(2n't-\omega-\omega')-\tfrac{1}{8}e^2\cos(3nt-n't-2\omega)\\
&+\tfrac{5}{8}e^2\cos(nt+n't-2\omega)+\tfrac{11}{8}e'^2\cos(nt+n't-2\omega')\\
&+\tfrac{53}{8}e'^2\cos(nt-3n't+2\omega')-\tfrac{1}{8}\gamma^2\cos(nt-n't-2\omega+2\Omega)\\
&+\tfrac{1}{8}\gamma^2\cos(nt-n't+2\omega-2\Omega)+\tfrac{5}{4}\gamma^2\cos(3nt-n't-2\Omega)\\
&+\tfrac{3}{2}\gamma^2\cos(nt+n't-2\Omega)\Big\}
\end{aligned}\right\} \quad . \ (211)
$$

Equations (184) and (203) will give

$$\frac{r_1^3}{r_1'^4}\cos^3\theta_1\cos 3(v_1-v_1')=$$

$$\left.\begin{array}{l}\frac{a^3}{a'^4}\Big\{\{1-\tfrac{15}{2}e^2-6e'^2-\tfrac{3}{4}\gamma^2\}\cos 3(nt-n't)+2e\cos(4nt-3n't-\omega)\\ \quad -4e\cos(2nt-3n't+\omega)+5e'\cos(3nt-4n't+\omega')\\ \quad -e'\cos(3nt-2n't-\omega')+\tfrac{25}{8}e^2\cos(5nt-3n't-2\omega)\\ \quad +\tfrac{43}{8}e^2\cos(nt-3n't+2\omega)+\tfrac{1}{8}e'^2\cos(3nt-n't-2\omega')\\ \quad +\tfrac{127}{8}e'^2\cos(3nt-5n't+2\omega')+10ee'\cos(4nt-4n't-\omega+\omega')\\ \quad +4ee'\cos(2nt-2n't+\omega-\omega')-2ee'\cos(4nt-2n't-\omega-\omega')\\ \quad -20ee'\cos(2nt-4n't+\omega+\omega')+\tfrac{3}{4}\gamma^2\cos(nt-3n't+2\Omega)\\ \quad +\tfrac{3}{8}\gamma^2\cos(3nt-3n't+2\omega-2\Omega)-\tfrac{3}{8}\gamma^2\cos(3nt-3n't-2\omega+2\Omega)\Big\}\end{array}\right\}. \quad (212)$$

Equations (183) and (204) will give

$$\frac{r_1^3}{r_1'^4}\{\tfrac{15}{8}\cos^2\theta_1-\tfrac{3}{2}\}\cos\theta_1\sin(v_1-v_1')=$$

$$\left.\begin{array}{l}\tfrac{3}{8}\frac{a^3}{a'^4}\Big\{\{1+2e^2+2e'^2-\tfrac{13}{2}\gamma^2\}\sin(nt-n't)-\tfrac{1}{2}e\sin(2nt-n't-\omega)\\ \quad +\tfrac{3}{2}e\sin(n't-\omega)+e'\sin(nt-\omega')+3e'\sin(nt-2n't+\omega')\\ \quad -\tfrac{1}{2}ee'\sin(2nt-\omega-\omega')-\tfrac{9}{2}ee'\sin(\omega-\omega')+\tfrac{15}{2}ee'\sin(2n't-\omega-\omega')\\ \quad -\tfrac{3}{2}ee'\sin(2nt-2n't-\omega+\omega')-\tfrac{3}{8}e^2\sin(3nt-n't-2\omega)\\ \quad -\tfrac{11}{8}e^2\sin(nt+n't-2\omega)+\tfrac{11}{8}e'^2\sin(nt+n't-2\omega')\\ \quad +\tfrac{53}{8}e'^2\sin(nt-3n't+2\omega')-\tfrac{27}{8}\gamma^2\sin(nt+n't-2\Omega)\\ \quad +\tfrac{25}{8}\gamma^2\sin(3nt-n't-2\Omega)-\tfrac{1}{8}\gamma^2\sin(nt-n't-2\omega+2\Omega)\\ \quad +\tfrac{1}{8}\gamma^2\sin(nt-n't+2\omega-2\Omega)\Big\}.\end{array}\right\}. \quad (213)$$

Equations (184), (197) and (205) will give

$$\frac{r_1^{\,3}}{r_1'^{\,4}}\cos^3\theta_1\sin 3(v_1-v_1')=$$

$$\left.\begin{aligned}&\frac{a^3}{a'^4}\Big\{\{1-6e^2-6e'^2-\tfrac{3}{4}\gamma^2\}\sin 3(nt-n't)+\tfrac{3}{2}e\sin(4nt-3n't-\omega)\\&\quad-\tfrac{3}{2}e\sin(2nt-3n't+\omega)+5e'\sin(3nt-4n't+\omega')\\&\quad-e'\sin(3nt-2n't-\omega')+\tfrac{15}{8}e^2\sin(5nt-3n't-2\omega)\\&\quad+\tfrac{57}{8}e^2\sin(nt-3n't+2\omega)+\tfrac{1}{8}e'^2\sin(3nt-n't-2\omega')\\&\quad+\tfrac{127}{8}e'^2\sin(3nt-5n't+2\omega')+\tfrac{15}{2}ee'\sin(4nt-4n't-\omega+\omega')\\&\quad+\tfrac{3}{2}ee'\sin(2nt-2n't+\omega-\omega')-\tfrac{3}{2}ee'\sin(4nt-2n't-\omega-\omega')\\&\quad-\tfrac{45}{2}ee'\sin(2nt-4n't+\omega+\omega')+\tfrac{3}{4}\gamma^2\sin(nt-3n't+2\Omega)\\&\quad+\tfrac{3}{8}\gamma^2\sin(3nt-3n't+2\omega-2\Omega)-\tfrac{3}{8}\gamma^2\sin(3nt-3n't-2\omega+2\Omega)\Big\}\end{aligned}\right\} \quad (214)$$

Equations (183) and (206) will give

$$\frac{r_1^{\,3}}{r_1'^{\,4}}\{\tfrac{15}{4}\cos^2\theta_1-1\}\sin\theta_1\cos(v_1-v_1')=$$

$$\left.\begin{aligned}&\tfrac{11}{4}\frac{a^3}{a'^4}\Big\{\tfrac{1}{2}\gamma\sin(2nt-n't-\Omega)+\tfrac{1}{2}\gamma\sin(n't-\Omega)\\&\qquad+\tfrac{1}{4}e\gamma\sin(3nt-n't-\omega-\Omega)+\tfrac{3}{4}e\gamma\sin(nt-n't-\omega+\Omega)\\&\qquad-\tfrac{3}{4}e\gamma\sin(nt+n't-\omega-\Omega)-\tfrac{7}{4}e\gamma\sin(nt-n't+\omega-\Omega)\\&\qquad+\tfrac{1}{2}e'\gamma\sin(\omega'-\Omega)+\tfrac{3}{2}e'\gamma\sin(2nt-2n't+\omega'-\Omega)\\&\qquad+\tfrac{3}{2}e'\gamma\sin(2n't-\omega'-\Omega)+\tfrac{1}{2}e'\gamma\sin(2nt-\omega'-\Omega)\Big\}\end{aligned}\right\} \quad (215)$$

Lastly, equations (184) and (207) will give

$$\frac{r_1^{\,3}}{r_1'^{\,4}}\sin\theta_1\cos^2\theta_1\cos 3(v_1-v_1')=$$

$$\left.\begin{aligned}&\frac{a^3}{a'^4}\Big\{\tfrac{1}{2}\gamma\sin(4nt-3n't-\Omega)-\tfrac{1}{2}\gamma\sin(2nt-3n't+\Omega)\\&\quad+\tfrac{5}{4}e\gamma\sin(5nt-3n't-\omega-\Omega)-\tfrac{1}{4}e\gamma\sin(3nt-3n't-\omega+\Omega)\\&\quad-\tfrac{11}{4}e\gamma\sin(3nt-3n't+\omega-\Omega)+\tfrac{7}{4}e\gamma\sin(nt-3n't+\omega+\Omega)\\&\quad+\tfrac{5}{2}e'\gamma\sin(4nt-4n't+\omega'-\Omega)-\tfrac{5}{2}e'\gamma\sin(2nt-4n't+\omega'+\Omega)\\&\quad-\tfrac{1}{2}e'\gamma\sin(4nt-2n't-\omega'-\Omega)+\tfrac{1}{2}e'\gamma\sin(2nt-2n't-\omega'+\Omega)\Big\}\end{aligned}\right\} \quad (216)$$

We have now completed the development of all the functions which enter into the expressions of the forces $\left(\frac{dR}{dr}\right)$, $\left(\frac{dR}{dv}\right)$ and $\left(\frac{dR}{d\theta}\right)$, when we neglect the square of the disturbing force; and if we now put

$$\overline{m}^2 = m'\frac{a^3}{a'^3}, \tag{217}$$

and substitute the different developments in equations (167–169), we shall obtain the following expressions of the forces arising from the sun's attraction:

$$
-\left(\frac{dR}{dr}\right)=
$$

$$
\left.\begin{aligned}
\frac{\overline{m}^2}{2a^2}\Big\{ &\{1 + \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2 + \tfrac{15}{8}e'^4 - \tfrac{3}{4}e^2\gamma^2 - \tfrac{9}{4}e'^2\gamma^2 + \tfrac{3}{2}\gamma^4\} \\
&- e\{1 - \tfrac{3}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2\}\cos(nt-\omega) \\
&+ 3e'\{1 + \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{2}\gamma^2\}\cos(n't-\omega') \\
&- \tfrac{1}{2}e^2\{1 - \tfrac{3}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2 + \tfrac{3}{4}\tfrac{\gamma^4}{e^2}\}\cos 2(nt-\omega) \\
&+ \tfrac{9}{2}e'^2\{1 + \tfrac{1}{2}e^2 + \tfrac{7}{9}e'^2 - \tfrac{3}{2}\gamma^2\}\cos 2(n't-\omega') \\
&- \tfrac{3}{2}ee'\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{2}\gamma^2\}\cos(nt+n't-\omega-\omega') \\
&- \tfrac{3}{2}ee'\{1 - \tfrac{3}{8}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{2}\gamma^2\}\cos(nt-n't-\omega+\omega') \\
&+ \tfrac{3}{2}\gamma^2\{1 - \tfrac{7}{2}e^2 + \tfrac{3}{2}e'^2 - \gamma^2\}\cos 2(nt-\Omega) \\
&- \tfrac{3}{8}e^3\cos 3(nt-\omega) + \tfrac{53}{8}e'^3\cos 3(n't-\omega') \\
&- \tfrac{9}{4}ee'^2\cos(nt+2n't-\omega-2\omega') - \tfrac{9}{4}ee'^2\cos(nt-2n't-\omega+2\omega') \\
&- \tfrac{3}{4}e^2e'\cos(2nt+n't-2\omega-\omega') - \tfrac{3}{4}e^2e'\cos(2nt-n't-2\omega+\omega') \\
&+ \tfrac{9}{4}e\gamma^2\cos(3nt-\omega-2\Omega) - \tfrac{15}{4}e\gamma^2\cos(nt+\omega-2\Omega) \\
&+ \tfrac{9}{4}e'\gamma^2\cos(2nt+n't-\omega'-2\Omega) + \tfrac{9}{4}e'\gamma^2\cos(2nt-n't+\omega'-2\Omega) \\
&- \tfrac{53}{16}ee'^3\cos(nt+3n't-\omega-3\omega') - \tfrac{53}{16}ee'^3\cos(nt-3n't-\omega+3\omega') \\
&- \tfrac{9}{16}e^3e'\cos(3nt+n't-3\omega-\omega') - \tfrac{9}{16}e^3e'\cos(3nt-n't-3\omega+\omega') \\
&- \tfrac{9}{8}e^2e'^2\cos(2nt+2n't-2\omega-2\omega') - \tfrac{9}{8}e^2e'^2\cos(2nt-2n't-2\omega+2\omega') \\
&+ \tfrac{27}{8}e'^2\gamma^2\cos(2nt+2n't-2\omega'-2\Omega) + \tfrac{27}{8}e'^2\gamma^2\cos(2nt-2n't+2\omega'-2\Omega) \\
&- \tfrac{1}{3}e^4\cos 4(nt-\omega) + \tfrac{77}{8}e'^4\cos 4(n't-\omega') + 3e^2\gamma^2\cos(4nt-2\omega-2\Omega) \\
&+ \tfrac{9}{4}e^2\gamma^2\cos 2(\omega-\Omega) + \tfrac{9}{2}ee'\gamma^2\cos(3nt-n't-\omega+\omega'-2\Omega) \\
&- \tfrac{9}{2}ee'\gamma^2\cos(nt+n't+\omega-\omega'-2\Omega) + \tfrac{27}{8}ee'\gamma^2\cos(3nt+n't-\omega-\omega'-2\Omega) \\
&- \tfrac{45}{8}ee'\gamma^2\cos(nt-n't+\omega+\omega'-2\Omega) + \tfrac{3}{8}\gamma^4\cos(2nt+2\omega-4\Omega) \\
&+ 3\{1 - \tfrac{7}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2 + \tfrac{47}{16}e^4 + \tfrac{85}{16}e'^4 \\
&\qquad + 8e^2e'^2 + \tfrac{7}{4}e^2\gamma^2 + \tfrac{5}{4}e'^2\gamma^2 + \tfrac{3}{8}\gamma^4\}\cos 2(nt-n't) \\
&+ \tfrac{9}{2}e\{1 - \tfrac{25}{8}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2\}\cos(3nt-2n't-\omega)
\end{aligned}\right\} \tag{218}
$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&-\tfrac{15}{2}e\left\{1-\tfrac{33}{40}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-2n't+\omega)\\
&-\tfrac{3}{2}e'\left\{1-\tfrac{7}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-n't-\omega')\\
&+\tfrac{21}{2}e'\left\{1-\tfrac{123}{56}e'^2-\tfrac{7}{2}e^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-3n't+\omega')\\
&+6e^2\left\{1-\tfrac{19}{6}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2-\tfrac{1}{32}\tfrac{\gamma^4}{e^2}\right\}\cos(4nt-2n't-2\omega)\\
&+\tfrac{9}{2}e^2\left\{1-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{1}{24}\tfrac{\gamma^4}{e^2}\right\}\cos 2(n't-\omega)\\
&+\tfrac{51}{2}e'^2\left\{1-\tfrac{119}{34}e^2-\tfrac{115}{51}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-4n't+2\omega')\\
&+\tfrac{3}{2}\gamma^2\left\{1-\gamma^2-\tfrac{1}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos 2(n't-\Omega)\\
&+\tfrac{3}{4}\gamma^2\left\{1-\gamma^2-\tfrac{59}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos(2nt-2n't+2\omega-2\Omega)\\
&-\tfrac{3}{4}\gamma^2\left\{1-\gamma^2-\tfrac{57}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos(2nt-2n't-2\omega+2\Omega)\\
&-\tfrac{9}{4}ee'\left\{1-\tfrac{25}{8}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(3nt-n't-\omega-\omega')\\
&-\tfrac{105}{4}ee'\left\{1-\tfrac{231}{280}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-3n't+\omega+\omega')\\
&+\tfrac{125}{16}e^3\cos(5nt-2n't-3\omega)\\
&+\tfrac{63}{4}ee'\left\{1-\tfrac{175}{56}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(3nt-3n't-\omega+\omega')\\
&+\tfrac{15}{4}ee'\left\{1-\tfrac{33}{40}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-n't+\omega-\omega')\\
&-\tfrac{1}{16}e^3\cos(nt+2n't-3\omega)+\tfrac{1}{16}e'^3\cos(2nt+n't-3\omega')\\
&+\tfrac{845}{16}e'^3\cos(2nt-5n't+3\omega')+\tfrac{3}{8}e\gamma^2\cos(3nt-2n't+\omega-2\Omega)\\
&+\tfrac{3}{8}e\gamma^2\cos(nt-2n't-\omega+2\Omega)-\tfrac{3}{8}e\gamma^2\cos(3nt-2n't-3\omega+2\Omega)\\
&-\tfrac{15}{8}e\gamma^2\cos(nt-2n't+3\omega-2\Omega)+\tfrac{21}{4}e'\gamma^2\cos(3n't-\omega'-2\Omega)\\
&-\tfrac{3}{4}e'\gamma^2\cos(n't+\omega'-2\Omega)+\tfrac{3}{8}e'\gamma^2\cos(2nt-n't-2\omega-\omega'+2\Omega)\\
&+\tfrac{21}{8}e'\gamma^2\cos(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-\tfrac{3}{8}e'\gamma^2\cos(2nt-n't+2\omega-\omega'-2\Omega)\\
&-\tfrac{255}{4}ee'^2\cos(nt-4n't+\omega+2\omega')-\tfrac{21}{8}e'\gamma^2\cos(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+\tfrac{153}{4}ee'^2\cos(3nt-4n't-\omega+2\omega')-3e^2e'\cos(4nt-n't-2\omega-\omega')\\
&+\tfrac{63}{4}e^2e'\cos(3n't-2\omega-\omega')+21e^2e'\cos(4nt-3n't-2\omega+\omega')\\
&-\tfrac{9}{4}e^2e'\cos(n't-2\omega+\omega')-\tfrac{3}{4}e\gamma^2\cos(nt+2n't-\omega-2\Omega)\\
&+\tfrac{81}{8}e^4\cos(6nt-2n't-4\omega)+\tfrac{1}{16}e^4\cos(2nt+2n't-4\omega)\\
&+\tfrac{1}{8}e'^4\cos(2nt+2n't-4\omega')+\tfrac{1599}{16}e'^4\cos(2nt-6n't+4\omega')\\
&+\tfrac{3}{16}\gamma^4\cos(2nt-2n't+4\omega-4\Omega)+\tfrac{3}{16}\gamma^4\cos(2nt+2n't-4\Omega)\\
&-\tfrac{3}{2}e^2\gamma^2\cos(4nt-2n't-4\omega+2\Omega)+\tfrac{9}{8}e^2\gamma^2\cos(2n't-4\omega+2\Omega)\\
&+\tfrac{3}{2}e^2\gamma^2\cos(4nt-2n't-2\Omega)-\tfrac{3}{8}e^2\gamma^2\cos(2nt+2n't-2\omega-2\Omega)\\
&+\tfrac{3}{32}ee'^3\cos(3nt+n't-\omega-3\omega')-\tfrac{5}{32}ee'^3\cos(nt+n't+\omega-3\omega')\\
&-\tfrac{4225}{32}ee'^3\cos(nt-5n't+\omega+3\omega')+\tfrac{647}{32}ee'^3\cos(3nt-5n't-\omega+3\omega')
\end{aligned}\right\}\quad(218)
$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&+\tfrac{7}{32}e^3e'\cos(nt+3n't-3\omega-\omega')-\tfrac{1}{32}e^3e'\cos(nt+n't-3\omega+\omega')\\
&-\tfrac{125}{32}e^3e'\cos(5nt-n't-3\omega-\omega')+\tfrac{2535}{32}e^3e'\cos(5nt-3n't-3\omega+\omega')\\
&+51e^2e'^2\cos(4nt-4n't-2\omega+2\omega')+\tfrac{153}{4}e^2e'^2\cos(4n't-2\omega-2\omega')\\
&+\tfrac{51}{8}e'^2\gamma^2\cos(2nt-4n't+2\omega+2\omega'-2\Omega)+\tfrac{51}{4}e'^2\gamma^2\cos(4n't-2\omega'-2\Omega)\\
&-\tfrac{51}{8}e'^2\gamma^2\cos(2nt-4n't-2\omega+2\omega'+2\Omega)\\
&+\tfrac{63}{16}ee'\gamma^2\cos(3nt-3n't+\omega+\omega'-2\Omega)\\
&-\tfrac{9}{16}ee'\gamma^2\cos(nt-n't-\omega-\omega'+2\Omega)\\
&+6ee'\gamma^2\cos(5nt-n't-\omega-\omega'-2\Omega)\\
&+\tfrac{63}{16}ee'\gamma^2\cos(nt-3n't-\omega+\omega'+2\Omega)\\
&-\tfrac{63}{16}ee'\gamma^2\cos(3nt-3n't-3\omega+\omega'+2\Omega)\\
&+\tfrac{15}{16}ee'\gamma^2\cos(nt-n't+3\omega-\omega'-2\Omega)\\
&+\tfrac{9}{16}ee'\gamma^2\cos(3nt-n't-3\omega-\omega'+2\Omega)\\
&-\tfrac{9}{16}ee'\gamma^2\cos(3nt-n't+\omega-\omega'-2\Omega)\\
&-\tfrac{105}{16}ee'\gamma^2\cos(nt-3n't+3\omega+\omega'-2\Omega)\\
&+\tfrac{3}{8}ee'\gamma^2\cos(nt+n't-\omega+\omega'-2\Omega)\\
&-\tfrac{69}{8}ee'\gamma^2\cos(nt+3n't-\omega-\omega'-2\Omega)\Big\}
\end{aligned}\right\} \quad (218)
$$

$$
\left.\begin{aligned}
&+\tfrac{9}{8}\frac{\overline{m}^2}{aa'}\Big\{\{1+\tfrac{1}{2}e^2+2e'^2-\tfrac{11}{4}\gamma^2\}\cos(nt-n't)+e'\cos(nt-\omega')\\
&+3e'\cos(nt-2n't+\omega')-2e\cos(n't-\omega)-\tfrac{1}{8}e^2\cos(3nt-n't-2\omega)\\
&+\tfrac{5}{8}e^2\cos(nt+n't-2\omega)+\tfrac{11}{8}e'^2\cos(nt+n't-2\omega')\\
&+\tfrac{53}{8}e'^2\cos(nt-3n't+2\omega')-2ee'\cos(\omega-\omega')\\
&-6ee'\cos(2n't-\omega-\omega')-\tfrac{1}{8}\gamma^2\cos(nt-n't-2\omega+2\Omega)\\
&+\tfrac{1}{8}\gamma^2\cos(nt-n't+2\omega-2\Omega)+\tfrac{5}{4}\gamma^2\cos(3nt-n't-2\Omega)\\
&+\tfrac{3}{2}\gamma^2\cos(nt+n't-2\Omega)+\tfrac{5}{8}\{1-\tfrac{15}{2}e^2-6e'^2-\tfrac{3}{4}\gamma^2\}\cos 3(nt-n't)\\
&+\tfrac{10}{3}e\cos(4nt-3n't-\omega)-\tfrac{20}{3}e\cos(2nt-3n't+\omega)\\
&+\tfrac{25}{3}e'\cos(3nt-4n't+\omega')-\tfrac{5}{3}e'\cos(3nt-2n't-\omega')\\
&+\tfrac{5}{24}e'^2\cos(3nt-n't-2\omega')+\tfrac{125}{24}e^2\cos(5nt-3n't-2\omega)\\
&+\tfrac{215}{24}e^2\cos(nt-3n't+2\omega)+\tfrac{635}{24}e'^2\cos(3nt-5n't+2\omega')\\
&+\tfrac{50}{3}ee'\cos(4nt-4n't-\omega+\omega')+\tfrac{20}{3}ee'\cos(2nt-2n't+\omega-\omega')\\
&-\tfrac{10}{3}ee'\cos(4nt-2n't-\omega-\omega')-\tfrac{100}{3}ee'\cos(2nt-4n't+\omega+\omega')\\
&+\tfrac{5}{4}\gamma^2\cos(nt-3n't+2\Omega)+\tfrac{5}{8}\gamma^2\cos(3nt-3n't+2\omega-2\Omega)\\
&-\tfrac{5}{8}\gamma^2\cos(3nt-3n't-2\omega+2\Omega)\Big\}
\end{aligned}\right\} \quad (218)
$$

Equations (184), (197) and (205) will give

$$\frac{r_1^3}{r_1'^4}\cos^3\theta_1 \sin 3(v_1 - v_1') =$$

$$\left.\begin{aligned}
&\frac{a^3}{a'^4}\Big\{ \{1 - 6e^2 - 6e'^2 - \tfrac{3}{4}\gamma^2\} \sin 3(nt - n't) + \tfrac{3}{2} e \sin(4nt - 3n't - \omega) \\
&\quad - \tfrac{3}{2} e \sin(2nt - 3n't + \omega) + 5e' \sin(3nt - 4n't + \omega') \\
&\quad - e' \sin(3nt - 2n't - \omega') + \tfrac{15}{8} e^2 \sin(5nt - 3n't - 2\omega) \\
&\quad + \tfrac{57}{8} e^2 \sin(nt - 3n't + 2\omega) + \tfrac{1}{8} e'^2 \sin(3nt - n't - 2\omega') \\
&\quad + \tfrac{127}{8} e'^2 \sin(3nt - 5n't + 2\omega') + \tfrac{15}{2} ee' \sin(4nt - 4n't - \omega + \omega') \\
&\quad + \tfrac{3}{2} ee' \sin(2nt - 2n't + \omega - \omega') - \tfrac{3}{2} ee' \sin(4nt - 2n't - \omega - \omega') \\
&\quad - \tfrac{45}{2} ee' \sin(2nt - 4n't + \omega + \omega') + \tfrac{3}{4}\gamma^2 \sin(nt - 3n't + 2\Omega) \\
&\quad + \tfrac{3}{8}\gamma^2 \sin(3nt - 3n't + 2\omega - 2\Omega) - \tfrac{3}{8}\gamma^2 \sin(3nt - 3n't - 2\omega + 2\Omega) \Big\}
\end{aligned}\right\} . \quad (214)$$

Equations (183) and (206) will give

$$\frac{r_1^3}{r_1'^4}\{\tfrac{15}{4}\cos^2\theta_1 - 1\} \sin\theta_1 \cos(v_1 - v_1') =$$

$$\left.\begin{aligned}
&\tfrac{11}{4}\frac{a^3}{a'^4}\Big\{ \tfrac{1}{2}\gamma \sin(2nt - n't - \Omega) + \tfrac{1}{2}\gamma \sin(n't - \Omega) \\
&\qquad + \tfrac{1}{4} e\gamma \sin(3nt - n't - \omega - \Omega) + \tfrac{3}{4} e\gamma \sin(nt - n't - \omega + \Omega) \\
&\qquad - \tfrac{3}{4} e\gamma \sin(nt + n't - \omega - \Omega) - \tfrac{7}{4} e\gamma \sin(nt - n't + \omega - \Omega) \\
&\qquad + \tfrac{1}{2} e'\gamma \sin(\omega' - \Omega) + \tfrac{3}{2} e'\gamma \sin(2nt - 2n't + \omega' - \Omega) \\
&\qquad + \tfrac{3}{2} e'\gamma \sin(2n't - \omega' - \Omega) + \tfrac{1}{2} e'\gamma \sin(2nt - \omega' - \Omega) \Big\}
\end{aligned}\right\} . \quad (215)$$

Lastly, equations (184) and (207) will give

$$\frac{r_1^3}{r_1'^4}\sin\theta_1 \cos^2\theta_1 \cos 3(v_1 - v_1') =$$

$$\left.\begin{aligned}
&\frac{a^3}{a'^4}\Big\{ \tfrac{1}{2}\gamma \sin(4nt - 3n't - \Omega) - \tfrac{1}{2}\gamma \sin(2nt - 3n't + \Omega) \\
&\quad + \tfrac{5}{4} e\gamma \sin(5nt - 3n't - \omega - \Omega) - \tfrac{1}{4} e\gamma \sin(3nt - 3n't - \omega + \Omega) \\
&\quad - \tfrac{11}{4} e\gamma \sin(3nt - 3n't + \omega - \Omega) + \tfrac{7}{4} e\gamma \sin(nt - 3n't + \omega + \Omega) \\
&\quad + \tfrac{5}{2} e'\gamma \sin(4nt - 4n't + \omega' - \Omega) - \tfrac{5}{2} e'\gamma \sin(2nt - 4n't + \omega' + \Omega) \\
&\quad - \tfrac{1}{2} e'\gamma \sin(4nt - 2n't - \omega' - \Omega) + \tfrac{1}{2} e'\gamma \sin(2nt - 2n't - \omega' + \Omega) \Big\}
\end{aligned}\right\} . \quad (216)$$

$$
\left.
\begin{array}{l}
+\frac{1}{48}e'^3\sin(2nt+n't-3\omega')+\frac{845}{48}e'^3\sin(2nt-5n't+3\omega') \\
+\frac{9}{8}e^4\sin(6nt-2n't-4\omega)+\frac{1}{16}e^4\sin(2nt+2n't-4\omega) \\
+\frac{1}{24}e'^4\sin(2nt+2n't-4\omega')+\frac{533}{16}e'^4\sin(2nt-6n't+4\omega') \\
+\frac{3}{128}\gamma^4\sin(6nt-2n't-4\Omega)+\frac{1}{16}\gamma^4\sin(2nt-2n't+4\omega-4\Omega) \\
-\frac{1}{16}\gamma^4\sin(4nt-2n't+2\omega-4\Omega)-\frac{3}{128}\gamma^4\sin(2nt+2n't-4\Omega) \\
-\frac{1}{4}e^2\gamma^2\sin(4nt-2n't-4\omega+2\Omega)-\frac{5}{8}e^2\gamma^2\sin(2n't-4\omega+2\Omega) \\
-\frac{25}{32}e^2\gamma^2\sin(6nt-2n't-2\omega-2\Omega)+\frac{3}{32}e^2\gamma^2\sin(2nt+2n't-2\omega-2\Omega) \\
-\frac{51}{16}e'^2\gamma^2\sin(4n't-2\omega'-2\Omega)+\frac{17}{2}e^2e'^2\sin(4nt-4n't-2\omega+2\omega') \\
-\frac{17}{16}e'^2\gamma^2\sin(4nt-4n't+2\omega'-2\Omega) \\
+\frac{17}{8}e'^2\gamma^2\sin(2nt-4n't+2\omega+2\omega'-2\Omega) \\
-\frac{17}{8}e'^2\gamma^2\sin(2nt-4n't-2\omega+2\omega'+2\Omega)-\frac{85}{4}e^2e'^2\sin(4n't-2\omega-2\omega') \\
+\frac{1}{48}ee'^3\sin(3nt+n't-\omega-3\omega')-\frac{1}{16}ee'^3\sin(nt+n't+\omega-3\omega') \\
-\frac{845}{16}ee'^3\sin(nt-5n't+\omega+3\omega')+\frac{845}{48}ee'^3\sin(3nt-5n't-\omega+3\omega') \\
+\frac{49}{48}e^3e'\sin(nt+3n't-3\omega-\omega')-\frac{7}{48}e^3e'\sin(nt+n't-3\omega+\omega') \\
-\frac{25}{48}e^3e'\sin(5nt-n't-3\omega-\omega')+\frac{175}{48}e^3e'\sin(5nt-3n't-3\omega+\omega') \\
-\frac{21}{16}ee'\gamma^2\sin(5nt-3n't-\omega+\omega'-2\Omega) \\
-\frac{3}{16}ee'\gamma^2\sin(nt+n't-\omega+\omega'-2\Omega) \\
+\frac{49}{16}ee'\gamma^2\sin(3nt-3n't+\omega+\omega'-2\Omega) \\
-\frac{3}{16}ee'\gamma^2\sin(nt-n't-\omega-\omega'+2\Omega) \\
+\frac{35}{16}ee'\gamma^2\sin(5nt-n't-\omega-\omega'-2\Omega) \\
+\frac{53}{16}ee'\gamma^2\sin(nt+3n't-\omega-\omega'-2\Omega) \\
-\frac{7}{16}ee'\gamma^2\sin(3nt-n't+\omega-\omega'-2\Omega) \\
+\frac{3}{2}ee'\gamma^2\sin(nt-3n't-\omega+\omega'+2\Omega) \\
-\frac{1}{8}ee'\gamma^2\sin(3nt-3n't-3\omega+\omega'+2\Omega) \\
+\frac{3}{8}ee'\gamma^2\sin(nt-n't+3\omega-\omega'-2\Omega) \\
+\frac{1}{8}ee'\gamma^2\sin(3nt-n't-3\omega-\omega'+2\Omega) \\
-\frac{45}{16}ee'\gamma^2\sin(nt-3n't+3\omega+\omega'-2\Omega)\Bigg\}
\end{array}
\right\} \quad (219)
$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&-\tfrac{15}{2}e\left\{1-\tfrac{33}{40}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-2n't+\omega)\\
&-\tfrac{3}{2}e'\left\{1-\tfrac{7}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-n't-\omega')\\
&+\tfrac{21}{2}e'\left\{1-\tfrac{123}{56}e'^2-\tfrac{7}{2}e^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-3n't+\omega')\\
&+6e^2\left\{1-\tfrac{19}{6}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2-\tfrac{1}{32}\tfrac{\gamma^4}{e^2}\right\}\cos(4nt-2n't-2\omega)\\
&+\tfrac{3}{2}e^2\left\{1-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2+\tfrac{1}{24}\tfrac{\gamma^4}{e^2}\right\}\cos 2(n't-\omega)\\
&+\tfrac{51}{2}e'^2\left\{1-\tfrac{119}{34}e^2-\tfrac{115}{51}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(2nt-4n't+2\omega')\\
&+\tfrac{3}{2}\gamma^2\left\{1-\gamma^2-\tfrac{1}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos 2(n't-\Omega)\\
&+\tfrac{3}{4}\gamma^2\left\{1-\gamma^2-\tfrac{59}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos(2nt-2n't+2\omega-2\Omega)\\
&-\tfrac{3}{4}\gamma^2\left\{1-\gamma^2-\tfrac{57}{4}e^2-\tfrac{5}{2}e'^2\right\}\cos(2nt-2n't-2\omega+2\Omega)\\
&-\tfrac{9}{4}ee'\left\{1-\tfrac{25}{8}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(3nt-n't-\omega-\omega')\\
&-\tfrac{105}{4}ee'\left\{1-\tfrac{281}{280}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-3n't+\omega+\omega')\\
&+\tfrac{125}{16}e^3\cos(5nt-2n't-3\omega)\\
&+\tfrac{63}{4}ee'\left\{1-\tfrac{175}{56}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(3nt-3n't-\omega+\omega')\\
&+\tfrac{15}{4}ee'\left\{1-\tfrac{33}{40}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\right\}\cos(nt-n't+\omega-\omega')\\
&-\tfrac{1}{16}e^3\cos(nt+2n't-3\omega)+\tfrac{1}{16}e'^3\cos(2nt+n't-3\omega')\\
&+\tfrac{845}{16}e'^3\cos(2nt-5n't+3\omega')+\tfrac{3}{8}e\gamma^2\cos(3nt-2n't+\omega-2\Omega)\\
&+\tfrac{3}{8}e\gamma^2\cos(nt-2n't-\omega+2\Omega)-\tfrac{3}{8}e\gamma^2\cos(3nt-2n't-3\omega+2\Omega)\\
&-\tfrac{15}{8}e\gamma^2\cos(nt-2n't+3\omega-2\Omega)+\tfrac{21}{4}e'\gamma^2\cos(3n't-\omega'-2\Omega)\\
&-\tfrac{3}{4}e'\gamma^2\cos(n't+\omega'-2\Omega)+\tfrac{3}{8}e'\gamma^2\cos(2nt-n't-2\omega-\omega'+2\Omega)\\
&+\tfrac{21}{8}e'\gamma^2\cos(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-\tfrac{3}{8}e'\gamma^2\cos(2nt-n't+2\omega-\omega'-2\Omega)\\
&-\tfrac{255}{4}ee'^2\cos(nt-4n't+\omega+2\omega')-\tfrac{21}{8}e'\gamma^2\cos(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+\tfrac{153}{4}ee'^2\cos(3nt-4n't-\omega+2\omega')-3e^2e'\cos(4nt-n't-2\omega-\omega')\\
&+\tfrac{63}{4}e^2e'\cos(3n't-2\omega-\omega')+21e^2e'\cos(4nt-3n't-2\omega+\omega')\\
&-\tfrac{9}{4}e^2e'\cos(n't-2\omega+\omega')-\tfrac{3}{4}e\gamma^2\cos(nt+2n't-\omega-2\Omega)\\
&+\tfrac{81}{8}e^4\cos(6nt-2n't-4\omega)+\tfrac{1}{16}e^4\cos(2nt+2n't-4\omega)\\
&+\tfrac{1}{8}e'^4\cos(2nt+2n't-4\omega')+\tfrac{1599}{16}e'^4\cos(2nt-6n't+4\omega')\\
&+\tfrac{3}{16}\gamma^4\cos(2nt-2n't+4\omega-4\Omega)+\tfrac{3}{16}\gamma^4\cos(2nt+2n't-4\Omega)\\
&-\tfrac{3}{2}e^2\gamma^2\cos(4nt-2n't-4\omega+2\Omega)+\tfrac{3}{8}e^2\gamma^2\cos(2n't-4\omega+2\Omega)\\
&+\tfrac{3}{2}e^2\gamma^2\cos(4nt-2n't-2\Omega)-\tfrac{3}{8}e^2\gamma^2\cos(2nt+2n't-2\omega-2\Omega)\\
&+\tfrac{3}{32}ee'^3\cos(3nt+n't-\omega-3\omega')-\tfrac{5}{32}ee'^3\cos(nt+n't+\omega-3\omega')\\
&-\tfrac{4225}{32}ee'^3\cos(nt-5n't+\omega+3\omega')+\tfrac{647}{32}ee'^3\cos(3nt-5n't-\omega+3\omega')
\end{aligned}\right\}\quad(218)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\tfrac{7}{32}e^3e'\cos(nt+3n't-3\omega-\omega')-\tfrac{1}{32}e^3e'\cos(nt+n't-3\omega+\omega')\\
&-\tfrac{125}{32}e^3e'\cos(5nt-n't-3\omega-\omega')+\tfrac{2535}{32}e^3e'\cos(5nt-3n't-3\omega+\omega')\\
&+51e^2e'^2\cos(4nt-4n't-2\omega+2\omega')+\tfrac{153}{4}e^2e'^2\cos(4n't-2\omega-2\omega')\\
&+\tfrac{51}{8}e'^2\gamma^2\cos(2nt-4n't+2\omega+2\omega'-2\Omega)+\tfrac{51}{4}e'^2\gamma^2\cos(4n't-2\omega'-2\Omega)\\
&-\tfrac{51}{8}e'^2\gamma^2\cos(2nt-4n't-2\omega+2\omega'+2\Omega)\\
&+\tfrac{63}{16}ee'\gamma^2\cos(3nt-3n't+\omega+\omega'-2\Omega)\\
&-\tfrac{9}{16}ee'\gamma^2\cos(nt-n't-\omega-\omega'+2\Omega)\\
&+6ee'\gamma^2\cos(5nt-n't-\omega-\omega'-2\Omega)\\
&+\tfrac{63}{16}ee'\gamma^2\cos(nt-3n't-\omega+\omega'+2\Omega)\\
&-\tfrac{63}{16}ee'\gamma^2\cos(3nt-3n't-3\omega+\omega'+2\Omega)\\
&+\tfrac{15}{16}ee'\gamma^2\cos(nt-n't+3\omega-\omega'-2\Omega)\\
&+\tfrac{9}{16}ee'\gamma^2\cos(3nt-n't-3\omega-\omega'+2\Omega)\\
&-\tfrac{9}{16}ee'\gamma^2\cos(3nt-n't+\omega-\omega'-2\Omega)\\
&-\tfrac{105}{16}ee'\gamma^2\cos(nt-3n't+3\omega+\omega'-2\Omega)\\
&+\tfrac{3}{8}ee'\gamma^2\cos(nt+n't-\omega+\omega'-2\Omega)\\
&-\tfrac{69}{8}ee'\gamma^2\cos(nt+3n't-\omega-\omega'-2\Omega)\Big\}
\end{aligned}\right\}. \quad (218)
$$

$$
\left.\begin{aligned}
&+\tfrac{9}{8}\frac{\overline{m}^2}{aa'}\Big\{\{1+\tfrac{1}{2}e^2+2e'^2-\tfrac{11}{4}\gamma^2\}\cos(nt-n't)+e'\cos(nt-\omega')\\
&+3e'\cos(nt-2n't+\omega')-2e\cos(n't-\omega)-\tfrac{1}{8}e^2\cos(3nt-n't-2\omega)\\
&+\tfrac{5}{8}e^2\cos(nt+n't-2\omega)+\tfrac{11}{8}e'^2\cos(nt+n't-2\omega')\\
&+\tfrac{53}{8}e'^2\cos(nt-3n't+2\omega')-2ee'\cos(\omega-\omega')\\
&-6ee'\cos(2n't-\omega-\omega')-\tfrac{1}{8}\gamma^2\cos(nt-n't-2\omega+2\Omega)\\
&+\tfrac{1}{8}\gamma^2\cos(nt-n't+2\omega-2\Omega)+\tfrac{5}{4}\gamma^2\cos(3nt-n't-2\Omega)\\
&+\tfrac{3}{2}\gamma^2\cos(nt+n't-2\Omega)+\tfrac{5}{8}\{1-\tfrac{15}{2}e^2-6e'^2-\tfrac{3}{4}\gamma^2\}\cos 3(nt-n't)\\
&+\tfrac{10}{3}e\cos(4nt-3n't-\omega)-\tfrac{20}{3}e\cos(2nt-3n't+\omega)\\
&+\tfrac{25}{8}e'\cos(3nt-4n't+\omega')-\tfrac{5}{8}e'\cos(3nt-2n't-\omega')\\
&+\tfrac{5}{24}e'^2\cos(3nt-n't-2\omega')+\tfrac{125}{24}e^2\cos(5nt-3n't-2\omega)\\
&+\tfrac{215}{24}e^2\cos(nt-3n't+2\omega)+\tfrac{635}{24}e'^2\cos(3nt-5n't+2\omega')\\
&+\tfrac{50}{3}ee'\cos(4nt-4n't-\omega+\omega')+\tfrac{20}{3}ee'\cos(2nt-2n't+\omega-\omega')\\
&-\tfrac{10}{3}ee'\cos(4nt-2n't-\omega-\omega')-\tfrac{100}{3}ee'\cos(2nt-4n't+\omega+\omega')\\
&+\tfrac{5}{4}\gamma^2\cos(nt-3n't+2\Omega)+\tfrac{5}{8}\gamma^2\cos(3nt-3n't+2\omega-2\Omega)\\
&-\tfrac{5}{8}\gamma^2\cos(3nt-3n't-2\omega+2\Omega)\Big\}
\end{aligned}\right\}. \quad (218)
$$

$$
\left.\begin{aligned}
&-\tfrac{1}{4}e\gamma^3\sin(2n't+\omega-3\Omega)-\tfrac{1}{4}e\gamma^3\sin(6nt-2n't-\omega-3\Omega)\\
&-\tfrac{3}{4}e\gamma^3\sin(2nt-2n't+3\omega-3\Omega)+\tfrac{1}{32}e'\gamma^3\sin(5nt-n't-\omega'-3\Omega)\\
&+\tfrac{21}{32}e'\gamma^3\sin(nt+3n't-\omega'-3\Omega)-\tfrac{7}{32}e'\gamma^3\sin(5nt-3n't+\omega'-3\Omega)\\
&-\tfrac{3}{32}e'\gamma^3\sin(nt+n't+\omega'-3\Omega)-\tfrac{1}{32}e'\gamma^3\sin(nt-n't-2\omega-\omega'+3\Omega)\\
&+\tfrac{21}{32}e'\gamma^3\sin(3nt-3n't+2\omega+\omega'-3\Omega)\\
&-\tfrac{3}{32}e'\gamma^3\sin(3nt-n't+2\omega-\omega'-3\Omega)\\
&+\tfrac{7}{32}e'\gamma^3\sin(nt-3n't-2\omega+\omega'+3\Omega)\\
&+\tfrac{1}{32}e'\gamma\{3\gamma^2-e^2\}\sin(3nt-n't-2\omega-\omega'+\Omega)\\
&+\tfrac{7}{32}e'\gamma\{43e^2-\gamma^2\}\sin(nt-3n't+2\omega+\omega'-\Omega)\\
&-\tfrac{1}{32}e'\gamma\{43e^2-\gamma^2\}\sin(nt-n't+2\omega-\omega'-\Omega)\\
&+\tfrac{7}{32}e'\gamma\{e^2-3\gamma^2\}\sin(3nt-3n't-2\omega+\omega'+\Omega)\\
&-17\,ee'^2\gamma\sin(2nt-4n't+\omega+2\omega'-\Omega)\\
&-\tfrac{17}{2}ee'^2\gamma\sin(4n't-\omega-2\omega'-\Omega)\\
&+\tfrac{17}{2}ee'^2\gamma\sin(4nt-4n't-\omega+2\omega'-\Omega)\\
&-\tfrac{25}{32}e^2e'\gamma\sin(5nt-n't-2\omega-\omega'-\Omega)\\
&+\tfrac{35}{32}e^2e'\gamma\sin(nt+3n't-2\omega-\omega'-\Omega)\\
&+\tfrac{175}{32}e^2e'\gamma\sin(5nt-3n't-2\omega+\omega'-\Omega)\\
&-\tfrac{5}{32}e^2e'\gamma\sin(nt+n't-2\omega+\omega'-\Omega)\Big\}
\end{aligned}\right\}\quad(220)
$$

$$
\left.\begin{aligned}
&+\tfrac{3}{2}\frac{\overline{m}^2}{a'}\Big\{\tfrac{11}{8}\gamma\sin(2nt-n't-\Omega)+\tfrac{11}{8}\gamma\sin(n't-\Omega)\\
&+\tfrac{11}{16}e\gamma\sin(3nt-n't-\omega-\Omega)+\tfrac{33}{16}e\gamma\sin(nt-n't-\omega+\Omega)\\
&-\tfrac{33}{16}e\gamma\sin(nt+n't-\omega-\Omega)-\tfrac{77}{16}e\gamma\sin(nt-n't+\omega-\Omega)\\
&+\tfrac{11}{8}e'\gamma\sin(\omega'-\Omega)+\tfrac{33}{8}e'\gamma\sin(2nt-2n't+\omega'-\Omega)\\
&+\tfrac{33}{8}e'\gamma\sin(2n't-\omega'-\Omega)+\tfrac{11}{8}e'\gamma\sin(2nt-\omega'-\Omega)\\
&+\tfrac{5}{8}\gamma\sin(4nt-3n't-\Omega)-\tfrac{5}{8}\gamma\sin(2nt-3n't+\Omega)\\
&+\tfrac{25}{16}e\gamma\sin(5nt-3n't-\omega-\Omega)-\tfrac{5}{16}e\gamma\sin(3nt-3n't-\omega+\Omega)\\
&-\tfrac{55}{16}e\gamma\sin(3nt-3n't+\omega-\Omega)+\tfrac{35}{16}e\gamma\sin(nt-3n't+\omega+\Omega)\\
&+\tfrac{25}{8}e'\gamma\sin(4nt-4n't+\omega'-\Omega)-\tfrac{25}{8}e'\gamma\sin(2nt-4n't+\omega'+\Omega)\\
&-\tfrac{5}{8}e'\gamma\sin(4nt-2n't-\omega'-\Omega)+\tfrac{5}{8}e'\gamma\sin(2nt-2n't-\omega'+\Omega)\Big\}
\end{aligned}\right\}.\quad(220)
$$

14. Having found the expressions for the forces which act upon the moon in the directions of the three polar co-ordinates r, v and θ, we must now compute the values of the factors by which these forces are multiplied in the differential equations of these co-ordinates. These equations are given on pages 39 and 40, and are designated by the letters (B), (C) and (D) respectively. We shall first develop the factors by which $\left(\frac{dR}{dr}\right)$, $\left(\frac{dR}{dv}\right)$ and $\left(\frac{dR}{d\theta}\right)$ are multiplied in equation (B).

Equation (104) will give

$$\left.\begin{aligned}
\sin v_1 = &\{1-e^2-\tfrac{7}{64}e^4-\tfrac{1}{32}\gamma^4\}\sin nt + e\{1-\tfrac{5}{4}e^2\}\sin(2nt-\omega)\\
&-e\sin\omega+\tfrac{9}{8}\{e^2-\tfrac{3}{2}e^4+\tfrac{1}{24}\gamma^4\}\sin(3nt-2\omega)\\
&\pm\tfrac{1}{8}e^2\{1-\tfrac{1}{6}e^2+\tfrac{1}{8}\tfrac{\gamma^4}{e^2}\}\sin(nt-2\omega)+\tfrac{4}{3}e^3\sin(4nt-3\omega)\\
&\pm\tfrac{1}{12}e^3\sin(2nt-3\omega)+\tfrac{625}{384}e^4\sin(5nt-4\omega)\pm\tfrac{9}{128}e^4\sin(3nt-4\omega)\\
&-\tfrac{1}{8}\gamma^2\{1-\tfrac{81}{8}e^2-\tfrac{1}{2}\gamma^2\}\sin(3nt-2\Omega)\\
&\mp\tfrac{1}{8}\gamma^2\{1-\tfrac{7}{8}e^2-\tfrac{1}{2}\gamma^2\}\sin(nt-2\Omega)\\
&+\tfrac{1}{8}\gamma^2\{1-\tfrac{29}{8}e^2-\tfrac{1}{2}\gamma^2\}\sin(nt+2\omega-2\Omega)\\
&-\tfrac{1}{8}\gamma^2\{1-\tfrac{7}{8}e^2-\tfrac{1}{2}\gamma^2\}\sin(nt-2\omega+2\Omega)-\tfrac{3}{8}e\gamma^2\sin(4nt-\omega-2\Omega)\\
&\mp\tfrac{1}{8}e\gamma^2\sin(2nt-\omega-2\Omega)+\tfrac{1}{2}e\gamma^2\sin(2nt+\omega-2\Omega)\\
&-\tfrac{1}{8}e\gamma^2\sin(2nt-3\omega+2\Omega)-\tfrac{1}{8}e\gamma^2\sin(3\omega-2\Omega)\\
&+\tfrac{3}{128}\gamma^4\sin(5nt-4\Omega)\pm\tfrac{1}{128}\gamma^4\sin(3nt-4\Omega)\\
&+\tfrac{3}{128}\gamma^4\sin(nt+4\omega-4\Omega)-\tfrac{1}{128}\gamma^4\sin(nt-4\omega+4\Omega)\\
&-\tfrac{3}{64}\gamma^4\sin(3nt+2\omega-4\Omega)\mp\tfrac{1}{64}\gamma^4\sin(nt+2\omega-4\Omega)\\
&-\tfrac{81}{64}e^2\gamma^2\sin(5nt-2\omega-2\Omega)\mp\tfrac{9}{64}e^2\gamma^2\sin(3nt-2\omega-2\Omega)\\
&-\tfrac{9}{64}e^2\gamma^2\sin(3nt-4\omega+2\Omega)\pm\tfrac{1}{64}e^2\gamma^2\sin(nt-4\omega+2\Omega)
\end{aligned}\right\}. \quad (221)$$

We may observe that Theorem III. applies to functions of the form of equation (104), which would correspond to the case where $i'=0$, $f'=0$ and $h'=0$. Consequently, equation (221) will give the value of $\cos v_1$ by changing *sin* to *cos*, and using the lower signs.

Equation (221) will give, by differentiation,

$$
\begin{aligned}
dv_1 \cos v_1 = {} & \\
ndt \Big\{ & \{1 - e^2 + \tfrac{7}{64}e^4 - \tfrac{1}{32}\gamma^4\} \cos nt + 2e\{1 - \tfrac{5}{4}e^2\} \cos(2nt - \omega) \\
& + \tfrac{27}{8}\{e^2 - \tfrac{3}{2}e^4 + \tfrac{1}{24}\gamma^4\} \cos(3nt - 2\omega) \\
& \pm \tfrac{1}{8}e^2\{1 - \tfrac{1}{6}e^2 + \tfrac{1}{8}\frac{\gamma^4}{e^2}\} \cos(nt - 2\omega) + \tfrac{16}{3}e^3 \cos(4nt - 3\omega) \\
& \pm \tfrac{1}{6}e^3 \cos(2nt - 3\omega) + \tfrac{3125}{384}e^4 \cos(5nt - 4\omega) \pm \tfrac{27}{128}e^4 \cos(3nt - 4\omega) \\
& - \tfrac{3}{8}\gamma^2\{1 - \tfrac{81}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(3nt - 2\Omega) \mp \gamma^2\{1 - \tfrac{7}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - 2\Omega) \\
& + \tfrac{1}{8}\gamma^2\{1 - \tfrac{29}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(nt + 2\omega - 2\Omega) - \tfrac{1}{4}e\gamma^2 \cos(2nt - 3\omega + 2\Omega) \\
& - \tfrac{1}{8}\gamma^2\{1 - \tfrac{7}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - 2\omega + 2\Omega) - \tfrac{3}{2}e\gamma^2 \cos(4nt - \omega - 2\Omega) \\
& \mp \tfrac{1}{4}e\gamma^2 \cos(2nt - \omega - 2\Omega) + e\gamma^2 \cos(2nt + \omega - 2\Omega) \\
& + \tfrac{15}{128}\gamma^4 \cos(5nt - 4\Omega) \pm \tfrac{3}{128}\gamma^4 \cos(3nt - 4\Omega) \\
& + \tfrac{3}{128}\gamma^4 \cos(nt + 4\omega - 4\Omega) - \tfrac{1}{128}\gamma^4 \cos(nt - 4\omega + 4\Omega) \\
& - \tfrac{9}{64}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \mp \tfrac{1}{64}\gamma^4 \cos(nt + 2\omega - 4\Omega) \\
& - \tfrac{255}{64}e^2\gamma^2 \cos(5nt - 2\omega - 2\Omega) \mp \tfrac{27}{64}e^2\gamma^2 \cos(3nt - 2\omega - 2\Omega) \\
& - \tfrac{27}{64}e^2\gamma^2 \cos(3nt - 4\omega + 2\Omega) \pm \tfrac{1}{64}e^2\gamma^2 \cos(nt - 4\omega + 2\Omega) \Big\}
\end{aligned} \quad . \quad (222)
$$

If in this equation we change *cos* to *sin*, and use the lower signs, we shall have the expression for $dv_1 \sin v_1$.

Equations (194) and (195) will give, by differentiation,

$$
\begin{aligned}
d\theta_1 \cos \theta_1 = {} & \\
ndt \Big\{ & \gamma\{1 - e^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - \Omega) + \tfrac{1}{8}\gamma^3 \cos(nt + 2\omega - 3\Omega) \\
& + 2e\gamma\{1 - \tfrac{5}{4}e^2 - \tfrac{1}{2}\gamma^2\} \cos(2nt - \omega - \Omega) + \tfrac{1}{4}e\gamma^3 \cos(2nt + \omega - 3\Omega) \\
& + \tfrac{27}{8}e^2\gamma \cos(3nt - 2\omega - \Omega) + \tfrac{1}{8}\gamma\{e^2 - \gamma^2\} \cos(nt - 2\omega + \Omega) \\
& + \tfrac{16}{3}e^3\gamma \cos(4nt - 3\omega - \Omega) + \tfrac{1}{4}e\gamma\{\tfrac{2}{3}e^2 - \gamma^2\} \cos(2nt + 3\omega + \Omega) \Big\}
\end{aligned} \quad . \quad (223)
$$

$$
\begin{aligned}
d\theta_1 \sin \theta_1 = ndt \Big\{ & \tfrac{1}{2}\gamma^2\{1 - 4e^2 - \tfrac{3}{4}\gamma^2\} \sin 2(nt - \Omega) \\
& + \tfrac{3}{2}e\gamma^2 \sin(3nt - \omega - 2\Omega) - \tfrac{1}{2}e\gamma^2 \sin(nt + \omega - 2\Omega) \\
& + \tfrac{1}{8}\gamma^4 \sin(2nt + 2\omega - 4\Omega) - \tfrac{1}{8}\gamma^4 \sin 2(nt - \omega) \\
& - \tfrac{1}{16}\gamma^4 \sin 4(nt - \Omega) + \tfrac{13}{4}e^2\gamma^2 \sin(4nt - 2\omega - 2\Omega) \Big\}
\end{aligned} \quad . \quad (224)
$$

If we multiply equations (221) and (224) together, we shall find,

$d\theta_1 \sin \theta_1 \sin v_1 =$

$$
\left.
\begin{array}{l}
ndt \Big\{ \mp \frac{1}{4}\gamma^2 \{1 - \frac{3}{4}\gamma^2 - 9e^2\} \cos(3nt - 2\Omega) \\
\quad + \frac{1}{4}\gamma^2 \{1 - \frac{3}{4}\gamma^2 - e^2\} \cos(nt - 2\Omega) \mp e\gamma^2 \cos(4nt - \omega - 2\Omega) \\
\quad \pm \frac{1}{2}e\gamma^2 \cos(2nt + \omega - 2\Omega) + \frac{1}{2}e\gamma^2 \cos(2nt - \omega - 2\Omega) \\
\quad \mp \frac{85}{32}e^2\gamma^2 \cos(5nt - 2\omega - 2\Omega) \pm \frac{1}{32}e^2\gamma^2 \cos(nt - 2\omega + 2\Omega) \\
\quad + \frac{27}{32}e^2\gamma^2 \cos(3nt - 2\omega - 2\Omega) \mp \frac{7}{32}e^2\gamma^2 \cos(nt + 2\omega - 2\Omega) \\
\quad \pm \frac{1}{16}\gamma^4 \cos(5nt - 4\Omega) \mp \frac{1}{16}\gamma^4 \cos nt \mp \frac{3}{32}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \\
\quad - \frac{1}{32}\gamma^4 \cos(nt - 2\omega) \pm \frac{3}{32}\gamma^4 \cos(3nt - 2\omega) + \frac{1}{32}\gamma^4 \cos(nt + 2\omega - 4\Omega) \Big\}
\end{array}
\right\} . \quad (225)
$$

This equation will give the value of $d\theta_1 \sin \theta_1 \cos v_1$ by using the lower signs and changing *cos* to *sin* in the second member.

Equations (195) and (222) will give,

$dv_1 \cos \theta_1 \cos v_1 =$

$$
\left.
\begin{array}{l}
ndt \Big\{ \{1 - e^2 - \frac{1}{4}\gamma^2 + \frac{1}{4}e^2\gamma^2 + \frac{7}{4}e^4 + \frac{7}{64}\gamma^4\} \cos nt + 2e\{1 - \frac{5}{4}e^2 - \frac{1}{4}\gamma^2\} \cos(2nt - \omega) \\
\quad + \frac{27}{8}e^2 \{1 - \frac{3}{2}e^2 + \frac{1}{36}\frac{\gamma^4}{e^2} - \frac{1}{4}\gamma^2\} \cos(3nt - 2\omega) \\
\quad \pm \frac{1}{8}e^2 \{1 - \frac{1}{6}e^2 - \frac{1}{4}\gamma^2\} \cos(nt - 2\omega) + \frac{16}{3}e^3 \cos(4nt - 3\omega) \\
\quad \pm \frac{1}{6}e^3 \cos(2nt - 3\omega) + \frac{3125}{384}e^4 \cos(5nt - 4\omega) \pm \frac{27}{128}e^4 \cos(3nt - 4\omega) \\
\quad - \frac{1}{4}\gamma^2 \{1 - \frac{3}{4}\gamma^2 - \frac{171}{16}e^2\} \cos(3nt - 2\Omega) \mp \frac{1}{64}e^2\gamma^2 \cos(nt - 2\Omega) \\
\quad + \frac{1}{8}\gamma^2 \{1 - \frac{11}{4}e^2 - \frac{3}{4}\gamma^2\} \cos(nt + 2\omega - 2\Omega) \\
\quad - \frac{1}{8}\gamma^2 \{1 - e^2 - \frac{3}{4}\gamma^2\} \sin(nt - 2\omega + 2\Omega) - e\gamma^2 \cos(4nt - \omega - 2\Omega) \\
\quad + \frac{3}{4}e\gamma^2 \cos(2nt + \omega - 2\Omega) - \frac{1}{4}e\gamma^2 \cos(2nt - 3\omega + 2\Omega) \\
\quad + \frac{1}{16}\gamma^4 \cos(5nt - 4\Omega) + \frac{3}{128}\gamma^4 \cos(nt + 4\omega - 4\Omega) \\
\quad - \frac{1}{128}\gamma^4 \cos(nt - 4\omega + 4\Omega) - \frac{3}{32}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \\
\quad - \frac{85}{32}e^2\gamma^2 \cos(5nt - 2\omega - 2\Omega) - \frac{27}{64}e^2\gamma^2 \cos(3nt - 4\omega + 2\Omega) \\
\quad \pm \frac{1}{64}e^2\gamma^2 \cos(nt - 4\omega + 2\Omega) \Big\}
\end{array}
\right\} . \quad (226)
$$

This equation will give the value of $dv_1 \cos \theta_1 \sin v_1$ by using the lower signs and changing *cos* to *sin* in the second member.

Equations (225) and (226) will give,

$$\left.\begin{aligned}
&\{dv_1 \cos\theta_1 \cos v_1 - d\theta_1 \sin\theta_1 \sin v_1\} = \\
&ndt\Big\{\{1 - e^2 - \tfrac{1}{4}\gamma^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{7}{64}e^4 + \tfrac{11}{64}\gamma^4\}\cos nt + 2e\{1 - \tfrac{5}{4}e^2 - \tfrac{1}{4}\gamma^2\}\cos(2nt - \omega) \\
&\quad + \tfrac{27}{8}e^2\{1 - \tfrac{3}{2}e^2 - \tfrac{1}{4}\gamma^2\}\cos(3nt - 2\omega) \\
&\quad \pm \tfrac{1}{8}e^2\{1 - \tfrac{1}{6}e^2 - \tfrac{1}{4}\gamma^2 + \tfrac{1}{4}\frac{\gamma^4}{e^2}\}\cos(nt - 2\omega) + \tfrac{16}{3}e^3\cos(4nt - 3\omega) \\
&\quad \pm \tfrac{1}{6}e^3\cos(2nt - 3\omega) + \tfrac{3125}{384}e^4\cos(5nt - 4\omega) \pm \tfrac{27}{128}e^4\cos(3nt - 4\omega) \\
&\quad + \tfrac{27}{64}e^2\gamma^2\cos(3nt - 2\Omega) \mp \tfrac{1}{4}\gamma^2\{1 - \tfrac{3}{4}\gamma^2 - \tfrac{15}{16}e^2\}\cos(nt - 2\Omega) \\
&\quad + \tfrac{1}{8}\gamma^2\{1 - e^2 - \tfrac{3}{4}\gamma^2\}\cos(nt + 2\omega - 2\Omega) \\
&\quad - \tfrac{1}{8}\gamma^2\{1 - \tfrac{3}{4}e^2 - \tfrac{3}{4}\gamma^2\}\cos(nt - 2\omega + 2\Omega) + \tfrac{1}{4}e\gamma^2\cos(2nt + \omega - 2\Omega) \\
&\quad - \tfrac{1}{4}e\gamma^2\cos(2nt - 3\omega + 2\Omega) + \tfrac{3}{128}\gamma^4\cos(nt + 4\omega - 4\Omega) \\
&\quad - \tfrac{1}{128}\gamma^4\cos(nt - 4\omega + 4\Omega) - \tfrac{27}{64}e^2\gamma^2\cos(3nt - 4\omega + 2\Omega) \\
&\quad \pm \tfrac{1}{64}e^2\gamma^2\cos(nt - 4\omega + 2\Omega) \mp \tfrac{27}{32}e^2\gamma^2\cos(3nt - 2\omega - 2\Omega) \\
&\quad \mp \tfrac{1}{32}\gamma^4\cos(nt + 2\omega - 4\Omega) \mp \tfrac{1}{2}e\gamma^2\cos(2nt - \omega - 2\Omega)\Big\}
\end{aligned}\right\} \quad (227)$$

This equation will give the value of $dv_1 \cos\theta_1 \sin v_1 + d\theta_1 \sin\theta_1 \cos v_1$ by using the lower signs and changing *cos* to *sin* in the second member.

If we multiply equation (227) by the value of r_1^2 given in equation (186) we shall obtain,

$$\left.\begin{aligned}
&r_1^2\{dv_1 \cos\theta_1 \cos v_1 - d\theta_1 \sin\theta_1 \sin v_1\} = \\
&a^2ndt\Big\{\{1 - \tfrac{3}{2}e^2 - \tfrac{1}{4}\gamma^2 + \tfrac{81}{64}e^4 + \tfrac{3}{8}e^2\gamma^2 + \tfrac{11}{64}\gamma^4\}\cos nt + e\{1 - \tfrac{7}{4}e^2 - \tfrac{1}{4}\gamma^2\}\cos(2nt - \omega) \\
&\quad - e\{1 - \tfrac{1}{2}e^2 - \tfrac{1}{4}\gamma^2\}\cos\omega + \tfrac{9}{8}e^2\{1 - 2e^2 - \tfrac{1}{4}\gamma^2\}\cos(3nt - 2\omega) \\
&\quad + \tfrac{4}{3}e^3\cos(4nt - 3\omega) \mp \tfrac{1}{8}e^2\{1 - \tfrac{2}{3}e^2 - \tfrac{1}{4}\gamma^2 - \tfrac{1}{4}\frac{\gamma^4}{e^2}\}\cos(nt - 2\omega) \\
&\quad \mp \tfrac{1}{12}e^3\cos(2nt - 3\omega) + \tfrac{625}{384}e^4\cos(5nt - 4\omega) \mp \tfrac{9}{128}e^4\cos(3nt - 4\omega) \\
&\quad + \tfrac{9}{64}e^2\gamma^2\cos(3nt - 2\Omega) \mp \tfrac{1}{4}\gamma^2\{1 - \tfrac{25}{16}e^2 - \tfrac{3}{4}\gamma^2\}\cos(nt - 2\Omega) \\
&\quad + \tfrac{1}{8}\gamma^2\{1 - \tfrac{3}{2}e^2 - \tfrac{3}{4}\gamma^2\}\cos(nt + 2\omega - 2\Omega) \\
&\quad - \tfrac{1}{8}\gamma^2\{1 - \tfrac{7}{4}e^2 - \tfrac{3}{4}\gamma^2\}\cos(nt - 2\omega + 2\Omega) - \tfrac{1}{8}e\gamma^2\cos(2nt - 3\omega + 2\Omega) \\
&\quad + \tfrac{1}{8}e\gamma^2\cos(2nt + \omega - 2\Omega) \pm \tfrac{3}{8}e\gamma^2\cos(\omega - 2\Omega) - \tfrac{1}{8}e\gamma^2\cos(3\omega - 2\Omega) \\
&\quad \mp \tfrac{1}{4}e\gamma^2\cos(2nt - \omega - 2\Omega) + \tfrac{3}{128}\gamma^4\cos(nt + 4\omega - 4\Omega) \\
&\quad - \tfrac{1}{128}\gamma^4\cos(nt - 4\omega + 4\Omega) \mp \tfrac{1}{32}\gamma^4\cos(nt + 2\omega - 4\Omega) \\
&\quad \mp \tfrac{1}{64}e^2\gamma^2\cos(nt - 4\omega + 2\Omega) \mp \tfrac{9}{32}e^2\gamma^2\cos(3nt - 2\omega - 2\Omega)\Big\}
\end{aligned}\right\} \quad (228)$$

This equation will give the value of $r_1^2\{dv_1 \cos\theta_1 \sin v_1 + d\theta_1 \sin\theta_1 \cos v_1\}$ by using the lower signs and changing *cos* to *sin* in the second member.

From equation (195) we get

$$\left.\begin{aligned}\frac{1}{\cos\theta_1} &= 1+\tfrac{1}{4}\gamma^2-\tfrac{7}{64}\gamma^4-\tfrac{1}{4}\gamma^2\{1-4e^2-\tfrac{1}{4}\gamma^2\}\cos 2(nt-\Omega)\\ &\quad -\tfrac{1}{2}e\gamma^2\cos(3nt-\omega-2\Omega)+\tfrac{1}{2}e\gamma^2\cos(nt+\omega-2\Omega)\\ &\quad -\tfrac{1}{16}\gamma^4\cos(2nt+2\omega-4\Omega)-\tfrac{13}{16}e^2\gamma^2\cos(4nt-2\omega-2\Omega)\\ &\quad +\tfrac{3}{64}\gamma^4\cos 4(nt-\Omega)+\tfrac{1}{16}\gamma^4\cos 2(nt-\omega)-\tfrac{3}{16}e^2\gamma^2\cos 2(\omega-\Omega)\end{aligned}\right\}. \quad (229)$$

By differentiating equation (185) we shall obtain

$$\left.\begin{aligned}dr_1 = andt\Big\{ & e(1-\tfrac{3}{8}e^2)\sin(nt-\omega)+e^2(1-\tfrac{2}{3}e^2)\sin 2(nt-\omega)\\ & +\tfrac{9}{8}e^3\sin 3(nt-\omega)+\tfrac{4}{3}e^4\sin 4(nt-\omega)\Big\}\end{aligned}\right\}. \quad (230)$$

From equations (221), (229) and (230) we obtain the two following:

$$\left.\begin{aligned}\frac{\cos v_1}{\cos\theta_1} &= \{1-e^2+\tfrac{1}{4}\gamma^2+\tfrac{7}{64}e^4-\tfrac{9}{64}\gamma^4-\tfrac{1}{4}e^2\gamma^2\}\cos nt+e\{1-\tfrac{5}{4}e^2+\tfrac{1}{4}\gamma^2\}\cos(2nt-\omega)\\ &\quad -e\{1+\tfrac{1}{4}\gamma^2\}\cos\omega+\tfrac{9}{8}e^2\{1-\tfrac{3}{2}e^2+\tfrac{1}{12}\frac{\gamma^4}{e^2}+\tfrac{1}{4}\gamma^2\}\cos(3nt-2\omega)\\ &\quad \mp\tfrac{1}{8}e^2\{1-\tfrac{1}{6}e^2+\tfrac{1}{4}\gamma^2\}\cos(nt-2\omega)+\tfrac{4}{3}e^3\cos(4nt-3\omega)\\ &\quad \mp\tfrac{1}{12}e^3\cos(nt-3\omega)+\tfrac{625}{384}e^4\cos(5nt-4\omega)\mp\tfrac{9}{128}e^4\cos(3nt-4\omega)\\ &\quad -\tfrac{1}{4}\gamma^2\{-\tfrac{153}{16}e^2-\tfrac{1}{4}\gamma^2\}\cos(3nt-2\Omega)\\ &\quad +\tfrac{1}{8}\gamma^2\{1-\tfrac{25}{4}e^2-\tfrac{1}{4}\gamma^2\}\cos(nt+2\omega-2\Omega)\\ &\quad -\tfrac{1}{8}\gamma^2\{1-e^2-\tfrac{1}{4}\gamma^2\}\cos(nt-2\omega+2\Omega)-\tfrac{3}{4}e\gamma^2\cos(4nt-\omega-2\Omega)\\ &\quad +\tfrac{7}{8}e\gamma^2\cos(2nt+\omega-2\Omega)\pm\tfrac{1}{8}e\gamma^2\cos(\omega-2\Omega)-\tfrac{1}{8}e\gamma^2\cos(3\omega-2\Omega)\\ &\quad -\tfrac{1}{8}e\gamma^2\cos(2nt-3\omega+2\Omega)+\tfrac{1}{64}e^2\gamma^2\cos(nt-2\Omega)\\ &\quad -\tfrac{51}{32}e^2\gamma^2\cos(5nt-2\omega-2\Omega)-\tfrac{9}{64}e^2\gamma^2\cos(3nt-4\omega+2\Omega)\\ &\quad \mp\tfrac{1}{64}e^2\gamma^2\cos(nt-4\omega+2\Omega)+\tfrac{1}{16}\gamma^4\cos(5nt-4\Omega)\\ &\quad +\tfrac{3}{128}\gamma^4\cos(nt+4\omega-4\Omega)-\tfrac{1}{128}\gamma^4\cos(nt-4\omega+4\Omega)\\ &\quad -\tfrac{3}{32}\gamma^4\cos(3nt+2\omega-4\Omega)\end{aligned}\right\}. \quad (231)$$

This equation will give the value of $\frac{\sin v_1}{\cos\theta_1}$ by using the lower signs and changing *cos* to *sin* in the second member.

$$
\left.
\begin{aligned}
&\frac{\cos v_1}{\cos \theta_1} dr_1 = \\
&andt \Big\{ \pm \tfrac{1}{2}e \{1 - \tfrac{7}{2}e^2 + \tfrac{1}{4}\gamma^2\} \sin (2nt - \omega) \mp \tfrac{1}{2}e \{1 - \tfrac{1}{4}e^2 + \tfrac{1}{4}\gamma^2\} \sin \omega \\
&\quad \pm e^2 \{1 - \tfrac{23}{8}e^2 + \tfrac{1}{4}\gamma^2\} \sin (3nt - 2\omega) \mp e^2 \{1 - \tfrac{3}{8}e^2 + \tfrac{1}{4}\gamma^2\} \sin nt \\
&\quad - \tfrac{1}{24}e^4 \sin (nt - 2\omega) \pm \tfrac{13}{8}e^3 \sin (4nt - 3\omega) \pm \tfrac{59}{24}e^4 \sin (5nt - 4\omega) \\
&\quad \mp \tfrac{1}{8}e\gamma^2 \sin (4nt - \omega - 2\Omega) \pm \tfrac{3}{16}e\gamma^2 \sin (2nt + \omega - 2\Omega) \\
&\quad \mp \tfrac{1}{16} e\gamma^2 \sin (3\omega - 2\Omega) - \tfrac{1}{16}e\gamma^2 \sin (\omega - 2\Omega) \mp \tfrac{1}{16}e\gamma^2 \sin (2nt - 3\omega + 2\Omega) \\
&\quad \mp \tfrac{1}{2}e^2\gamma^2 \sin (5nt - 2\omega - 2\Omega) \pm \tfrac{7}{8}e^2\gamma^2 \sin (3nt - 2\Omega) \\
&\quad \mp \tfrac{3}{8}e^2\gamma^2 \sin (nt + 2\omega - 2\Omega) \pm \tfrac{1}{8}e^2\gamma^2 \sin (nt - 2\omega + 2\Omega) \\
&\quad \mp \tfrac{1}{8}e^2\gamma^2 \sin (3nt - 4\omega + 2\Omega) \Big\}
\end{aligned}
\right\} . \quad (232)
$$

This equation will give the value of $\frac{\sin v_1}{\cos \theta_1} dr_1$ by using the lower signs and changing *sin* to *cos* in the second member.

Equations (185) and (226) will give

$$
\left.
\begin{aligned}
&r_1\, dv_1 \cos \theta_1 \sin v_1 = \\
&andt \Big\{ \{1 - \tfrac{3}{2}e^2 - \tfrac{1}{4}\gamma^2 + \tfrac{3}{8}e^2\gamma^2 + \tfrac{23}{64}e^4 + \tfrac{7}{64}\gamma^4\} \sin nt + \tfrac{3}{2}e \{1 - \tfrac{5}{3}e^2 - \tfrac{1}{4}\gamma^2\} \sin (2nt - \omega) \\
&\quad - \tfrac{1}{2}e \{1 - \tfrac{1}{4}e^2 - \tfrac{1}{4}\gamma^2\} \sin \omega + \{\tfrac{17}{8}e^2 - 4e^4 + \tfrac{3}{32}\gamma^4 - \tfrac{17}{32}e^2\gamma^2\} \sin (3nt - 2\omega) \\
&\quad \pm \tfrac{1}{8}e^2 \{1 - \tfrac{1}{4}\gamma^2\} \sin (nt - 2\omega) + \tfrac{71}{24}e^3 \sin (4nt - 3\omega) \\
&\quad \pm \tfrac{1}{12}e^3 \sin (2nt - 3\omega) - \tfrac{1}{4}\gamma^2 \{1 - \tfrac{169}{16}e^2 - \tfrac{3}{4}\gamma^2\} \sin (3nt - 2\Omega) \\
&\quad + \tfrac{1}{8}\gamma^2 \{1 - \tfrac{19}{4}e^2 - \tfrac{3}{4}\gamma^2\} \sin (nt + 2\omega - 2\Omega) \\
&\quad - \tfrac{1}{8}\gamma^2 \{1 - \tfrac{3}{2}e^2 - \tfrac{3}{4}\gamma^2\} \sin (nt - 2\omega + 2\Omega) - \tfrac{7}{8}e\gamma^2 \sin (4nt - \omega - 2\Omega) \\
&\quad + \tfrac{13}{16}e\gamma^2 \sin (2nt + \omega - 2\Omega) - \tfrac{3}{16}e\gamma^2 \sin (2nt - 3\omega + 2\Omega) \\
&\quad \mp \tfrac{1}{16}e\gamma^2 \sin (\omega - 2\Omega) - \tfrac{1}{16}e\gamma^2 \sin (3\omega - 2\Omega) + \tfrac{523}{128}e^4 \sin (5nt - 4\omega) \\
&\quad \pm \tfrac{9}{128}e^4 \sin (3nt - 4\omega) \mp \tfrac{1}{64}e^2\gamma^2 \sin (nt - 2\Omega) + \tfrac{1}{16}\gamma^4 \sin (5nt - 4\Omega) \\
&\quad + \tfrac{3}{128}\gamma^4 \sin (nt + 4\omega - 4\Omega) - \tfrac{1}{128}\gamma^4 \sin (nt - 4\omega + 4\Omega) \\
&\quad - \tfrac{3}{32}\gamma^4 \sin (3nt + 2\omega - 4\omega) - \tfrac{67}{32}e^2\gamma^2 \sin (5nt - 2\omega - 2\Omega) \\
&\quad - \tfrac{17}{64}e^2\gamma^2 \sin (3nt - 4\omega + 2\Omega) \pm \tfrac{1}{64}e^2\gamma^2 \sin (nt - 4\omega + 2\Omega) \Big\}
\end{aligned}
\right\} . \quad (233)
$$

This equation will give the value of $r_1\, dv_1 \cos \theta_1 \cos v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

From equations (232) and (233) we get

$$\left.\begin{aligned}
&\left\{2r_1\, dv_1 \cos\theta_1 \sin v_1 - \frac{\cos v_1}{\cos\theta_1}\, dr_1\right\} = \\
&2andt\Big\{ \{1 - e^2 - \tfrac{1}{4}\gamma^2 + \tfrac{1}{2}e^2\gamma^2 + \tfrac{11}{64}e^4 + \tfrac{7}{64}\gamma^4\} \sin nt \\
&\quad + \tfrac{5}{4}e\, \{1 - \tfrac{13}{10}e^2 - \tfrac{7}{20}\gamma^2\} \sin(2nt - \omega) - \tfrac{1}{4}e\, \{1 - \tfrac{1}{4}e^2 - \tfrac{3}{4}\gamma^2\} \sin\omega \\
&\quad + \{\tfrac{13}{8}e^2 - \tfrac{41}{16}e^4 + \tfrac{3}{32}\gamma^4 - \tfrac{21}{32}e^2\gamma^2\} \sin(3nt - 2\omega) \\
&\quad \pm \tfrac{1}{8}e^2\, \{1 + \tfrac{1}{6}e^2 - \tfrac{1}{4}\gamma^2\} \sin(nt - 2\omega) + \tfrac{103}{48}e^3 \sin(4nt - 3\omega) \\
&\quad \pm \tfrac{1}{12}e^3 \sin(2nt - 3\omega) - \tfrac{1}{4}\gamma^2\, \{1 - \tfrac{141}{16}e^2 - \tfrac{3}{4}\gamma^2\} \sin(3nt - 2\Omega) \\
&\quad + \tfrac{1}{8}\gamma^2\, \{1 - \tfrac{13}{4}e^2 - \tfrac{3}{4}\gamma^2\} \sin(nt + 2\omega - 2\Omega) \\
&\quad - \tfrac{1}{8}\gamma^2\, \{1 - e^2 - \tfrac{3}{4}\gamma^2\} \sin(nt - 2\omega + 2\Omega) - \tfrac{13}{16}e\gamma^2 \sin(4nt - \omega - 2\Omega) \\
&\quad + \tfrac{23}{32}e\gamma^2 \sin(2nt + \omega - 2\Omega) - \tfrac{5}{32}e\gamma^2 \sin(2nt - 3\omega + 2\Omega) \\
&\quad \mp \tfrac{1}{32}e\gamma^2 \sin(\omega - 2\Omega) - \tfrac{1}{32}e\gamma^2 \sin(3\omega - 2\Omega) + \tfrac{1097}{384}e^4 \sin(5nt - 4\omega) \\
&\quad \pm \tfrac{9}{128}e^4 \sin(3nt - 4\omega) \mp \tfrac{1}{64}e^2\gamma^2 \sin(nt - 2\Omega) \\
&\quad - \tfrac{59}{32}e^2\gamma^2 \sin(5nt - 2\omega - 2\Omega) - \tfrac{13}{64}e^2\gamma^2 \sin(3nt - 4\omega + 2\Omega) \\
&\quad \pm \tfrac{1}{64}e^2\gamma^2 \sin(nt - 4\omega + 2\Omega) + \tfrac{1}{16}\gamma^4 \sin(5nt - 4\Omega) \\
&\quad + \tfrac{3}{128}\gamma^4 \sin(nt + 4\omega - 4\Omega) - \tfrac{1}{128}\gamma^4 \sin(nt - 4\omega + 4\Omega) \\
&\quad - \tfrac{3}{32}\gamma^4 \sin(3nt + 2\omega - 4\Omega) \Big\}
\end{aligned}\right\} . \quad (234)$$

This equation will give the value of $\left\{2r_1\, dv_1 \cos\theta_1 \cos v_1 + \dfrac{\sin v_1}{\cos\theta_1}\, dr_1\right\}$ by using the lower signs and changing *sin* to *cos* in the second member.

From equations (221) and (223) we get

$$\left.\begin{aligned}
&d\theta_1 \cos\theta_1 \sin v_1 = \\
&ndt\Big\{ \tfrac{1}{2}\gamma\, \{1 - 4e^2 - \tfrac{5}{8}\gamma^2\} \sin(2nt - \Omega) + \tfrac{1}{2}\gamma\, \{1 - \tfrac{3}{8}\gamma^2\} \sin\Omega \\
&\quad + \tfrac{3}{2}e\gamma \sin(3nt - \omega - \Omega) + \tfrac{1}{2}e\gamma \sin(nt - \omega + \Omega) - \tfrac{1}{2}e\gamma \sin(nt + \omega - \Omega) \\
&\quad \mp \tfrac{1}{2}e\gamma \sin(nt - \omega - \Omega) + \tfrac{13}{4}e^2\gamma \sin(4nt - 2\omega - \Omega) \\
&\quad + \tfrac{1}{8}\gamma\, \{5e^2 - \gamma^2\} \sin(2nt - 2\omega + \Omega) \mp \tfrac{5}{8}e^2\gamma \sin(2nt - 2\omega - \Omega) \\
&\quad - \tfrac{1}{16}\gamma^3 \sin(4nt - 3\Omega) \mp \tfrac{1}{16}\gamma^3 \sin(2nt - 3\Omega) + \tfrac{1}{8}\gamma^3 \sin(2nt + 2\omega - 3\Omega) \Big\}
\end{aligned}\right\} . \quad (235)$$

This equation will give the value of $d\theta_1 \cos\theta_1 \cos v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

Multiplying equation (235) by the value of $2r_1$ it becomes

$$\left.\begin{array}{l} 2r_1\, d\theta_1 \cos\theta_1 \sin v_1 = \\ 2andt\Big\{ \tfrac{1}{2}\gamma\,\{1 - \tfrac{9}{2}e^2 - \tfrac{5}{8}\gamma^2\}\sin(2nt - \Omega) + \tfrac{1}{2}\gamma\,\{1 - \tfrac{1}{2}e^2 - \tfrac{3}{8}\gamma^2\}\sin\Omega \\ \quad + \tfrac{5}{4}e\gamma\sin(3nt - \omega - \Omega) + \tfrac{1}{4}e\gamma\sin(nt - \omega + \Omega) - \tfrac{3}{4}e\gamma\sin(nt + \omega - \Omega) \\ \quad \mp \tfrac{1}{4}e\gamma\sin(nt - \omega - \Omega) + \tfrac{19}{8}e^2\gamma\sin(4nt - 2\omega - \Omega) \\ \quad + \tfrac{1}{4}\gamma\,\{e^2 - \tfrac{1}{2}\gamma^2\}\sin(2nt - 2\omega + \Omega) \mp \tfrac{1}{4}e^2\gamma\sin(2nt - 2\omega - \Omega) \\ \quad - \tfrac{1}{16}\gamma^3\sin(4nt - 3\Omega) \mp \tfrac{1}{16}\gamma^3\sin(2nt - 3\Omega) + \tfrac{1}{8}\gamma^3\sin(2nt + 2\omega - 3\Omega) \\ \quad + \tfrac{1}{8}e^2\gamma\sin(2\omega - \Omega)\Big\} \end{array}\right\}. \quad (236)$$

This equation will give the value of $r_1\, d\theta_1 \cos\theta_1 \cos v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

Equations (194) and (230) will give

$$dr_1 \sin\theta_1 = andt\left\{\begin{array}{l} \tfrac{1}{2}e\gamma\cos(\omega - \Omega) - \tfrac{1}{2}e\gamma\cos(2nt - \omega - \Omega) \\ \qquad + e^2\gamma\cos(nt - \Omega) - e^2\gamma\cos(3nt - 2\omega - \Omega) \end{array}\right\}. \quad (237)$$

If we multiply this by the value of $\sin v_1$ given in equation (221), we shall obtain

$$\left.\begin{array}{l} dr_1 \sin\theta_1 \sin v_1 = \\ andt\Big\{ \tfrac{1}{4}e\gamma\sin(nt + \omega - \Omega) + \tfrac{1}{4}e\gamma\sin(nt - \omega + \Omega) - \tfrac{1}{4}e\gamma\sin(3nt - \omega - \Omega) \\ \quad \pm \tfrac{1}{4}e\gamma\sin(nt - \omega - \Omega) + e^2\gamma\sin(2nt - \Omega) - \tfrac{3}{4}e^2\gamma\sin(4nt - 2\omega - \Omega) \\ \quad - \tfrac{1}{4}e^2\gamma\sin(2\omega - \Omega) \pm \tfrac{1}{4}e^2\gamma\sin(2nt - 2\omega - \Omega) + \tfrac{1}{4}e^2\gamma\sin(2nt - 2\omega + \Omega)\Big\} \end{array}\right\}. \quad (238)$$

This equation will give the value of $dr_1 \sin\theta_1 \cos v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

Adding equations (236) and (228), we get

$$\left.\begin{array}{l} \{2r_1\, d\theta_1 \cos\theta_1 \sin v_1 + dr_1 \sin\theta_1 \sin v_1\} = \\ 2andt\Big\{ \tfrac{1}{2}\gamma\,\{1 - \tfrac{7}{2}e^2 - \tfrac{5}{8}\gamma^2\}\sin(2nt - \Omega) + \tfrac{1}{2}\gamma\,\{1 - \tfrac{1}{2}e^2 - \tfrac{3}{8}\gamma^2\}\sin\Omega \\ \quad + \tfrac{9}{8}e\gamma\sin(3nt - \omega - \Omega) + \tfrac{3}{8}e\gamma\sin(nt - \omega + \Omega) - \tfrac{5}{8}e\gamma\sin(nt + \omega - \Omega) \\ \quad \mp \tfrac{1}{8}e\gamma\sin(nt - \omega - \Omega) + 2e^2\gamma\sin(4nt - 2\omega - \Omega) \\ \quad + \tfrac{1}{8}\gamma\,\{3e^2 - \gamma^2\}\sin(2nt - 2\omega + \Omega) \mp \tfrac{1}{8}e^2\gamma\sin(2nt - 2\omega - \Omega) \\ \quad - \tfrac{1}{16}\gamma^3\sin(4nt - 3\Omega) \mp \tfrac{1}{16}\gamma^3\sin(2nt - 3\Omega) + \tfrac{1}{8}\gamma^3\sin(2nt + 2\omega - 3\Omega)\Big\} \end{array}\right\}. \quad (239)$$

This equation will give the value of $2r_1\, d\theta_1 \cos\theta_1 \cos v_1 + dr_1 \sin\theta_1 \sin v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

We have thus developed all the factors of equation (B) which are affected by the signs of integration, and equations (195) and (221) will give for the value of the factors, without the sign of integration, the following expression:

$$
\begin{aligned}
&\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\cos\theta_1\sin v_1 = \\
&\left.\begin{aligned}
&\{1-\tfrac{1}{2}e^2+\tfrac{1}{4}\gamma^2-\tfrac{1}{64}e^4-\tfrac{5}{64}\gamma^4-\tfrac{1}{8}e^2\gamma^2\}\sin nt+e\{1-\tfrac{3}{4}e^2+\tfrac{1}{4}\gamma^2\}\sin(2nt-\omega)\\
&-e\{1+\tfrac{1}{2}e^2+\tfrac{1}{4}\gamma^2\}\sin\omega+\tfrac{9}{8}e^2\{1-e^2+\tfrac{1}{4}\gamma^2\}\sin(3nt-2\omega)\\
&\pm\tfrac{1}{8}e^2\{1+\tfrac{1}{3}e^2+\tfrac{1}{4}\frac{\gamma^4}{e^2}+\tfrac{1}{4}\gamma^2\}\sin(nt-2\omega)\mp\tfrac{1}{4}\gamma^2\{1-\tfrac{7}{16}e^2-\tfrac{1}{4}\gamma^2\}\sin(nt-2\Omega)\\
&+\tfrac{1}{8}\gamma^2\{1-\tfrac{1}{2}e^2-\tfrac{1}{4}\gamma^2\}\sin(nt+2\omega-2\Omega)-\tfrac{1}{8}\gamma^2\{1-\tfrac{1}{4}e^2-\tfrac{1}{4}\gamma^2\}\sin(nt-2\omega+2\Omega)\\
&+\tfrac{4}{3}e^3\sin(4nt-3\omega)\pm\tfrac{1}{12}e^3\sin(2nt-3\omega)\\
&\pm\tfrac{1}{8}e\gamma^2\sin(\omega-2\Omega)-\tfrac{1}{8}e\gamma^2\sin(3\omega-2\Omega)\\
&\mp\tfrac{1}{4}e\gamma^2\sin(2nt-\omega-2\Omega)+\tfrac{1}{8}e\gamma^2\sin(2nt+\omega-2\Omega)-\tfrac{1}{8}e\gamma^2\sin(2nt-3\omega+2\Omega)\\
&+\tfrac{625}{384}e^4\sin(5nt-4\omega)\pm\tfrac{9}{128}e^4\sin(3nt-4\omega)+\tfrac{9}{64}e^2\gamma^2\sin(3nt-2\Omega)\\
&-\tfrac{9}{64}e^2\gamma^2\sin(3nt-4\omega+2\Omega)\pm\tfrac{1}{64}e^2\gamma^2\sin(nt-4\omega+2\Omega)\\
&\mp\tfrac{9}{32}e^2\gamma^2\sin(3nt-2\omega-2\Omega)\mp\tfrac{1}{32}\gamma^4\sin(nt+2\omega-4\Omega)\\
&-\tfrac{1}{128}\gamma^4\sin(nt-4\omega+4\Omega)+\tfrac{3}{128}\gamma^4\sin(nt+4\omega-4\Omega)
\end{aligned}\right\}. \quad (240)
\end{aligned}
$$

This equation will give the value of $\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\cos\theta_1\cos v_1$ by using the lower signs and changing *sin* to *cos* in the second member.

Equations (120) and (221) will give

$$
\left.\begin{aligned}
&\tan\theta_1\sin v_1 = \\
&\pm\tfrac{1}{2}\gamma\{1-\tfrac{1}{4}\gamma^2\}\cos\Omega\mp\tfrac{1}{2}\gamma\{1-4e^2\}\cos(2nt-\Omega)\\
&\mp e\gamma\{1-\tfrac{27}{8}e^2\}\cos(3nt-\omega-\Omega)\pm e\gamma\{1-\tfrac{7}{8}e^2\}\cos(nt+\omega-\Omega)\\
&\pm\tfrac{1}{8}\gamma^3\cos(2nt-2\omega+\Omega)\mp\tfrac{1}{8}\gamma^3\cos(2nt+2\omega-3\Omega)\pm\tfrac{1}{8}\gamma^3\cos(4nt-3\Omega)\\
&\mp\tfrac{13}{8}e^2\gamma\cos(4nt-2\omega-\Omega)\mp\tfrac{3}{8}e^2\gamma\cos(2\omega-\Omega)\mp\tfrac{1}{4}e\gamma^3\cos(nt-\omega+\Omega)\\
&\pm\tfrac{1}{4}e\gamma^3\cos(3nt-3\omega+\Omega)\pm\tfrac{1}{2}e\gamma^3\cos(5nt-\omega-3\Omega)\\
&\mp\tfrac{3}{4}e\gamma^3\cos(3nt+\omega-3\Omega)\pm\tfrac{1}{4}e\gamma^3\cos(nt+3\omega-3\Omega)\\
&\mp\tfrac{59}{24}e^3\gamma\cos(5nt-3\omega-\Omega)-\tfrac{1}{24}e^3\gamma\cos(nt-3\omega+\Omega)
\end{aligned}\right\}. \quad (241)
$$

This equation will give the value of $\tan\theta_1\cos v_1$ by using the lower signs and changing *cos* to *sin* in the second member.

Lastly, we get from equation (126) and (221)

$$\left.\begin{aligned}\frac{\sin v_1}{r_1^2} = \\ \frac{1}{a^2}\Big\{ &\{1-\tfrac{1}{2}e^2\}\sin nt + 2e\{1-\tfrac{3}{4}e^2\}\sin(2nt-\omega) \mp \tfrac{1}{8}e^2\sin(nt-2\omega) \\ &+\tfrac{15}{8}e^3\sin(4nt-3\omega)+\tfrac{27}{8}e^2\sin(3nt-2\omega)\mp\tfrac{1}{6}e^3\sin(2nt-3\omega) \\ &-\tfrac{1}{8}\gamma^2\sin(3nt-2\Omega)\mp\tfrac{1}{8}\gamma^2\sin(nt-2\Omega)+\tfrac{1}{8}\gamma^2\sin(nt+2\omega-2\Omega) \\ &-\tfrac{1}{8}\gamma^2\sin(nt-2\omega+2\Omega)-\tfrac{1}{2}e\gamma^2\sin(4nt-\omega-2\Omega) \\ &\mp\tfrac{1}{4}e\gamma^2\sin(2nt-\omega-2\Omega)+\tfrac{5}{8}e\gamma^2\sin(2nt+\omega-2\Omega) \\ &-\tfrac{1}{4}e\gamma^2\sin(2nt-3\omega+2\Omega)\pm\tfrac{1}{8}e\gamma^2\sin(\omega-2\Omega)\Big\}\end{aligned}\right\}. \quad (242)$$

This equation will give the value of $\frac{\cos v_1}{r_1^2}$ by using the lower signs and changing *sin* to *cos* in the second member.

15. Having completed the development of all the functions which enter into the values of $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$, we shall now develop equations (B), (C), and (D) in a more convenient form for actual computation of the perturbations. For this purpose we shall observe that equation (B) contains the term $\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e\cos\theta_0$ $\cos\theta\sin(v-\omega)$, which is not multiplied by the disturbing forces; but since v and θ denote the true values of the longitude and latitude of the moon, they must be equal to the elliptical values of these quantities increased by the effects of the disturbing forces. We shall therefore put $v=v_1+(\delta v)$, and $\theta=\theta_1+(\delta\theta)$, (δv) and $(\delta\theta)$ denoting the complete values of the disturbing forces. If we now put $\frac{d\delta_2 r}{dt}$, and $\frac{d\delta_3 r}{dt}$ for the parts of the preceding term arising from the values of (δv) and $(\delta\theta)$, respectively, we shall have

$$\left.\begin{aligned}\frac{d\delta_2 r}{dt} &= \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e\cos\theta_0\cos\theta_1\{\cos(v_1-\omega)(\delta v)-\tfrac{1}{2}\sin(v_1-\omega)(\delta v)^2\} \\ \frac{d\delta_3 r}{dt} &= -\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e\cos\theta_0\sin\theta_1\sin(v_1-\omega)(\delta\theta)\end{aligned}\right\}. \quad (243)$$

In like manner if the corresponding parts of equations (C) and (D) be represented by $\frac{d\delta_1 v}{dt}$, $\frac{d\delta_2 v}{dt}$, $\frac{d\delta_1\theta}{dt}$, and $\frac{d\delta_2\theta}{dt}$, we shall have

$$\left.\begin{aligned}\frac{d\delta_1 v}{dt} &= -2\frac{dv_1}{r_1 dt}(\delta r) + 3\frac{dv_1}{r_1^2 dt}(\delta r)^2 - 4\frac{dv_1}{r_1^3 dt}(\delta r)^3\\ \frac{d\delta_2 v}{dt} &= +2\frac{dv_1}{dt}\tan\theta_1(\delta\theta) + \frac{dv_1}{dt}(\delta\theta)^2 - 4\frac{dv_1}{dt}\tan\theta_1(\delta r)(\delta\theta)\end{aligned}\right\} . \quad (244)$$

$$\left.\begin{aligned}\frac{d\delta_1\theta}{dt} &= -2\frac{d\theta_1}{r_1 dt}(\delta r) + 3\frac{d\theta_1}{r_1^2 dt}(\delta r)^2 - 4\frac{d\theta_1}{r_1^3 dt}(\delta r)^3 - 3\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\frac{\tan\theta_1}{r_1^4}(\delta r)^2(\delta v)\\ \frac{d\delta_2\theta}{dt} &= -\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\frac{\tan\theta_1}{r_1^2}(\delta v) - \tfrac{1}{2}\frac{d\theta_1}{dt}(\delta v)^2 + 2\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\frac{\tan\theta_1}{r_1^3}(\delta r)(\delta v)\\ &\quad + \tfrac{1}{6}\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\frac{\tan\theta_1}{r_1^2}(\delta v)^3 + \frac{d\theta_1}{r_1 dt}(\delta r)(\delta v)^2\end{aligned}\right\} . \quad (245)$$

In the calculations it will be found convenient to denote by δr, δv, and $\delta\theta$ the terms of perturbation arising from the first power of the disturbing force; and by $\delta^2 r$, $\delta^2 v$, and $\delta^2\theta$, the terms arising from the square of the disturbing force, and so on; then we shall have

$$\left.\begin{aligned}(\delta r) &= \delta r + \delta^2 r + \delta^3 r + \text{etc.},\\ (\delta v) &= \delta v + \delta^2 v + \delta^3 v + \text{etc.},\\ (\delta\theta) &= \delta\theta + \delta^2\theta + \delta^3\theta + \text{etc.}\end{aligned}\right\} . \quad (246)$$

We shall now put

$$\left.\begin{aligned}\frac{d\delta_0 r}{dt} = \left\{\frac{dr_1}{dt} + \frac{d\delta_2 r}{dt} + \frac{d\delta_3 r}{dt}\right\}\Bigg\{&\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\int\left(\frac{dR}{dv}\right)dt\\ &+ \frac{1+\gamma^2}{a\mu(1-e^2)}\left[\int\left(\frac{dR}{dv}\right)dt\right]^2 + \text{etc.}\Bigg\}\end{aligned}\right\} . \quad (247)$$

Also,
$$\left.\begin{aligned}\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\cos\theta\cos v &= c_1\cos\beta\\ \frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\cos\theta\sin v &= c_1\sin\beta\end{aligned}\right\} . \quad (248)$$

$$\left.\begin{aligned}r^2\{d\theta\sin\theta\sin v - dv\cos\theta\cos v\} &= c_2\cos\beta\\ r^2\{d\theta\sin\theta\cos v + dv\cos\theta\sin v\} &= c_2\sin\beta\end{aligned}\right\} . \quad (249)$$

$$\left.\begin{aligned}2rdv\cos\theta\sin v - \frac{\cos v}{\cos\theta}dr &= c_3\sin\beta\\ 2rdv\cos\theta\cos v + \frac{\sin v}{\cos\theta}dr &= c_3\cos\beta\end{aligned}\right\} . \quad (250)$$

$$\left.\begin{aligned}\{2rd\theta\cos\theta + dr\sin\theta\}\sin v &= c_4\sin\beta\\ \{2rd\theta\cos\theta + dr\sin\theta\}\cos v &= c_4\cos\beta\end{aligned}\right\} . \quad (251)$$

If we substitute these quantities in equation (B), and put

$$\left.\left(\frac{d\delta_1 r}{dt}\right) = \frac{\begin{array}{l}\frac{1}{\sqrt{a\mu}} c_1 \cos\beta \int \left\{ c_2 \cos\beta\left(\frac{dR}{dr}\right) + c_3 \sin\beta\left(\frac{dR}{dv}\right) + c_4 \sin\beta\left(\frac{dR}{d\theta}\right)\right\} \\ -\frac{1}{\sqrt{a\mu}} c_1 \sin\beta \int \left\{ c_2 \sin\beta\left(\frac{dR}{dr}\right) + c_3 \cos\beta\left(\frac{dR}{dv}\right) + c_4 \cos\beta\left(\frac{dR}{d\theta}\right)\right\}\end{array}}{1 - \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\int\left(\frac{dR}{dv}\right)dt}\right\} . \quad (252)$$

$$\left(\frac{d\delta_1 r}{dt}\right) = \frac{d\delta_1 r}{dt}\left\{1 - \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\int\left(\frac{dR}{dv}\right)dt\right\}^{-1}. \quad (252')$$

$$= \frac{d\delta_1 r}{dt} + \frac{d\delta_4 r}{dt}. \quad (252'')$$

we shall have

$$\frac{dr}{dt} = \frac{dr_1}{dt} + \frac{d\delta_0 r}{dt} + \left(\frac{d\delta_1 r}{dt}\right) + \frac{d\delta_2 r}{dt} + \frac{d\delta_3 r}{dt} = \frac{dr_1}{dt} + \frac{d\delta r}{dt}, \quad (253)$$

for the complete value of $\frac{dr}{dt}$; r_1 denoting the elliptical value of the radius vector.

If we now put

$$\frac{d\delta_0 v}{dt} = -\frac{1}{r^2\cos^2\theta}\int\left(\frac{dR}{dv}\right)dt, \quad (254)$$

we shall have

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{d\delta_0 v}{dt} + \frac{d\delta_1 v}{dt} + \frac{d\delta_2 v}{dt} = \frac{dv_1}{dt} + \frac{d\delta v}{dt}. \quad (255)$$

Lastly we shall put

$$\left.\begin{array}{l}\frac{d\delta_0\theta}{dt} = \frac{\sin v}{r^2}\int\left\{\tan\theta\cos v\left(\frac{dR}{dv}\right) - \sin v\left(\frac{dR}{d\theta}\right)\right\}dt \\ \qquad -\frac{\cos v}{r^2}\int\left\{\tan\theta\sin v\left(\frac{dR}{dv}\right) + \cos v\left(\frac{dR}{d\theta}\right)\right\}dt\end{array}\right\} . \quad (256)$$

and we shall have

$$\frac{d\theta}{dt} = \frac{d\theta_1}{dt} + \frac{d\delta_0\theta}{dt} + \frac{d\delta_1\theta}{dt} + \frac{d\delta_2\theta}{dt} = \frac{d\theta_1}{dt} + \frac{d\delta\theta}{dt}. \quad (257)$$

CHAPTER III.

PERTURBATIONS OF r, v, AND θ ARISING FROM THE FIRST POWER OF THE DISTURBING FORCE.

16. We shall in this chapter develop the perturbations of the moon's radius vector, longitude and latitude, in so far as they depend on the first power of the disturbing force. The following terms of the equations given in the preceding article will therefore give the required precision.

$$\frac{d\delta_0 r}{dt} = \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}} \frac{dr_1}{dt} \int \left(\frac{dR}{dv}\right) dt; \tag{258}$$

$$\left.\begin{aligned} \frac{d\delta_1 r}{dt} = & \frac{1}{\sqrt{a\mu}} c_1 \cos\beta \int \left\{ c_2 \cos\beta \left(\frac{dR}{dr}\right) + c_3 \sin\beta \left(\frac{dR}{dv}\right) + c_4 \sin\beta \left(\frac{dR}{d\theta}\right) \right\} \\ & - \frac{1}{\sqrt{a\mu}} c_1 \sin\beta \int \left\{ c_2 \sin\beta \left(\frac{dR}{dr}\right) + c_3 \cos\beta \left(\frac{dR}{dv}\right) + c_4 \cos\beta \left(\frac{dR}{d\theta}\right) \right\} \end{aligned}\right\}; \tag{259}$$

$$\left.\begin{aligned} \frac{d\delta_2 r}{dt} &= \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}} \mu e \cos\theta_0 \{\cos\theta_1 \cos(v_1 - \omega)\delta v\} \\ \frac{d\delta_3 r}{dt} &= -\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}} \mu e \cos\theta_0 \sin\theta_1 \sin(v_1 - \omega)\delta\theta \end{aligned}\right\}; \tag{260}$$

$$\frac{d\delta_0 v}{dt} = -\frac{1}{r_1^2 \cos^2\theta_1} \int \left(\frac{dR}{dv}\right) dt; \tag{261}$$

$$\frac{d\delta_1 v}{dt} = -2\frac{dv_1}{r_1 dt}\delta r, \qquad \frac{d\delta_2 v}{dt} = 2\frac{dv_1}{dt}\tan\theta_1 \delta\theta; \tag{262}$$

$$\left.\begin{aligned} \frac{d\delta_0 \theta}{dt} = & \frac{\sin v_1}{r_1^2} \int \left\{ \tan\theta_1 \cos v_1 \left(\frac{dR}{dv}\right) - \sin v_1 \left(\frac{dR}{d\theta}\right) \right\} dt \\ & - \frac{\cos v_1}{r_1^2} \int \left\{ \tan\theta_1 \sin v_1 \left(\frac{dR}{dv}\right) + \cos v_1 \left(\frac{dR}{d\theta}\right) \right\} dt \end{aligned}\right\}; \tag{263}$$

$$\frac{d\delta_1 \theta}{dt} = -2\frac{d\theta_1}{r_1 dt}\delta r, \qquad \frac{d\delta_2 \theta}{dt} = -\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}} \frac{\tan\theta_1}{r_1^2}\delta v. \tag{264}$$

17. We shall first give the development of equation (258). The value of $\left(\frac{dR}{dv}\right)$ is given in equation (219); and if we multiply it by dt and take the integral, we shall obtain

$$\int\left(\frac{dR}{dv}\right)dt =$$

$$\left.\begin{array}{l}
\frac{\overline{m}^2}{an}\Big\{ -\{1-\frac{5}{2}e^2-\frac{5}{2}e'^2-\frac{1}{4}\gamma^2+\frac{23}{16}e^4+\frac{13}{16}e'^4+\frac{5}{64}\gamma^4 \\
\qquad\qquad +\frac{25}{4}e^2e'^2+\frac{5}{8}e^2\gamma^2+\frac{5}{8}e'^2\gamma^2\}[9.9088263]\cos 2(nt-n't) \\
\quad -e\{1-\frac{19}{8}e^2-\frac{5}{2}e'^2-\frac{1}{4}\gamma^2\}[9.7211859]\cos(3nt-2n't-\omega) \\
\quad +e\{1-\frac{13}{24}e^2-\frac{5}{2}e'^2-\frac{1}{4}\gamma^2\}[0.7235905]\cos(nt-2n't+\omega) \\
\quad +e'\{1-\frac{5}{2}e^2-\frac{1}{8}e'^2-\frac{1}{4}\gamma^2\}[9.5905857]\cos(2nt-n't-\omega') \\
\quad -e'\{1-\frac{5}{2}e^2-\frac{123}{56}e'^2-\frac{1}{4}\gamma^2\}[0.4708152]\cos(2nt-3n't+\omega') \\
\quad -e^2\{1-\frac{5}{2}e^2-\frac{5}{2}e'^2-\frac{1}{4}\gamma^2\}[9.5905858]\cos(4nt-2n't-2\omega) \\
\quad +e^2\{1-\frac{5}{2}e'^2-\frac{1}{4}\gamma^2+\frac{1}{40}\frac{\gamma^4}{e^2}\}[1.3990922]\cos 2(n't-\omega) \\
\quad -e'^2\{1-\frac{5}{2}e^2-\frac{115}{51}e'^2-\frac{1}{4}\gamma^2\}[0.8748583]\cos(2nt-4n't+2\omega') \\
\quad +\gamma^2\{1-\frac{33}{2}e^2-\frac{5}{2}e'^2-\frac{3}{4}\gamma^2\}[8.6874958]\cos(4nt-2n't-2\Omega) \\
\quad +\gamma^2\{1-\frac{1}{8}e^2-\frac{5}{2}e'^2-\frac{3}{4}\gamma^2\}[0.5751834]\cos 2(n't-\Omega) \\
\quad -\gamma^2\{1-\frac{147}{8}e^2-\frac{5}{2}e'^2-\frac{3}{4}\gamma^2\}[9.3067663]\cos(2nt-2n't+2\omega-2\Omega) \\
\quad +\gamma^2\{1-\frac{107}{8}e^2-\frac{5}{2}e'^2-\frac{3}{4}\gamma^2\}[9.3067663]\cos(2nt-2n't-2\omega+2\Omega) \\
\quad +ee'\{1-\frac{19}{8}e^2-\frac{1}{8}e'^2-\frac{1}{4}\gamma^2\}[9.4089059]\cos(3nt-n't-\omega-\omega') \\
\quad -ee'\{1-\frac{13}{24}e^2-\frac{1}{8}e'^2-\frac{1}{4}\gamma^2\}[0.3859475]\cos(nt-n't+\omega-\omega') \\
\quad +ee'\{1-\frac{91}{168}e^2-\frac{123}{56}e'^2-\frac{1}{4}\gamma^2\}[1.3076450]\cos(nt-3n't+\omega+\omega') \\
\quad -ee'\{1-\frac{19}{8}e^2-\frac{123}{56}e'^2-\frac{1}{4}\gamma^2\}[0.2768230]\cos(3nt-3n't-\omega+\omega') \\
\quad -e\gamma^2[9.8205005]\cos(nt-2n't-\omega+2\Omega) \\
\quad +e\gamma^2[9.0643451]\cos(5nt-2n't-\omega-2\Omega) \\
\quad +e\gamma^2[9.1191259]\cos(3nt-2n't-3\omega+2\Omega) \\
\quad +e\gamma^2[0.1215305]\cos(nt-2n't+3\omega-2\Omega) \\
\quad -e\gamma^2[9.6631939]\cos(3nt-2n't+\omega-2\Omega) \\
\quad -e\gamma^2[9.6895747]\cos(nt+2n't-\omega-2\Omega) \\
\quad -e'\gamma^2[8.3781096]\cos(4nt-n't-\omega'-2\Omega) \\
\quad +e'\gamma^2[0.9431601]\cos(3n't-\omega'-2\Omega) \\
\quad +e'\gamma^2[9.2400838]\cos(4nt-3n't+\omega'-2\Omega) \\
\quad -e'\gamma^2[0.5751834]\cos(n't+\omega'-2\Omega) \\
\quad -e'\gamma^2[8.9885257]\cos(2nt-n't-2\omega-\omega'+2\Omega) \\
\quad -e'\gamma^2[9.8687552]\cos(2nt-3n't+2\omega+\omega'-2\Omega)
\end{array}\right\}. \quad (265)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ e'\gamma^2 [8.9885257] \cos (2nt - n't + 2\omega - \omega' - 2\Omega) \\
&+ e'\gamma^2 [9.8687552] \cos (2nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&+ ee'^2 [1.7370405] \cos (nt - 4n't + \omega + 2\omega') \\
&- ee'^2 [0.6740186] \cos (3nt - 4n't - \omega + 2\omega') \\
&+ e^2e' [9.2811996] \cos (4nt - n't - 2\omega - \omega') \\
&+ e^2e' [1.7670689] \cos (3n't - 2\omega - \omega') \\
&- e^2e' [0.1431738] \cos (4nt - 3n't - 2\omega + \omega') \\
&- e^2e' [1.3990922] \cos (n't - 2\omega + \omega') \\
&- e^3 [9.5080426] \cos (5nt - 2n't - 3\omega) - e^3 [9.5804302] \cos (nt + 2n't - 3\omega) \\
&- e'^3 [8.1778736] \cos (2n't + n't - 3\omega') \\
&- e'^3 [1.2105879] \cos (2nt - 5n't + 3\omega') \\
&- e^4 [9.4600584] \cos (6nt - 2n't - 4\omega) - e^4 [8.6396131] \cos (2nt + 2n't - 4\omega) \\
&- e'^4 [8.4635218] \cos (2nt + 2n't - 4\omega') \\
&- e'^4 [1.5080329] \cos (2nt - 6n't + 4\omega') \\
&- \gamma^4 [7.7788171] \cos (6nt - 2n't - 4\Omega) \\
&+ \gamma^4 [8.3864658] \cos (4nt - 2n't + 2\omega - 4\Omega) \\
&- \gamma^4 [8.7047063] \cos (2nt - 2n't + 4\omega - 4\Omega) \\
&+ \gamma^4 [8.2136443] \cos (2nt + 2n't - 4\Omega) \\
&+ e^2\gamma^2 [8.9885258] \cos (4nt - 2n't - 4\omega + 2\Omega) \\
&+ e^2\gamma^2 [0.7970322] \cos (2n't - 4\omega + 2\Omega) \\
&+ e^2\gamma^2 [9.3016959] \cos (6nt - 2n't - 2\omega - 2\Omega) \\
&- e^2\gamma^2 [8.8157043] \cos (2nt + 2n't - 2\omega - 2\Omega) \\
&+ e'^2\gamma^2 [1.2035723] \cos (4n't - 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 [9.6341252] \cos (4nt - 4n't + 2\omega' - 2\Omega) \\
&- e'^2\gamma^2 [0.2727983] \cos (2nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 [0.2727983] \cos (2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- e^2e'^2 [0.5372152] \cos (4nt - 4n't - 2\omega + 2\omega') \\
&+ e^2e'^2 [2.0274811] \cos (4n't - 2\omega - 2\omega') \\
&- ee'^3 [8.0070329] \cos (3nt + n't - \omega - 3\omega') \\
&+ ee'^3 [8.9406431] \cos (nt + n't + \omega - 3\omega') \\
&+ ee'^3 [2.1022582] \cos (nt - 5n't + \omega + 3\omega') \\
&- ee'^3 [1.0024131] \cos (3nt - 5n't - \omega + 3\omega') \\
&- e^3e' [0.0971214] \cos (nt + 3n't - 3\omega - \omega') \\
&+ e^3e' [9.3086199] \cos (nt + n't - 3\omega + \omega') \\
&+ e^3e' [9.2003663] \cos (5nt - n't - 3\omega - \omega') \\
&- e^3e' [0.0588606] \cos (5nt - 3n't - 3\omega + \omega')
\end{aligned}\right\} . \quad (265)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ ee'\gamma^2\,[9.6151630]\cos(5nt - 3n't - \omega + \omega' - 2\Omega)\\
&- ee'\gamma^2\,[0.2188210]\cos(3nt - 3n't + \omega + \omega' - 2\Omega)\\
&+ ee'\gamma^2\,[9.4828575]\cos(nt - n't - \omega - \omega' + 2\Omega)\\
&- ee'\gamma^2\,[9.8236155]\cos(5nt - n't - \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2\,[0.6083225]\cos(nt + 3n't - \omega - \omega' - 2\Omega)\\
&+ ee'\gamma^2\,[9.3509139]\cos(3nt - n't + \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2\,[0.4625469]\cos(nt - 3n't - \omega + \omega' + 2\Omega)\\
&+ ee'\gamma^2\,[9.6747430]\cos(3nt - 3n't - 3\omega + \omega' + 2\Omega)\\
&- ee'\gamma^2\,[9.7838875]\cos(nt - n't + 3\omega - \omega' - 2\Omega)\\
&- ee'\gamma^2\,[8.8068459]\cos(3nt - n't - 3\omega - \omega' + 2\Omega)\\
&+ ee'\gamma^2\,[0.7355482]\cos(nt - 3n't + 3\omega + \omega' - 2\Omega)\\
&+ ee'\gamma^2\,[9.4177643]\cos(nt + n't - \omega + \omega' - 2\Omega)\Big\}\\
&+\frac{\overline{m}^2}{a'n}\Big\{-\{1 + 2e^2 + 2e'^2 - \tfrac{13}{2}\gamma^2\}[9.6077963]\cos(nt - n't)\\
&- \{1 - 6e^2 - 6e'^2 - \tfrac{3}{4}\gamma^2\}[9.8269450]\cos 3(nt - n't)\\
&+ e\,[8.9885257]\cos(2nt - n't - \omega)\\
&- e\,[1.0980622]\cos(n't - \omega) - e\,[9.8721070]\cos(4nt - 3n't - \omega)\\
&+ e\,[0.6768697]\cos(2nt - 3n't + \omega) - e'\,[9.5740313]\cos(nt - \omega')\\
&- e'\,[0.1215305]\cos(nt - 2n't + \omega') - e'\,[0.5404797]\cos(3nt - 4n't + \omega')\\
&+ e'\,[9.8180959]\cos(3nt - 2n't - \omega') + ee'\,[8.9719713]\cos(2nt - \omega - \omega')\\
&- ee'\,[1.2741534]\cos(2n't - \omega - \omega')\\
&+ ee'\,[9.4828575]\cos(2nt - 2n't - \omega + \omega')\\
&- ee'\,[0.5797675]\cos(4nt - 4n't - \omega + \omega')\\
&- ee'\,[0.6589488]\cos(2nt - 2n't + \omega - \omega')\\
&+ ee'\,[9.8635870]\cos(4nt - 2n't - \omega - \omega')\\
&+ ee'\,[1.3945319]\cos(2nt - 4n't + \omega + \omega')\\
&+ e^2\,[9.2839671]\cos(3nt - n't - 2\omega) + e^2\,[9.6810058]\cos(nt + n't - 2\omega)\\
&- e^2\,[9.8669751]\cos(5nt - 3n't - 2\omega) - e^2\,[1.2361504]\cos(nt - 3n't + 2\omega)\\
&- e'^2\,[9.6810058]\cos(nt + n't - 2\omega') - e'^2\,[0.5055816]\cos(nt - 3n't + 2\omega')\\
&- e'^2\,[8.9037558]\cos(3nt - n't - 2\omega')\\
&- e'^2\,[1.0544214]\cos(3nt - 5n't + 2\omega')\\
&+ \gamma^2\,[0.0709766]\cos(nt + n't - 2\Omega) - \gamma^2\,[9.6027260]\cos(3nt - n't - 2\Omega)\\
&- \gamma^2\,[8.7447063]\cos(nt - n't - 2\omega + 2\Omega)\\
&- \gamma^2\,[8.7047063]\cos(nt - n't + 2\omega - 2\Omega)
\end{aligned}\right\} . \quad (265)
$$

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$$\left.\begin{aligned}
&-\gamma^2[0.2584269]\cos(nt-3n't+2\Omega)\\
&-\gamma^2[9.4036762]\cos(3nt-3n't+2\omega-2\Omega)\\
&+\gamma^2[9.4036762]\cos(3nt-3n't-2\omega+2\Omega)\\
&+\tfrac{15}{16}h''\frac{n}{\alpha''}\cos(\alpha''t-\beta'')\Big\}
\end{aligned}\right\} . \quad (265)$$

In finding the preceding integral we have supposed that $n' = 0.0748013\,n$, according to observation. The numbers in brackets are logarithms.

If we multiply the value of $\int\left(\frac{dR}{dv}\right)dt$ by $\frac{\sqrt{1+\gamma^2}}{\sqrt{au(1-e^2)}}\frac{dv_1}{dt}$ we shall have the value of $\frac{d\delta_0 r}{dt}$. Now we have

$$\left.\begin{aligned}
\frac{\sqrt{1+\gamma^2}}{\sqrt{1+e^2}}\frac{dv_1}{dt} = an\Big\{&e\{1+\tfrac{1}{8}e^2+\tfrac{1}{2}\gamma^2\}\sin(nt-\omega)\\
&+e^2\{1-\tfrac{1}{6}e^2+\tfrac{1}{2}\gamma^2\}\sin 2(nt-\omega)+\tfrac{9}{8}e^3\sin 3(nt-\omega)\\
&+\tfrac{4}{3}e^3\sin 4(nt-\omega)
\end{aligned}\right\} . \quad (266)$$

Hence we deduce

$$\left.\begin{aligned}
\frac{d\delta_0 r}{dt} = a\frac{\overline{m}^2}{\mu}n\Big\{&-e\{0.4053184-3.8027898e^2-1.013296e'^2\\
&\qquad\qquad+0.1013296\gamma^2\}\sin(3nt-2n't-\omega)\\
&+e\{0.4053184+11.83369e^2-1.013296e'^2+0.1013296\gamma^2\}\sin(nt-2n't+\omega)\\
&+e^2\{2.9089427+11.033309e^2-7.272357e'^2+0.7272357\gamma^2\}\sin 2(nt-n't)\\
&-e^2\{0.6684396-4.810491e^2-1.671099e'^2\\
&\qquad\qquad+0.1671099\gamma^2\}\sin(4nt-2n't-2\omega)\\
&+e^2\{2.2405031-0.127305e^2-5.601258e'^2+0.5601258\gamma^2\}\sin 2(n't-\omega)\\
&+ee'\{0.1943371-1.773042e^2-0.0242921e'^2\\
&\qquad\qquad+0.0485842\gamma^2\}\sin(3nt-n't-\omega-\omega')\\
&-ee'\{0.1943371+12.19986e^2-0.0242921e'^2\\
&\qquad\qquad+0.0485842\gamma^2\}\sin(nt-n't+\omega-\omega')\\
&-ee'\{1.478377-14.359879e^2-3.24715e'^2\\
&\qquad\qquad+0.369594\gamma^2\}\sin(3nt-3n't-\omega+\omega')\\
&+ee'\{1.478377+26.67878e^2-3.24715e'^2\\
&\qquad\qquad+0.369594\gamma^2\}\sin(nt-3n't+\omega+\omega')\\
&-e^3(0.9134414)\sin(5nt-2n't-3\omega)+e^3(14.72304)\sin(nt+2n't-3\omega)\\
&-e\gamma^2(0.1256777)\sin(3nt-2n't+\omega-2\Omega)\\
&+e\gamma^2(1.778650)\sin(nt-2n't-\omega+2\Omega)
\end{aligned}\right\} . \quad (267)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ e\gamma^2\,(0.02434814)\sin(5nt - 2n't - \omega - 2\Omega)\\
&+ e\gamma^2\,(1.879980)\sin(nt + 2n't - \omega - 2\Omega)\\
&+ e\gamma^2\,(0.1013296)\sin(3nt - 2n't - 3\omega + 2\Omega)\\
&+ e\gamma^2\,(0.1013296)\sin(nt - 2n't + 3\omega - 2\Omega)\\
&- ee'^2\,(3.748248)\sin(3nt - 4n't - \omega + 2\omega')\\
&+ ee'^2\,(3.748248)\sin(nt - 4n't + \omega + 2\omega')\\
&- e^2e'\,(1.344151)\sin(2nt - n't - \omega') + e^2e'\,(11.09927)\sin(2nt - 3n't + \omega')\\
&+ e^2e'\,(8.67510)\sin(3n't - 2\omega - \omega') - e^2e'\,(1.021618)\sin(n't - 2\omega + \omega')\\
&+ e^2e'\,(0.3225335)\sin(4nt - n't - 2\omega - \omega')\\
&- e^2e'\,(2.424164)\sin(4nt - 3n't - 2\omega + \omega')\\
&- e^2e'^2\,(6.108664)\sin(4nt - 4n't - 2\omega + 2\omega')\\
&+ e^2e'^2\,(29.65086)\sin(2nt - 4n't + 2\omega')\\
&+ e^2e'^2\,(23.54219)\sin(4n't - 2\omega - 2\omega')\\
&+ e^3e'\,(0.4423621)\sin(5nt - n't - 3\omega - \omega')\\
&+ e^3e'\,(37.73444)\sin(nt + 3n't - 3\omega - \omega')\\
&- e^3e'\,(3.304215)\sin(5nt - 3n't - 3\omega + \omega')\\
&- e^3e'\,(13.53053)\sin(nt + n't - 3\omega + \omega')\\
&- ee'^3\,(0.00753084)\sin(3nt + n't - \omega - 3\omega')\\
&+ ee'^3\,(0.00753084)\sin(nt + n't + \omega - 3\omega')\\
&- ee'^3\,(8.120035)\sin(3nt - 5n't - \omega + 3\omega')\\
&+ ee'^3\,(8.120035)\sin(nt - 5n't + \omega + 3\omega')\\
&- e^4\,(1.1928420)\sin(6nt - 2n't - 4\omega) + e^4\,(14.77905)\sin(2nt + 2n't - 4\omega)\\
&+ e^2\gamma^2\,(0.3895455)\sin(4nt - 2n't - 2\Omega) + e^2\gamma^2\,(0.0152517)\sin 2(n't - \Omega)\\
&+ e^2\gamma^2\,(0.0823330)\sin(6nt - 2n't - 2\omega - 2\Omega)\\
&+ e^2\gamma^2\,(0.867338)\sin(2nt - 2n't + 2\omega - 2\Omega)\\
&+ e^2\gamma^2\,(1.635330)\sin(2nt + 2n't - 2\omega - 2\Omega)\\
&+ e^2\gamma^2\,(1.483472)\sin(2nt - 2n't - 2\omega + 2\Omega)\\
&+ e^2\gamma^2\,(0.1671099)\sin(4nt - 2n't - 4\omega + 2\Omega)\\
&+ e^2\gamma^2\,(0.560125)\sin(2n't - 4\omega + 2\Omega)\\
&+ ee'\gamma^2\,(0.06063835)\sin(3nt - n't + \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2\,(1.831284)\sin(nt - n't - \omega - \omega' + 2\Omega)\\
&- ee'\gamma^2\,(0.4565011)\sin(3nt - 3n't + \omega + \omega' - 2\Omega)\\
&+ ee'\gamma^2\,(4.017027)\sin(nt - 3nt - \omega + \omega' + 2\Omega)\\
&- ee'\gamma^2\,(0.01194207)\sin(5nt - n't - \omega - \omega' - 2\Omega)\\
&+ ee'\gamma^2\,(4.386621)\sin(nt + 3n't - \omega - \omega' - 2\Omega)\\
&+ ee'\gamma^2\,(0.0869068)\sin(5nt - 3n't - \omega + \omega' - 2\Omega)
\end{aligned}\right\} \quad . \quad (267)
$$

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$$
\left.\begin{array}{l}
-ee'\gamma^2(1.879980)\sin(nt+n't-\omega+\omega'-2\Omega)\\
-ee'\gamma^2(0.04869628)\sin(3nt-n't-3\omega-\omega'+2\Omega)\\
+ee'\gamma^2(0.3695943)\sin(nt-3n't+3\omega+\omega'-2\Omega)\\
-ee'\gamma^2(0.04869628)\sin(nt-n't+3\omega-\omega'-2\Omega)\\
+ee'\gamma^2(0.3695943)\sin(3nt-3n't-3\omega+\omega'+2\Omega)\Big\}\\
\\
+a\dfrac{\overline{m}^2}{\mu}\dfrac{a}{a'}n\Big\{-e(0.2026592)\sin(2nt-n\,t-\omega)-e(0.2026592)\sin(n't-\omega)\\
-e(0.3377653)\sin(4nt-3n't-\omega)+e(0.3377653)\sin(2nt-3n't+\omega)\\
-e^2(6.315301)\sin(nt-n't)+e^2(2.748421)\sin 3(nt-n't)\\
-e^2(0.1539629)\sin(3nt-n't-2\omega)-e^2(6.469264)\sin(nt+n't-2\omega)\\
-e^2(0.7102231)\sin(5nt-3n't-2\omega)-e^2(2.038198)\sin(nt-3n't+2\omega)\\
-ee'(0.1875000)\sin(2nt-\omega-\omega')+ee'(0.1875000)\sin(\omega-\omega')\\
-ee'(0.666456)\sin(2nt-2n't-\omega+\omega')\\
-ee'(0.666456)\sin(2n't-\omega-\omega')\\
-ee'(1.735600)\sin(4nt-4n't-\omega+\omega')\\
+ee'(1.735600)\sin(2nt-4n't+\omega+\omega')\\
+ee'(0.3289015)\sin(4nt-2n't-\omega-\omega')\\
-ee'(0.3289015)\sin(2nt-2n't+\omega-\omega')\Big\}
\end{array}\right\} . \quad (267)
$$

18. We shall now develop equation (259). The value of $\left(\dfrac{dR}{dr}\right)$ is given by equation (218). It contains the terms $-\frac{3}{4}e'^2$ and $-\frac{3}{8}e^2\gamma^2\cos 2(\omega-\Omega)$, which are independent of the co-ordinates of the sun and moon. These terms are functions of the time, and may be represented generally by the single term $h'\cos(\alpha' t-\beta')$, in which h', α', and β' are constant. We shall also put

$$
\left.\begin{aligned}
ee'\cos(\omega-\omega') &= h''\cos(\alpha'' t-\beta''),\\
ee'\sin(\omega-\omega') &= h''\sin(\alpha'' t-\beta''),\\
e\gamma\{1+\tfrac{1}{4}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(\omega-\Omega) &= h_{,}\sin(\alpha_{,}t-\beta_{,}),\\
e'\gamma\sin(\omega'-\Omega) &= h_{,,}\sin(\alpha_{,,}t-\beta_{,,});
\end{aligned}\right\} \quad (267')
$$

and shall use these values instead of the equivalent terms in the expressions of the forces $\left(\dfrac{dR}{dr}\right)$, $\left(\dfrac{dR}{dv}\right)$, and $\left(\dfrac{dR}{d\theta}\right)$.

The value of $c_2 \cos\beta$ is given by equation (228). If we therefore multiply equations (218) and (228) together, we shall obtain the following development:

$$c_2 \cos\beta\left(\frac{dR}{dr}\right)=$$

$$\left.\begin{aligned}
\overline{m}^2 ndt \Big\{ & \pm \tfrac{1}{2}\{1 - e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2 + \tfrac{7}{64}e^4 + \tfrac{15}{8}e'^4 - \tfrac{9}{4}e^2e'^2 \\
& \qquad\qquad + \tfrac{7}{4}e^2\gamma^2 - \tfrac{21}{8}e'^2\gamma^2 + \tfrac{119}{64}\gamma^4\} \cos nt \\
& \pm \tfrac{1}{4}e\,\{1 - \tfrac{5}{4}e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(2nt - \omega) \mp \tfrac{3}{4}e\,\{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2\} \cos\omega \\
& \pm \tfrac{3}{4}e'\,\{1 - e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(nt + n't - \omega') \\
& \pm \tfrac{3}{4}e'\,\{1 - e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(nt - n't + \omega') \\
& \pm \tfrac{3}{16}e^2\,\{1 - \tfrac{3}{2}e^2 + \tfrac{3}{2}e'^2 - 2\gamma^2 - \tfrac{1}{2}\frac{\gamma^4}{e^2}\} \cos(3nt - 2\omega) \\
& + \tfrac{1}{16}e^2\,\{1 + \tfrac{25}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2 - \tfrac{1}{2}\frac{\gamma^4}{e^2}\} \cos(nt - 2\omega) \\
& \pm \tfrac{9}{8}e'^2\,\{1 - e^2 + \tfrac{7}{9}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(nt + 2n't - 2\omega') \\
& \pm \tfrac{9}{8}e'^2\,\{1 - e^2 + \tfrac{7}{9}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(nt - 2n't + 2\omega') \\
& \pm \tfrac{3}{8}ee'\,\{1 - \tfrac{5}{4}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(2nt + n't - \omega - \omega') \\
& - \tfrac{9}{8}ee'\,\{1 - \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(n't - \omega - \omega') \\
& \pm \tfrac{3}{8}ee'\,\{1 - \tfrac{5}{4}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(2nt - n't - \omega + \omega') \\
& \mp \tfrac{9}{8}ee'\,\{1 - \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\} \cos(n't + \omega - \omega') \\
& \pm \tfrac{3}{8}\gamma^2\,\{1 - \tfrac{143}{16}e^2 + \tfrac{3}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(3nt - 2\Omega) \\
& + \tfrac{1}{4}\gamma^2\,\{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt - 2\Omega) \\
& \mp \tfrac{1}{16}\gamma^2\,\{1 + \tfrac{53}{4}e^2 + \tfrac{3}{2}e'^2\, \tfrac{9}{4}\gamma^2\} \cos(nt + 2\omega - 2\Omega) \\
& \mp \tfrac{1}{16}\gamma^2\,\{1 + \tfrac{15}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\} \cos(nt - 2\omega + 2\Omega) \\
& \pm \tfrac{1}{6}e^3 \cos(4nt - 3\omega) + \tfrac{1}{48}e^3 \cos(2nt - 3\omega) \pm \tfrac{53}{32}e'^3 \cos(nt + 3n't - 3\omega') \\
& \pm \tfrac{53}{32}e'^3 \cos(nt - 3n't + 3\omega') \pm \tfrac{9}{16}ee'^2 \cos(2nt + 2n't - \omega - 2\omega') \\
& - \tfrac{27}{16}ee'^2 \cos(2n't - \omega - 2\omega') \pm \tfrac{9}{16}ee'^2 \cos(2nt - 2n't - \omega + 2\omega') \\
& \mp \tfrac{27}{16}ee'^2 \cos(2n't + \omega - 2\omega') \pm \tfrac{9}{32}e^2e' \cos(3nt + n't - 2\omega - \omega') \\
& + \tfrac{3}{32}e^2e' \cos(nt + n't - 2\omega - \omega') \pm \tfrac{9}{32}e^2e' \cos(3nt - n't - 2\omega + \omega') \\
& + \tfrac{3}{32}e^2e' \cos(nt - n't - 2\omega + \omega') \pm \tfrac{15}{16}e\gamma^2 \cos(4nt - \omega - 2\Omega) \\
& + \tfrac{1}{8}e\gamma^2 \cos(2nt - \omega - 2\Omega) \mp \tfrac{41}{32}e\gamma^2 \cos(2nt + \omega - 2\Omega) \\
& - \tfrac{9}{32}e\gamma^2 \cos(\omega - 2\Omega) \mp \tfrac{3}{32}e\gamma^2 \cos(3\omega - 2\Omega) \mp \tfrac{1}{32}e\gamma^2 \cos(2nt - 3\omega + 2\Omega) \\
& \pm \tfrac{9}{16}e'\gamma^2 \cos(3nt + n't - \omega' - 2\Omega) + \tfrac{3}{8}e'\gamma^2 \cos(nt + n't - \omega' - 2\Omega) \\
& \pm \tfrac{9}{16}e'\gamma^2 \cos(3nt - n't + \omega' - 2\Omega) + \tfrac{3}{8}e'\gamma^2 \cos(nt - n't + \omega' - 2\Omega) \\
& \mp \tfrac{3}{32}e'\gamma^2 \cos(nt + n't - 2\omega - \omega' + 2\Omega) \\
& \mp \tfrac{3}{32}e'\gamma^2 \cos(nt - n't - 2\omega + \omega' + 2\Omega)
\end{aligned}\right\} \tag{268}$$

(*Continued on the next page.*)

$$\left.\begin{aligned}
&\pm \tfrac{3}{32}\, e'\gamma^2 \cos(nt + n't + 2\omega - \omega' - 2\Omega) \\
&\pm \tfrac{3}{32} e'\gamma^2 \cos(nt - n't + 2\omega + \omega' - 2\Omega) \pm \tfrac{53}{64}\, ee'^3 \cos(2nt + 3n't - \omega - 3\omega') \\
&- \tfrac{159}{64}\, ee'^3 \cos(3n't - \omega - 3\omega') \pm \tfrac{53}{64} ee'^3 \cos(2nt - 3n't - \omega + 3\omega') \\
&\mp \tfrac{159}{64}\, ee'^3 \cos(3n't + \omega - 3\omega') \pm \tfrac{1}{4} e^3 e' \cos(4nt + n't - 3\omega - \omega') \\
&+ \tfrac{1}{32}\, e^3 e' \cos(2nt + n't - 3\omega - \omega') \pm \tfrac{1}{4} e^3 e' \cos(4nt - n't - 3\omega + \omega') \\
&+ \tfrac{1}{32} e^3 e' \cos(2nt - n't - 3\omega + \omega') \pm \tfrac{27}{64} e^2 e'^2 \cos(3nt + 2n't - 2\omega - 2\omega') \\
&+ \tfrac{9}{64} e^2 e'^2 \cos(nt + 2n't - 2\omega - 2\omega') \\
&\pm \tfrac{27}{64} e^2 e'^2 \cos(3nt - 2n't - 2\omega + 2\omega') \\
&+ \tfrac{9}{64} e^2 e'^2 \cos(nt - 2n't - 2\omega + 2\omega') \pm \tfrac{27}{64} e'^2\gamma^2 \cos(3nt + 2n't - 2\omega' - 2\Omega) \\
&+ \tfrac{9}{16} e'^2\gamma^2 \cos(nt + 2n't - 2\omega' - 2\Omega) \pm \tfrac{27}{32} e'^2\gamma^2 \cos(3nt - 2n't + 2\omega' - 2\Omega) \\
&+ \tfrac{9}{16} e'^2\gamma^2 \cos(nt - 2n't + 2\omega' - 2\Omega) \\
&\mp \tfrac{9}{64}\, e'^2\gamma^2 \cos(nt + 2n't - 2\omega - 2\omega' + 2\Omega) \\
&\mp \tfrac{9}{64} e'^2\gamma^2 \cos(nt - 2n't - 2\omega + 2\omega' + 2\Omega) \\
&\pm \tfrac{9}{64} e'^2\gamma^2 \cos(nt + 2n't + 2\omega - 2\omega' - 2\Omega) \\
&\pm \tfrac{9}{64} e'^2\gamma^2 \cos(nt - 2n't + 2\omega + 2\omega' - 2\Omega) \pm \tfrac{125}{768} e^4 \cos(5nt - 4\omega) \\
&+ \tfrac{3}{256} e^4 \cos(3nt - 4\omega) \pm \tfrac{77}{32} e'^4 \cos(nt + 4n't - 4\omega') \\
&\pm \tfrac{77}{32} e'^4 \cos(nt - 4n't + 4\omega') \pm \tfrac{111}{64} e^2\gamma^2 \cos(5nt - 2\omega - 2\Omega) \\
&+ \tfrac{3}{32} e^2\gamma^2 \cos(3nt - 2\omega - 2\Omega) + \tfrac{1}{128} e^2\gamma^2 \cos(nt - 4\omega + 2\Omega) \\
&\pm \tfrac{3}{64} e^2\gamma^2 \cos(3nt - 4\omega + 2\Omega) \pm \tfrac{27}{16} ee'\gamma^2 \cos(4nt - n't - \omega + \omega' - 2\Omega) \\
&+ \tfrac{15}{32} ee'\gamma^2 \cos(2nt - n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{105}{64} ee'\gamma^2 \cos(2nt + n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{9}{64} ee'\gamma^2 \cos(n't + \omega - \omega' - 2\Omega) \pm \tfrac{45}{32} ee'\gamma^2 \cos(4nt + n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{3}{16} ee'\gamma^2 \cos(2nt + n't - \omega - \omega' - 2\Omega) \\
&\mp \tfrac{123}{64} ee'\gamma^2 \cos(2nt - n't + \omega + \omega' - 2\Omega) \\
&\mp \tfrac{27}{64} ee'\gamma^2 \cos(n't - \omega - \omega' + 2\Omega) \mp \tfrac{3}{64} ee'\gamma^2 \cos(2nt + n't - 3\omega - \omega' + 2\Omega) \\
&\mp \tfrac{3}{64} ee'\gamma^2 \cos(2nt - n't - 3\omega + \omega' + 2\Omega) \pm \tfrac{9}{64}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \\
&+ \tfrac{1}{32}\gamma^4 \cos(nt + 2\omega - 4\Omega) - \tfrac{3}{32}\gamma^4 \cos(3nt - 4\Omega) \\
&\pm \tfrac{3}{256}\gamma^4 \cos(nt + 4\omega - 4\Omega) \mp \tfrac{1}{256}\gamma^4 \cos(nt - 4\omega + 4\Omega) \\
&\pm \tfrac{3}{4}\{1 - 9e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2 + 12e^4 + \tfrac{35}{16}e'^4 + \tfrac{87}{4}e^2e'^2 \\
&\qquad\qquad + \tfrac{15}{8}e'^2\gamma^2 + \tfrac{27}{4}e^2\gamma^2 + \tfrac{39}{64}\gamma^4\} \cos(3nt - 2n't) \\
&+ \tfrac{3}{4}\{1 - e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2 + \tfrac{7}{64}e^4 + \tfrac{35}{16}e'^4 + \tfrac{7}{4}e^2e'^2 \\
&\qquad\qquad + \tfrac{15}{8}e'^2\gamma^2 + \tfrac{3}{4}e^2\gamma^2 + \tfrac{39}{64}\gamma^4\} \cos(nt - 2n't) \\
&\pm \tfrac{15}{8}e\,\{1 - \tfrac{34}{5}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(4nt - 2n't - \omega) \\
&+ \tfrac{3}{8}e\,\{1 - \tfrac{5}{4}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - 2n't - \omega) \\
&\mp \tfrac{21}{8}e\,\{1 - \tfrac{89}{28}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - 2n't + \omega) \\
&\mp \tfrac{9}{8}e\,\{1 - \tfrac{1}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2n't - \omega)
\end{aligned}\right\} . \quad (268)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp \tfrac{3}{8}e' \{1 - 9e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - n't - \omega') \\
&- \tfrac{3}{8}e' \{1 - e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt - n't - \omega') \\
&\pm \tfrac{21}{8}e' \{1 - 9e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 3n't + \omega') \\
&+ \tfrac{21}{8}e' \{1 - e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt - 3n't + \omega') \\
&\pm \tfrac{111}{32}e^2 \{1 - \tfrac{427}{74}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2 - \tfrac{1}{74}\frac{\gamma^4}{e^2}\} \cos(5nt - 2n't - 2\omega) \\
&+ \tfrac{9}{32}e^2 \{1 - \tfrac{3}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 2n't - 2\omega) \\
&\pm \tfrac{3}{32}e^2 \{1 + \tfrac{101}{6}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt + 2n't - 2\omega) \\
&\pm \tfrac{93}{32}e^2 \{1 - \tfrac{83}{62}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2 + \tfrac{1}{62}\frac{\gamma^4}{e^2}\} \cos(nt - 2n't + 2\omega) \\
&\pm \tfrac{51}{8}e'^2 \{1 - 9e^2 - \tfrac{115}{51}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 4n't + 2\omega') \\
&+ \tfrac{51}{8}e'^2 \{1 - e^2 - \tfrac{115}{51}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt - 4n't + 2\omega') \\
&\pm \tfrac{3}{8}\gamma^2 \{1 - \tfrac{33}{32}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt + 2n't - 2\Omega) \\
&\pm \tfrac{3}{16}\gamma^2 \{1 - \tfrac{19}{3}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - 2n't + 2\Omega) \\
&\pm \tfrac{9}{32}\gamma^2 \{1 - \tfrac{33}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(3nt - 2n't + 2\omega - 2\Omega) \\
&+ \tfrac{3}{32}\gamma^2 \{1 - \tfrac{125}{4}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - 2n't + 2\omega - 2\Omega) \\
&\mp \tfrac{9}{32}\gamma^2 \{1 - \tfrac{67}{4}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(3nt - 2n't - 2\omega + 2\Omega) \\
&- \tfrac{3}{32}\gamma^2 \{1 - 24e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - 2n't - 2\omega + 2\Omega) \\
&- \tfrac{3}{16}\gamma^2 \{1 - \tfrac{147}{16}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(3nt - 2n't - 2\Omega) \\
&\mp \tfrac{15}{16}ee' \{1 - \tfrac{34}{5}e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(4nt - n't - \omega - \omega') \\
&- \tfrac{3}{16}ee' \{1 - \tfrac{5}{4}e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - n't - \omega - \omega') \\
&\mp \tfrac{147}{16}ee' \{1 - \tfrac{623}{196}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - 3n't + \omega + \omega') \\
&\mp \tfrac{63}{16}ee' \{1 - \tfrac{29}{63}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3n't - \omega - \omega') \\
&\pm \tfrac{105}{16}ee' \{1 - \tfrac{34}{5}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(4nt - 3n't - \omega + \omega') \\
&+ \tfrac{21}{16}ee' \{1 - \tfrac{5}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - 3n't - \omega + \omega') \\
&\pm \tfrac{21}{16}ee' \{1 - \tfrac{89}{28}e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - n't + \omega - \omega') \\
&\pm \tfrac{9}{16}ee' \{1 - \tfrac{1}{2}e^2 - \tfrac{1}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(n't - \omega + \omega') \\
&\pm \tfrac{183}{32}e^3 \cos(6nt - 2n't - 3\omega) + \tfrac{1}{4}e^3 \cos(4nt - 2n't - 3\omega) \\
&- \tfrac{31}{32}e^3 \cos(2n't - 3\omega) \pm \tfrac{1}{64}e'^3 \cos(3nt + n't - 3\omega') \\
&+ \tfrac{1}{64}e'^3 \cos(nt + n't - 3\omega') \pm \tfrac{845}{64}e'^3 \cos(3nt - 5n't + 3\omega') \\
&+ \tfrac{845}{64}e'^3 \cos(nt - 5n't + 3\omega') \pm \tfrac{45}{64}e\gamma^2 \cos(4nt - 2n't + \omega - 2\Omega) \\
&+ \tfrac{45}{64}e\gamma^2 \cos(2nt - 2n't + \omega - 2\Omega) \pm \tfrac{69}{64}e\gamma^2 \cos(2nt - 2n't - \omega + 2\Omega) \\
&\mp \tfrac{27}{64}e\gamma^2 \cos(2n't + \omega - 2\Omega) \mp \tfrac{45}{64}e\gamma^2 \cos(4nt - 2n't - 3\omega + 2\Omega) \\
&- \tfrac{3}{64}e\gamma^2 \cos(2nt - 2n't - 3\omega + 2\Omega) \mp \tfrac{63}{64}e\gamma^2 \cos(2nt - 2n't + 3\omega - 2\Omega) \\
&\mp \tfrac{9}{64}e\gamma^2 \cos(2n't - 3\omega + 2\Omega) \pm \tfrac{3}{16}e\gamma^2 \cos(2nt + 2n't - \omega - 2\Omega) \\
&- \tfrac{9}{32}e\gamma^2 \cos(2n't - \omega - 2\Omega) - \tfrac{15}{32}e\gamma^2 \cos(4nt - 2n't - \omega - 2\Omega)
\end{aligned}
\right\} \; . \qquad (268)
$$

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$$
\left.\begin{array}{l}
\pm \frac{21}{16} e'\gamma^2 \cos(nt + 3n't - \omega' - 2\Omega) \pm \frac{21}{32} e'\gamma^2 \cos(nt - 3n't + \omega' + 2\Omega) \\
\mp \frac{3}{16} e'\gamma^2 \cos(nt + n't + \omega' - 2\Omega) \\
\mp \frac{3}{32} e'\gamma^2 \cos(nt - n't - \omega' + 2\Omega) \pm \frac{9}{64} e'\gamma^2 \cos(3nt - n't - 2\omega - \omega' + 2\Omega) \\
+ \frac{3}{64} e'\gamma^2 \cos(nt - n't - 2\omega - \omega' + 2\Omega) \\
\pm \frac{63}{64} e'\gamma^2 \cos(3nt - 3n't + 2\omega + \omega' - 2\Omega) \\
+ \frac{21}{64} e'\gamma^2 \cos(nt - 3n't + 2\omega + \omega' - 2\Omega) \\
\mp \frac{9}{64} e'\gamma^2 \cos(3nt - n't + 2\omega - \omega' - 2\Omega) \\
- \frac{3}{64} e'\gamma^2 \cos(nt - n't + 2\omega - \omega' - 2\Omega) \\
\mp \frac{63}{64} e'\gamma^2 \cos(3nt - 3n't - 2\omega + \omega' + 2\Omega) \\
- \frac{21}{64} e'\gamma^2 \cos(nt - 3n't - 2\omega + \omega' + 2\Omega) \\
- \frac{21}{32} e'\gamma^2 \cos(3nt - n't + \omega' - 2\Omega) + \frac{3}{32} e'\gamma^2 \cos(3nt - n't - \omega' - 2\Omega) \\
\mp \frac{357}{16} ee'^2 \cos(2nt - 4n't + \omega + 2\omega') \mp \frac{153}{16} ee'^2 \cos(4n't - \omega - 2\omega') \\
\pm \frac{255}{16} ee'^2 \cos(4nt - 4n't - \omega + 2\omega') + \frac{51}{16} ee'^2 \cos(2nt - 4n't - \omega + 2\omega') \\
\mp \frac{111}{64} e^2e' \cos(5nt - n't - 2\omega - \omega') - \frac{9}{64} e^2e' \cos(3nt - n't - 2\omega - \omega') \\
\pm \frac{21}{64} e^2e' \cos(nt + 3n't - 2\omega - \omega') \pm \frac{651}{64} e^2e' \cos(nt - 3n't + 2\omega + \omega') \\
\pm \frac{771}{64} e^2e' \cos(5nt - 3n't - 2\omega + \omega') + \frac{63}{64} e^2e' \cos(3nt - 3n't - 2\omega + \omega') \\
\mp \frac{3}{64} e^2e' \cos(nt + n't - 2\omega + \omega') \mp \frac{93}{64} e^2e' \cos(nt - n't + 2\omega - \omega') \\
\pm \frac{4553}{512} e^4 \cos(7nt - 2n't - 4\omega) + \frac{125}{512} e^4 \cos(5nt - 2n't - 4\omega) \\
\mp \frac{7}{512} e^4 \cos(3nt + 2n't - 4\omega) - \frac{3}{512} e^4 \cos(nt + 2n't - 4\omega) \\
\pm \frac{1}{32} e'^4 \cos(3nt + 2n't - 4\omega') + \frac{1}{32} e'^4 \cos(nt + 2n't - 4\omega') \\
\pm \frac{1599}{64} e'^4 \cos(3nt - 6n't + 4\omega') + \frac{1599}{64} e'^4 \cos(nt - 6n't + 4\omega') \\
\pm \frac{45}{512} \gamma^4 \cos(3nt - 2n't + 4\omega - 4\Omega) + \frac{21}{512} \gamma^4 \cos(nt - 2n't + 4\omega - 4\Omega) \\
\pm \frac{3}{64} \gamma^4 \cos(3nt + 2n't - 4\Omega) - \frac{3}{64} \gamma^4 \cos(nt + 2n't - 4\Omega) \\
- \frac{9}{128} \gamma^4 \cos(3nt - 2n't + 2\omega - 4\Omega) \mp \frac{3}{128} \gamma^4 \cos(nt - 2n't - 2\omega + 4\Omega) \\
\pm \frac{3}{64} \gamma^4 \cos(nt + 2n't + 2\omega - 4\Omega) - \frac{3}{512} \gamma^4 \cos(nt - 2n't - 4\omega + 4\Omega) \\
\pm \frac{3}{256} \gamma^4 \cos(3nt - 2n't - 4\omega + 4\Omega) \mp \frac{153}{128} e^2\gamma^2 \cos(5nt - 2n't - 4\omega + 2\Omega) \\
- \frac{9}{256} e^2\gamma^2 \cos(3nt - 2n't - 4\omega + 2\Omega) \pm \frac{15}{128} e^2\gamma^2 \cos(nt + 2n't - 4\omega + 2\Omega) \\
\pm \frac{141}{128} e^2\gamma^2 \cos(nt - 2n't + 4\omega - 2\Omega) \pm \frac{333}{256} e^2\gamma^2 \cos(5nt - 2n't - 2\Omega) \\
\pm \frac{9}{64} e^2\gamma^2 \cos(3nt + 2n't - 2\omega - 2\Omega) + \frac{3}{128} e^2\gamma^2 \cos(nt + 2n't - 2\omega - 2\Omega) \\
- \frac{111}{128} e^2\gamma^2 \cos(5nt - 2n't - 2\omega - 2\Omega) \pm \frac{5}{128} ee'^3 \cos(4nt + n't - \omega - 3\omega') \\
+ \frac{1}{128} ee'^3 \cos(2nt + n't - \omega - 3\omega') \mp \frac{7}{128} ee'^3 \cos(2nt + n't + \omega - 3\omega') \\
- \frac{3}{128} ee'^3 \cos(n't + \omega - 3\omega') \mp \frac{5915}{128} ee'^3 \cos(2nt - 5n't + \omega + 3\omega') \\
\mp \frac{2585}{128} ee'^3 \cos(5n't - \omega - 3\omega') \pm \frac{2387}{128} ee'^3 \cos(4nt - 5n't - \omega + 3\omega') \\
- \frac{1043}{128} ee'^3 \cos(2nt - 5n't - \omega + 3\omega') \pm \frac{7}{64} e^3e' \cos(2nt + 3n't - 3\omega - \omega') \\
- \frac{105}{32} e^3e' \cos(3n't - 3\omega - \omega') \mp \frac{1}{64} e^3e' \cos(2nt + n't - 3\omega + \omega') \\
+ \frac{15}{32} e^3e' \cos(n't - 3\omega + \omega') \mp \frac{183}{64} e^3e' \cos(6nt - n't - 3\omega - \omega')
\end{array}\right\} . \quad (268)
$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&-\tfrac{1}{8}e^3e'\cos(4nt-n't-3\omega-\omega')\pm\tfrac{2111}{64}e^3e'\cos(6nt-3n't-3\omega+\omega')\\
&+\tfrac{443}{32}e^3e'\cos(4nt-3n't-3\omega+\omega')\\
&\pm\tfrac{1887}{64}e^2e'^2\cos(5nt-4n't-2\omega+2\omega')\\
&+\tfrac{153}{64}e^2e'^2\cos(3nt-4n't-2\omega+2\omega')\pm\tfrac{51}{64}e^2e'^2\cos(nt+4n't-2\omega-2\omega')\\
&\pm\tfrac{1581}{64}e^2e'^2\cos(nt-4n't+2\omega+2\omega')\\
&\pm\tfrac{153}{64}e'^2\gamma^2\cos(3nt-4n't+2\omega+2\omega'-2\Omega)\\
&+\tfrac{51}{64}e'^2\gamma^2\cos(nt-4n't+2\omega+2\omega'-2\Omega)\\
&\pm\tfrac{51}{16}e'^2\gamma^2\cos(nt+4n't-2\omega'-2\Omega)\pm\tfrac{51}{32}e'^2\gamma^2\cos(nt-4n't+2\omega'+2\Omega)\\
&\mp\tfrac{153}{64}e'^2\gamma^2\cos(3nt-4n't-2\omega+2\omega'+2\Omega)\\
&-\tfrac{51}{64}e'^2\gamma^2\cos(nt-4n't-2\omega+2\omega'+2\Omega)\\
&-\tfrac{51}{32}e'^2\gamma^2\cos(3nt-4n't+2\omega'-2\Omega)\\
&\pm\tfrac{315}{128}ee'\gamma^2\cos(4nt-3n't+\omega+\omega'-2\Omega)\\
&+\tfrac{315}{128}ee'\gamma^2\cos(2nt-3n't+\omega+\omega'-2\Omega)\\
&\mp\tfrac{69}{128}ee'\gamma^2\cos(2nt-n't-\omega-\omega'+2\Omega)\\
&\pm\tfrac{27}{128}ee'\gamma^2\cos(n't+\omega+\omega'-2\Omega)\\
&\pm\tfrac{3}{2}ee'\gamma^2\cos(6nt-n't-\omega-\omega'-2\Omega)\\
&+\tfrac{111}{64}ee'\gamma^2\cos(4nt-n't-\omega-\omega'-2\Omega)\\
&\pm\tfrac{483}{128}ee'\gamma^2\cos(2nt-3n't-\omega+\omega'+2\Omega)\\
&\mp\tfrac{189}{128}ee'\gamma^2\cos(3n't+\omega-\omega'-2\Omega)\\
&\mp\tfrac{315}{128}ee'\gamma^2\cos(4nt-3n't-3\omega+\omega'+2\Omega)\\
&-\tfrac{21}{128}ee'\gamma^2\cos(2nt-3n't-3\omega+\omega'+2\Omega)\\
&\pm\tfrac{63}{128}ee'\gamma^2\cos(2nt-n't+3\omega-\omega'-2\Omega)\\
&\pm\tfrac{9}{128}ee'\gamma^2\cos(n't-3\omega+\omega'+2\Omega)\\
&\pm\tfrac{45}{128}ee'\gamma^2\cos(4nt-n't-3\omega-\omega'+2\Omega)\\
&+\tfrac{3}{128}ee'\gamma^2\cos(2nt-n't-3\omega-\omega'+2\Omega)\\
&\mp\tfrac{45}{128}ee'\gamma^2\cos(4nt-n't+\omega-\omega'-2\Omega)\\
&-\tfrac{45}{128}ee'\gamma^2\cos(2nt-n't+\omega-\omega'-2\Omega)\\
&\mp\tfrac{441}{128}ee'\gamma^2\cos(2nt-3n't+3\omega+\omega'-2\Omega)\\
&\mp\tfrac{63}{128}ee'\gamma^2\cos(3n't-3\omega-\omega'+2\Omega)\\
&\mp\tfrac{3}{32}ee'\gamma^2\cos(2nt+n't-\omega+\omega'-2\Omega)+\tfrac{9}{64}ee'\gamma^2\cos(n't-\omega+\omega'-2\Omega)\\
&\mp\tfrac{27}{32}ee'\gamma^2\cos(2nt+3n't-\omega-\omega'-2\Omega)\\
&-\tfrac{159}{64}ee'\gamma^2\cos(3n't-\omega-\omega'-2\Omega)\\
&-\tfrac{105}{64}ee'\gamma^2\cos(4nt-3n't-\omega+\omega'-2\Omega)\\
&-\tfrac{9}{64}ee'\gamma^2\cos(n't-3\omega-\omega'+2\Omega)\mp\tfrac{9}{64}ee'\gamma^2\cos(n't+3\omega-\omega'-2\Omega).\\
&\left.\pm\tfrac{1}{2}h'\{\cos(nt+\alpha't-\beta')+\cos(nt-\alpha't+\beta')\}\right\}
\end{aligned}\right\}\quad . \tag{268}
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\overline{m}^2\frac{a}{a'}ndt\Big\{\pm\tfrac{9}{16}\{1-3e^2+2e'^2-3\gamma^2\}\cos(2nt-n't)\\
&\pm\tfrac{9}{16}\{1+e^2+3e'^2-3\gamma^2\}\cos n't\pm\tfrac{9}{16}e'\cos(2nt-\omega')\\
&\pm\tfrac{9}{16}e'\cos\omega'\pm\tfrac{27}{16}e'\cos(2nt-2n't+\omega')\pm\tfrac{27}{16}e'\cos(2n't-\omega')\\
&\mp\tfrac{9}{16}e\cos(nt+n't-\omega)\mp\tfrac{27}{16}e\cos(nt-n't+\omega)\\
&\pm\tfrac{9}{16}e\cos(3nt-n't-\omega)-\tfrac{9}{16}e\cos(nt-n't-\omega)\\
&-\tfrac{9}{64}e^2\cos(2nt-n't-2\omega)\mp\tfrac{9}{16}e^2\cos(2nt+n't-2\omega)\\
&+\tfrac{45}{32}e^2\cos(n't-2\omega)\pm\tfrac{99}{128}e'^2\cos(2nt+n't-2\omega')\\
&+\tfrac{99}{128}e'^2\cos(n't-2\omega')\pm\tfrac{477}{128}e'^2\cos(2nt-3n't+2\omega')\\
&\pm\tfrac{477}{128}e'^2\cos(3n't-2\omega')\mp\tfrac{9}{16}ee'\cos(nt+\omega-\omega')\\
&\pm\tfrac{9}{16}ee'\cos(nt-\omega+\omega')\pm\tfrac{9}{16}ee'\cos(3nt-\omega-\omega')\\
&-\tfrac{9}{16}ee'\cos(nt-\omega-\omega')\mp\tfrac{9}{64}\gamma^2\cos(2nt-n't-2\omega+2\Omega)\\
&\pm\tfrac{9}{64}\gamma^2\cos(2nt-n't+2\omega-2\Omega)\pm\tfrac{45}{64}\gamma^2\cos(4nt-n't-2\Omega)\\
&+\tfrac{9}{16}\gamma^2\cos(2nt-n't-2\Omega)\pm\tfrac{27}{32}\gamma^2\cos(2nt+n't-2\Omega)\\
&+\tfrac{45}{64}\gamma^2\cos(n't-2\Omega)\mp\tfrac{9}{8}h''\cos(nt+\alpha''t-\beta'')\\
&\mp\tfrac{9}{8}h''\cos(nt-\alpha''t+\beta'')\pm\tfrac{15}{16}\{1-15e^2-6e'^2-\gamma^2\}\cos(4nt-3n't)\\
&+\tfrac{15}{16}\{1-3e^2-6e'^2-\gamma^2\}\cos(2nt-3n't)\pm\tfrac{45}{16}e\cos(5nt-2n't-\omega)\\
&+\tfrac{15}{16}e\cos(3nt-3n't-\omega)\mp\tfrac{75}{16}e\cos(3nt-3n't+\omega)\\
&-\tfrac{45}{16}e\cos(nt-3n't+\omega)\pm\tfrac{75}{16}e'\cos(4nt-4n't+\omega')\\
&+\tfrac{75}{16}e'\cos(2nt-4n't+\omega')\mp\tfrac{15}{16}e'\cos(4nt-2n't-\omega')\\
&-\tfrac{15}{16}e'\cos(2nt-2n't-\omega')\pm\tfrac{375}{64}e^2\cos(6nt-3n't-2\omega)\\
&+\tfrac{15}{16}e^2\cos(4nt-3n't-2\omega)\pm\tfrac{555}{64}e^2\cos(2nt-3n't+2\omega)\\
&\pm\tfrac{75}{32}e^2\cos(3n't-2\omega)\pm\tfrac{15}{128}e'^2\cos(4nt-n't-2\omega')\\
&+\tfrac{15}{128}e'^2\cos(2nt-n't-2\omega')\pm\tfrac{1905}{128}e'^2\cos(4nt-5n't+2\omega')\\
&+\tfrac{1905}{128}e'^2\cos(2nt-5n't+2\omega')\pm\tfrac{225}{16}ee'\cos(5nt-4n't-\omega+\omega')\\
&+\tfrac{75}{16}ee'\cos(3nt-4n't-\omega+\omega')\pm\tfrac{75}{16}ee'\cos(3nt-2n't+\omega-\omega')\\
&+\tfrac{45}{16}ee'\cos(nt-2n't+\omega-\omega')\mp\tfrac{45}{16}ee'\cos(5nt-2n't-\omega-\omega')\\
&-\tfrac{15}{16}ee'\cos(3nt-2n't-\omega-\omega')\mp\tfrac{375}{16}ee'\cos(nt-4n't+\omega+\omega')\\
&-\tfrac{225}{16}ee'\cos(nt-4n't+\omega+\omega')\pm\tfrac{27}{16}ee'\cos(3nt-2n't-\omega+\omega')\\
&\mp\tfrac{27}{16}ee'\cos(nt+2n't-\omega-\omega')\mp\tfrac{81}{16}ee'\cos(nt-2n't+\omega+\omega')\\
&-\tfrac{27}{16}ee'\cos(nt-2n't-\omega+\omega')\pm\tfrac{15}{32}\gamma^2\cos(2nt-3n't+2\Omega)\\
&\pm\tfrac{45}{64}\gamma^2\cos(3n't-2\Omega)\pm\tfrac{15}{32}\gamma^2\cos(4nt-3n't+2\omega-2\Omega)\\
&+\tfrac{15}{64}\gamma^2\cos(2nt-3n't+2\omega-2\Omega)\mp\tfrac{15}{16}\gamma^2\cos(4nt-3n't-2\omega+2\Omega)\\
&-\tfrac{15}{64}\gamma^2\cos(2nt-3n't-2\omega+2\Omega)-\tfrac{15}{64}\gamma^2\cos(4nt-3n't-2\Omega)\Big\}
\end{aligned}\right\} \tag{268}
$$

This equation will give the value of $c_2 \sin\beta\left(\frac{dR}{dr}\right)$ by using the lower signs and changing *cos* to *sin* in the second member.

The development of $c_3 \sin \beta \left(\frac{dR}{dv}\right)$ is obtained by multiplying equations (219) and (234) together. Therefore we get

$$
\begin{aligned}
&c_3 \sin \beta \left(\frac{dR}{dv}\right) = \\
&\overline{m}^2 ndt \Big\{ \mp \tfrac{3}{2}\{1 - \tfrac{15}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2 + \tfrac{999}{64}e^4 + \tfrac{18}{16}e'^4 + \tfrac{11}{32}\gamma^4 \\
&\qquad\qquad + \tfrac{75}{4}e^2e'^2 + \tfrac{17}{4}e^2\gamma^2 + \tfrac{5}{4}e'^2\gamma^2\} \cos(3nt - 2n't) \\
&\quad + \tfrac{3}{2}\{1 - \tfrac{3}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2 + \tfrac{15}{64}e^4 + \tfrac{13}{16}e'^4 + \tfrac{19}{32}\gamma^4 \\
&\qquad\qquad + \tfrac{15}{4}e^2e'^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{5}{4}e'^2\gamma^2\} \cos(nt - 2n't) \\
&\quad \mp \tfrac{27}{8}e\{1 - \tfrac{58}{9}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{9}\gamma^2\} \cos(4nt - 2n't - \omega) \\
&\quad + \tfrac{9}{8}e\{1 - \tfrac{17}{12}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{3}\gamma^2\} \cos(2nt - 2n't - \omega) \\
&\quad \pm \tfrac{39}{8}e\{1 - \tfrac{133}{52}e^2 - \tfrac{5}{2}e'^2 - \tfrac{7}{13}\gamma^2\} \cos(2nt - 2n't + \omega) \\
&\quad \mp \tfrac{21}{8}e\{1 - \tfrac{1}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{9}{7}\gamma^2\} \cos(2n't - \omega) \\
&\quad \pm \tfrac{3}{4}e'\{1 - \tfrac{15}{2}e^2 - \tfrac{1}{8}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(3nt - n't - \omega') \\
&\quad - \tfrac{3}{4}e'\{1 - \tfrac{3}{2}e^2 - \tfrac{1}{8}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - n't - \omega') \\
&\quad \mp \tfrac{21}{4}e'\{1 - \tfrac{105}{14}e^2 - \tfrac{123}{56}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(3nt - 3n't + \omega') \\
&\quad + \tfrac{21}{4}e'\{1 - \tfrac{3}{2}e^2 - \tfrac{123}{56}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - 3n't + \omega') \\
&\quad \mp \tfrac{93}{16}e^2\{1 - \tfrac{514}{93}e^2 - \tfrac{155}{62}e'^2 - \tfrac{481}{806}\gamma^2 + \tfrac{881}{12648}\frac{\gamma^4}{e^2}\} \cos(5nt - 2n't - 2\omega) \\
&\quad + \tfrac{15}{16}e^2\{1 - \tfrac{3}{5}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{10}\gamma^2 - \tfrac{1}{40}\frac{\gamma^4}{e^2}\} \cos(3nt - 2n't - 2\omega) \\
&\quad \pm \tfrac{9}{16}e^2\{1 - \tfrac{5}{2}e'^2 - \tfrac{1}{6}\gamma^2 + \tfrac{1}{8}\frac{\gamma^4}{e^2}\} \cos(nt + 2n't - 2\omega) \\
&\quad \mp \tfrac{75}{16}e^2\{1 - \tfrac{277}{300}e^2 - \tfrac{5}{2}e'^2 - \tfrac{31}{50}\gamma^2 + \tfrac{7}{200}\frac{\gamma^4}{e^2}\} \cos(nt - 2n't + 2\omega) \\
&\quad \mp \tfrac{51}{4}e'^2\{1 - \tfrac{15}{2}e^2 - \tfrac{115}{51}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(3nt - 4n't + 2\omega') \\
&\quad + \tfrac{51}{4}e'^2\{1 - \tfrac{3}{2}e^2 - \tfrac{115}{51}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(nt - 4n't + 2\omega') \\
&\quad \pm \tfrac{9}{16}\gamma^2\{1 - \tfrac{159}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(5nt - 2n't - 2\Omega) \\
&\quad - \tfrac{3}{16}\gamma^2\{1 - \tfrac{57}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(3nt - 2n't - 2\Omega) \\
&\quad \pm \tfrac{3}{16}\gamma^2\{1 - \tfrac{7}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(nt + 2n't - 2\Omega) \\
&\quad \mp \tfrac{9}{16}\gamma^2\{1 - \tfrac{31}{24}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(nt - 2n't + 2\Omega) \\
&\quad \mp \tfrac{9}{16}\gamma^2\{1 - \tfrac{203}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(3nt - 2n't + 2\omega - 2\Omega) \\
&\quad + \tfrac{3}{16}\gamma^2\{1 - \tfrac{217}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(nt - 2n't + 2\omega - 2\Omega) \\
&\quad \pm \tfrac{9}{16}\gamma^2\{1 - \tfrac{487}{36}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(3nt - 2n't - 2\omega + 2\Omega) \\
&\quad - \tfrac{3}{16}\gamma^2\{1 - \tfrac{191}{8}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(nt - 2n't - 2\omega + 2\Omega)
\end{aligned}
\qquad (269)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\pm \tfrac{27}{16} ee' \{1 - \tfrac{447}{72} e^2 - \tfrac{1}{8} e'^2 - \tfrac{10}{9} \gamma^2\} \cos (4nt - n't - \omega - \omega') \\
&- \tfrac{9}{16} ee' \{1 - \tfrac{17}{12} e^2 - \tfrac{1}{8} e'^2 - \tfrac{1}{3} \gamma^2\} \cos (2nt - n't - \omega - \omega') \\
&\pm \tfrac{273}{16} ee' \{1 - \tfrac{133}{52} e^2 - \tfrac{123}{56} e'^2 - \tfrac{7}{13} \gamma^2\} \cos (2nt - 3n't + \omega + \omega') \\
&\mp \tfrac{147}{16} ee' \{1 - \tfrac{1}{2} e^2 - \tfrac{123}{56} e'^2 - \tfrac{6}{7} \gamma^2\} \cos (3n't - \omega - \omega') \\
&\mp \tfrac{189}{16} ee' \{1 - \tfrac{53}{9} e^2 - \tfrac{123}{56} e'^2 - \tfrac{5}{9} \gamma^2\} \cos (4nt - 3n't - \omega + \omega') \\
&+ \tfrac{63}{16} ee' \{1 - \tfrac{23}{12} e^2 - \tfrac{123}{56} e'^2 - \tfrac{1}{5} \gamma^2\} \cos (2nt - 3n't - \omega + \omega') \\
&\mp \tfrac{39}{16} ee' \{1 - \tfrac{133}{52} e^2 - \tfrac{1}{8} e'^2 - \tfrac{7}{13} \gamma^2\} \cos (2nt - n't + \omega - \omega') \\
&\pm \tfrac{21}{16} ee' \{1 - \tfrac{5}{56} e^2 - \tfrac{1}{8} e'^2 - \tfrac{3}{7} \gamma^2\} \cos (n't - \omega + \omega') \\
&\mp \tfrac{15}{8} e\gamma^2 \cos (2nt - 2n't - \omega + 2\Omega) \pm \tfrac{3}{32} e\gamma^2 \cos (2n't + \omega - 2\Omega) \\
&\pm \tfrac{153}{64} e\gamma^2 \cos (6nt - 2n't - \omega - 2\Omega) - \tfrac{33}{64} e\gamma^2 \cos (4nt - 2n't - \omega - 2\Omega) \\
&\pm \tfrac{81}{64} e\gamma^2 \cos (4nt - 2n't - 3\omega + 2\Omega) - \tfrac{9}{64} e\gamma^2 \cos (2nt - 2n't - 3\omega + 2\Omega) \\
&\pm \tfrac{117}{64} e\gamma^2 \cos (2nt - 2n't + 3\omega - 2\Omega) \mp \tfrac{21}{64} e\gamma^2 \cos (2n't - 3\omega + 2\Omega) \\
&\mp \tfrac{135}{32} e\gamma^2 \cos (4nt - 2n't + \omega - 2\Omega) + \tfrac{27}{32} e\gamma^2 \cos (2nt - 2n't + \omega - 2\Omega) \\
&\pm \tfrac{3}{64} e\gamma^2 \cos (2nt + 2n't - \omega - 2\Omega) + \tfrac{45}{64} e\gamma^2 \cos (2n't - \omega - 2\Omega) \\
&\mp \tfrac{9}{32} e'\gamma^2 \cos (5nt - n't - \omega' - 2\Omega) + \tfrac{3}{32} e'\gamma^2 \cos (3nt - n't - \omega' - 2\Omega) \\
&\pm \tfrac{21}{32} e'\gamma^2 \cos (nt + 3n't - \omega' - 2\Omega) \mp \tfrac{63}{32} e'\gamma^2 \cos (nt - 3n't + \omega' + 2\Omega) \\
&\pm \tfrac{63}{32} e'\gamma^2 \cos (5nt - 3n't + \omega' - 2\Omega) - \tfrac{21}{32} e'\gamma^2 \cos (3nt - 3n't + \omega' - 2\Omega) \\
&\mp \tfrac{3}{32} e'\gamma^2 \cos (nt + n't + \omega' - 2\Omega) \pm \tfrac{9}{32} e'\gamma^2 \cos (nt - n't - \omega' + 2\Omega) \\
&\mp \tfrac{9}{32} e'\gamma^2 \cos (3nt - n't - 2\omega - \omega' + 2\Omega) \\
&+ \tfrac{3}{32} e'\gamma^2 \cos (nt - n't - 2\omega - \omega' + 2\Omega) \\
&\mp \tfrac{63}{32} e'\gamma^2 \cos (3nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&+ \tfrac{21}{32} e'\gamma^2 \cos (nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&\pm \tfrac{9}{32} e'\gamma^2 \cos (3nt - n't + 2\omega - \omega' - 2\Omega) \\
&- \tfrac{3}{32} e'\gamma^2 \cos (nt - n't + 2\omega - \omega' - 2\Omega) \\
&\pm \tfrac{63}{32} e'\gamma^2 \cos (3nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&- \tfrac{21}{32} e'\gamma^2 \cos (nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&\pm \tfrac{663}{16} ee'^2 \cos (2nt - 4n't + \omega + 2\omega') \mp \tfrac{357}{16} ee'^2 \cos (4n't - \omega - 2\omega') \\
&\mp \tfrac{459}{16} ee'^2 \cos (4nt - 4n't - \omega + 2\omega') \\
&+ \tfrac{153}{16} ee'^2 \cos (2nt - 4n't - \omega + 2\omega') \\
&\pm \tfrac{93}{32} e^2e' \cos (5nt - n't - 2\omega - \omega') - \tfrac{15}{32} e^2e' \cos (3nt - n't - 2\omega - \omega') \\
&\pm \tfrac{63}{32} e^2e' \cos (nt + 3n't - 2\omega - \omega') \mp \tfrac{525}{32} e^2e' \cos (nt - 3n't + 2\omega + \omega') \\
&\mp \tfrac{651}{32} e^2e' \cos (5nt - 3n't - 2\omega + \omega') \\
&+ \tfrac{105}{32} e^2e' \cos (3nt - 3n't - 2\omega + \omega') \\
&\mp \tfrac{9}{32} e^2e' \cos (nt + n't - 2\omega + \omega') \pm \tfrac{75}{32} e^2e' \cos (nt - n't + 2\omega - \omega') \\
&\mp \tfrac{291}{32} e^3 \cos (6nt - 2n't - 3\omega) + \tfrac{7}{8} e^3 \cos (4nt - 2n't - 3\omega) \\
&\pm \tfrac{5}{32} e^3 \cos (2nt + 2n't - 3\omega) + \tfrac{15}{16} e^3 \cos (2n't - 3\omega)
\end{aligned}
\right\} . \quad (269)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp \tfrac{1}{32}e'^3 \cos(3nt + n't - 3\omega') + \tfrac{1}{32}e'^3 \cos(nt + n't - 3\omega') \\
&\mp \tfrac{845}{32}e'^3 \cos(3nt - 5n't + 3\omega') + \tfrac{845}{32}e'^3 \cos(nt - 5n't + 3\omega') \\
&\mp \tfrac{3477}{256}e^4 \cos(7nt - 2n't - 4\omega) + \tfrac{225}{256}e^4 \cos(5nt - 2n't - 4\omega) \\
&\pm \tfrac{21}{256}e^4 \cos(3nt + 2n't - 4\omega) + \tfrac{47}{256}e^4 \cos(nt + 2n't - 4\omega) \\
&\mp \tfrac{1}{16}e'^4 \cos(3nt + 2n't - 4\omega') + \tfrac{1}{16}e'^4 \cos(nt + 2n't - 4\omega') \\
&\mp \tfrac{1599}{32}e'^4 \cos(3nt - 6n't + 4\omega') + \tfrac{1599}{32}e'^4 \cos(nt - 6n't + 4\omega') \\
&\mp \tfrac{45}{256}\gamma^4 \cos(7nt - 2n't - 4\Omega) + \tfrac{9}{256}\gamma^4 \cos(5nt - 2n't - 4\Omega) \\
&\mp \tfrac{45}{256}\gamma^4 \cos(3nt - 2n't + 4\omega - 4\Omega) + \tfrac{9}{256}\gamma^4 \cos(nt - 2n't + 4\omega - 4\Omega) \\
&\pm \tfrac{45}{128}\gamma^4 \cos(5nt - 2n't + 2\omega - 4\Omega) - \tfrac{9}{128}\gamma^4 \cos(3nt - 2n't + 2\omega - 4\Omega) \\
&\mp \tfrac{3}{256}\gamma^4 \cos(3nt + 2n't - 4\Omega) - \tfrac{9}{256}\gamma^4 \cos(nt + 2n't - 4\Omega) \\
&\pm \tfrac{3}{128}\gamma^4 \cos(nt + 2n't + 2\omega - 4\Omega) - \tfrac{3}{256}\gamma^4 \cos(nt - 2n't - 4\omega + 4\Omega) \\
&\pm \tfrac{9}{128}\gamma^4 \cos(nt - 2n't - 2\omega + 4\Omega) \mp \tfrac{9}{256}\gamma^4 \cos(3nt - 2n't - 4\omega + 4\Omega) \\
&\pm \tfrac{279}{128}e^2\gamma^2 \cos(5nt - 2n't - 4\omega + 2\Omega) \\
&- \tfrac{15}{128}e^2\gamma^2 \cos(3nt - 2n't - 4\omega + 2\Omega) \\
&\pm \tfrac{9}{128}e^2\gamma^2 \cos(nt + 2n't - 4\omega + 2\Omega) \mp \tfrac{225}{128}e^2\gamma^2 \cos(nt - 2n't + 4\omega - 2\Omega) \\
&\pm \tfrac{837}{128}e^2\gamma^2 \cos(7nt - 2n't - 2\omega - 2\Omega) \\
&- \tfrac{129}{128}e^2\gamma^2 \cos(5nt - 2n't - 2\omega - 2\Omega) \\
&\pm \tfrac{3}{128}e^2\gamma^2 \cos(3nt + 2n't - 2\omega - 2\Omega) \\
&+ \tfrac{9}{128}e^2\gamma^2 \cos(nt + 2n't - 2\omega - 2\Omega) \pm \tfrac{51}{32}e'^2\gamma^2 \cos(nt + 4n't - 2\omega' - 2\Omega) \\
&\mp \tfrac{153}{32}e'^2\gamma^2 \cos(nt - 4n't + 2\omega' + 2\Omega) \\
&\mp \tfrac{153}{32}e'^2\gamma^2 \cos(5nt - 4n't + 2\omega' - 2\Omega) \\
&+ \tfrac{51}{32}e'^2\gamma^2 \cos(3nt - 4n't + 2\omega' - 2\Omega) \\
&\mp \tfrac{153}{32}e'^2\gamma^2 \cos(3nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&- \tfrac{51}{32}e'^2\gamma^2 \cos(nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&\pm \tfrac{153}{32}e'^2\gamma^2 \cos(3nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- \tfrac{51}{32}et'^2\gamma^2 \cos(nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&\mp \tfrac{1581}{32}e^2e'^2 \cos(5nt - 4n't - 2\omega + 2\omega') \\
&+ \tfrac{255}{32}e^2e'^2 \cos(3nt - 4n't - 2\omega + 2\omega') \\
&\pm \tfrac{153}{32}e^2e'^2 \cos(nt + 4n't - 2\omega - 2\omega') \\
&\mp \tfrac{1275}{32}e^2e'^2 \cos(nt - 4n't + 2\omega + 2\omega') \\
&\mp \tfrac{9}{128}ee'^3 \cos(4nt + n't - \omega - 3\omega') + \tfrac{3}{128}ee'^3 \cos(2nt + n't - \omega - 3\omega') \\
&\pm \tfrac{13}{128}ee'^3 \cos(2nt + n't + \omega - 3\omega') - \tfrac{7}{128}ee'^3 \cos(nt + \omega - 3\omega') \\
&\pm \tfrac{10985}{128}ee'^3 \cos(2nt - 5n't + \omega + 3\omega') \mp \tfrac{5915}{128}ee'^3 \cos(5nt - \omega - 3\omega') \\
&\mp \tfrac{7605}{128}ee'^3 \cos(4nt - 5n't - \omega + 3\omega') \\
&+ \tfrac{2535}{128}ee'^3 \cos(2nt - 5n't - \omega + 3\omega') \\
&\pm \tfrac{35}{64}e^3e' \cos(2nt + 3n't - 3\omega - \omega') + \tfrac{105}{32}e^3e' \cos(3n't - 3\omega - \omega')
\end{aligned}
\right\} \quad . \qquad (269)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp \tfrac{5}{64}e^3e' \cos(2nt + n't - 3\omega + \omega') - \tfrac{15}{32}e^3e' \cos(n't - 3\omega + \omega') \\
&\pm \tfrac{291}{64}e^3e' \cos(6nt - n't - 3\omega - \omega') - \tfrac{7}{16}e^3e' \cos(4nt - n't - 3\omega + \omega') \\
&\mp \tfrac{2037}{64}e^3e' \cos(6nt - 3n't - 3\omega + \omega') \\
&+ \tfrac{49}{16}e^3e' \cos(4nt - 3n't - 3\omega + \omega') \\
&\pm \tfrac{1071}{128}ee'\gamma^2 \cos(6nt - 3n't - \omega + \omega' - 2\Omega) \\
&- \tfrac{231}{128}ee'\gamma^2 \cos(4nt - 3n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{3}{128}ee'\gamma^2 \cos(2nt + n't - \omega + \omega' - 2\Omega) \\
&- \tfrac{45}{128}ee'\gamma^2 \cos(n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{945}{64}ee'\gamma^2 \cos(4nt - 3n't + \omega + \omega' - 2\Omega) \\
&+ \tfrac{189}{64}ee'\gamma^2 \cos(2nt - 3n't + \omega + \omega' - 2\Omega) \\
&\pm \tfrac{63}{64}ee'\gamma^2 \cos(2nt - n't - \omega - \omega' + 2\Omega) \mp \tfrac{3}{64}ee'\gamma^2 \cos(n't + \omega + \omega' - 2\Omega) \\
&\mp \tfrac{537}{128}ee'\gamma^2 \cos(6nt - n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{417}{128}ee'\gamma^2 \cos(4nt - n't - \omega - \omega' - 2\Omega) \\
&\mp \tfrac{363}{128}ee'\gamma^2 \cos(2nt + 3n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{699}{128}ee'\gamma^2 \cos(3n't - \omega - \omega' - 2\Omega) \\
&\pm \tfrac{185}{64}ee'\gamma^2 \cos(4nt - n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{27}{64}ee'\gamma^2 \cos(2nt - n't + \omega - \omega' - 2\Omega) \\
&\mp \tfrac{459}{64}ee'\gamma^2 \cos(2nt - 3n't - \omega + \omega' + 2\Omega) \\
&\pm \tfrac{39}{64}ee'\gamma^2 \cos(3n't + \omega - \omega' - 2\Omega) \\
&\pm \tfrac{567}{128}ee'\gamma^2 \cos(4nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- \tfrac{63}{128}ee'\gamma^2 \cos(2nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{117}{128}ee'\gamma^2 \cos(2nt - n't + 3\omega - \omega' - 2\Omega) \\
&\pm \tfrac{21}{128}ee'\gamma^2 \cos(n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{81}{128}ee'\gamma^2 \cos(4nt - n't - 3\omega - \omega' + 2\Omega) \\
&+ \tfrac{9}{128}ee'\gamma^2 \cos(2nt - n't - 3\omega - \omega' + 2\Omega) \\
&\pm \tfrac{855}{128}ee'\gamma^2 \cos(2nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&\mp \tfrac{183}{128}ee'\gamma^2 \cos(3n't - 3\omega - \omega' + 2\Omega) \Big\} \\
&+ \overline{m}^2 \frac{a}{a'}\, ndt \Big\{ \mp \tfrac{3}{8}\{1 - 2e^2 + 2e'^2 - \tfrac{27}{4}\gamma^2\} \cos(2nt - n't) \\
&\pm \tfrac{3}{8}\{1 + e^2 + 2e'^2 - \tfrac{27}{4}\gamma^2\} \cos n't \mp \tfrac{9}{32}e \cos(3nt - n't - \omega) \\
&- \tfrac{9}{32}e \cos(nt - n't - \omega) \mp \tfrac{15}{32}e \cos(nt + n't - \omega) \\
&\pm \tfrac{33}{32}e \cos(nt - n't + \omega) \mp \tfrac{3}{8}e' \cos(2nt - \omega') \pm \tfrac{3}{8}e' \cos\omega' \\
&\mp \tfrac{9}{8}e' \cos(nt - 2n't + \omega') \pm \tfrac{9}{8}e' \cos(2n't - \omega') \\
&\mp \tfrac{9}{32}ee' \cos(3nt - \omega - \omega') - \tfrac{9}{32}ee' \cos(nt - \omega - \omega') \\
&\mp \tfrac{45}{32}ee' \cos(nt + 2n't - \omega - \omega') \pm \tfrac{99}{32}ee' \cos(nt - 2n't + \omega + \omega')
\end{aligned}
\right\} . \quad (269)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\pm \tfrac{3}{32}ee' \cos(nt + \omega - \omega') \pm \tfrac{15}{32}ee' \cos(nt - \omega + \omega') \\
&\mp \tfrac{27}{32}ee' \cos(3nt - 2n't - \omega + \omega') - \tfrac{27}{32}ee' \cos(nt - 2n't - \omega + \omega') \\
&+ \tfrac{3}{16}e^2 \cos(4nt - n't - 2\omega) - \tfrac{9}{16}e^2 \cos(2nt - n't - 2\omega) \\
&- \tfrac{3}{64}e^2 \cos(2nt + n't - 2\omega) - \tfrac{45}{64}e^2 \cos(n't - 2\omega) \\
&\mp \tfrac{33}{64}e'^2 \cos(2nt + n't - 2\omega') + \tfrac{33}{64}e'^2 \cos(n't - 2\omega') \\
&\mp \tfrac{159}{64}e'^2 \cos(2nt - 3n't + 2\omega') \pm \tfrac{159}{64}e'^2 \cos(3n't - 2\omega') \\
&\pm \tfrac{75}{64}\gamma^2 \cos(2nt + n't - 2\Omega) - \tfrac{81}{64}\gamma^2 \cos(n't - 2\Omega) \\
&\mp \tfrac{69}{64}\gamma^2 \cos(4nt - n't - 2\Omega) + \tfrac{75}{64}\gamma^2 \cos(2nt - n't - 2\Omega) \\
&\pm \tfrac{3}{32}\gamma^2 \cos(n't + 2\omega - 2\Omega) \mp \tfrac{3}{32}\gamma^2 \cos(2nt - n't + 2\omega - 2\Omega) \\
&\mp \tfrac{15}{8}\{1 - 13e^2 - 6e'^2 - \gamma^2\} \cos(4nt - 3n't) \\
&+ \tfrac{15}{8}\{1 - 4e^2 - 6e'^2 - \gamma^2\} \cos(2nt - 3n't) \\
&\mp \tfrac{165}{32}e \cos(5nt - 3n't - \omega) + \tfrac{75}{32}e \cos(3nt - 3n't - \omega) \\
&\pm \tfrac{285}{32}e \cos(3nt - 3n't + \omega) - \tfrac{195}{32}e \cos(nt - 3n't + \omega) \\
&\mp \tfrac{75}{8}e' \cos(4nt - 4n't + \omega') + \tfrac{75}{8}e' \cos(2nt - 4n't + \omega') \\
&\pm \tfrac{15}{8}e' \cos(4nt - 2n't - \omega') - \tfrac{15}{8}e' \cos(2nt - 2n't - \omega') \\
&\mp \tfrac{645}{64}e^2 \cos(6nt - 3n't - 2\omega) + \tfrac{165}{64}e^2 \cos(4nt - 3n't - 2\omega) \\
&\mp \tfrac{975}{64}e^2 \cos(2nt - 3n't + 2\omega) \pm \tfrac{375}{64}e^2 \cos(3n't - 2\omega) \\
&\mp \tfrac{15}{64}e'^2 \cos(4nt - n't - 2\omega') + \tfrac{15}{64}e'^2 \cos(2nt - n't - 2\omega') \\
&\mp \tfrac{1905}{64}e'^2 \cos(4nt - 5n't + 2\omega') + \tfrac{1905}{64}e'^2 \cos(2nt - 5n't + 2\omega') \\
&\mp \tfrac{825}{32}ee' \cos(5nt - 4n't - \omega + \omega') + \tfrac{375}{32}ee' \cos(3nt - 4n't - \omega + \omega') \\
&\mp \tfrac{285}{32}ee' \cos(3nt - 2n't + \omega - \omega') + \tfrac{195}{32}ee' \cos(nt - 2n't + \omega - \omega') \\
&\pm \tfrac{165}{32}ee' \cos(5nt - 2n't - \omega - \omega') - \tfrac{75}{32}ee' \cos(3nt - 2n't - \omega - \omega') \\
&\pm \tfrac{1425}{32}ee' \cos(3nt - 4n't + \omega + \omega') - \tfrac{975}{32}ee' \cos(nt - 4n't + \omega + \omega') \\
&\pm \tfrac{15}{16}h'' \cos(nt + a''t - \beta'') \mp \tfrac{15}{16}h'' \cos(nt - a''t + \beta'') \\
&\mp \tfrac{45}{32}\gamma^2 \cos(2nt - 3n't + 2\Omega) \pm \tfrac{15}{16}\gamma^2 \cos(3nt - 2\Omega) \\
&\mp \tfrac{15}{16}\gamma^2 \cos(4nt - 3n't + 2\omega - 2\Omega) \\
&+ \tfrac{15}{32}\gamma^2 \cos(2nt - 3n't + 2\omega - 2\Omega) \\
&\pm \tfrac{15}{16}\gamma^2 \cos(4nt - 3n't - 2\omega + 2\Omega) \\
&- \tfrac{15}{32}\gamma^2 \cos(2nt - 3n't - 2\omega + 2\Omega) \pm \tfrac{15}{32}\gamma^2 \cos(6nt - 3n't - 2\Omega) \Big\}
\end{aligned}\right\} \quad (269)
$$

This equation will give the value of $c_3 \cos\beta\left(\frac{dR}{dv}\right)$ by using the lower signs and changing *cos* to *sin* in the second member.

In like manner equations (220) and (239) will give

$$
\begin{aligned}
&c_4 \sin\beta \left(\frac{dR}{d\theta}\right) = \\
&\overline{m}^2 n dt \Big\{ \mp \tfrac{3}{4}\gamma^2\{1 - \tfrac{15}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{11}{4}\gamma^2\} \cos(3nt - 2\Omega) \\
&\quad + \tfrac{3}{4}\gamma^2\{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - 2\Omega) \pm \tfrac{3}{4}\{e^2\gamma^2 - \tfrac{1}{4}\gamma^4\} \cos nt \\
&\quad \mp \tfrac{39}{16}e\gamma^2 \cos(2nt + \omega - 2\Omega) \mp \tfrac{3}{8}e\gamma^2 \cos(2nt - \omega) \pm \tfrac{3}{8}e\gamma^2 \cos\omega \\
&\quad - \tfrac{15}{16}e\gamma^2 \cos(\omega - 2\Omega) \mp \tfrac{9}{8}e'\gamma^2 \cos(3nt + n't - \omega' - 2\Omega) \\
&\quad \mp \tfrac{9}{8}e'\gamma^2 \cos(3nt - n't + \omega' - 2\Omega) + \tfrac{9}{8}e'\gamma^2 \cos(nt + n't - \omega' - 2\Omega) \\
&\quad + \tfrac{9}{8}e'\gamma^2 \cos(nt - n't + \omega' - 2\Omega) \mp \tfrac{27}{16}e\gamma^2 \cos(4nt - \omega - 2\Omega) \\
&\quad + \tfrac{3}{16}e\gamma^2 \cos(2nt - \omega - 2\Omega) - \tfrac{3}{8}\gamma^2 \{e^2 + \tfrac{1}{4}\gamma^2\} \cos(nt - 2\omega) \\
&\quad \mp \tfrac{93}{32}e^2\gamma^2 \cos(5nt - 2\omega - 2\Omega) \mp \tfrac{3}{8}\gamma^2\{e^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 2\omega) \\
&\quad \mp \tfrac{75}{32}e^2\gamma^2 \cos(nt + 2\omega - 2\Omega) \mp \tfrac{3}{32}e^2\gamma^2 \cos(nt - 2\omega + 2\Omega) \\
&\quad + \tfrac{3}{32}e^2\gamma^2 \cos(3nt - 2\omega - 2\Omega) + \tfrac{3}{16}\gamma^4 \cos(3nt - 4\Omega) \\
&\quad + \tfrac{3}{32}\gamma^4 \cos(nt + 2\omega - 4\Omega) \mp \tfrac{9}{32}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \\
&\quad \mp \tfrac{27}{16}e'^2\gamma^2 \cos(3nt + 2n't - 2\omega' - 2\Omega) \mp \tfrac{27}{16}e'^2\gamma^2 \cos(3nt - 2n't + 2\omega' - 2\Omega) \\
&\quad + \tfrac{27}{16}e'^2\gamma^2 \cos(nt + 2n't - 2\omega' - 2\Omega) \\
&\quad + \tfrac{27}{16}e'^2\gamma^2 \cos(nt - 2n't + 2\omega' - 2\Omega) \\
&\quad \mp \tfrac{9}{16} ee'\gamma^2 \cos(2nt + n't - \omega - \omega') \mp \tfrac{9}{16}ee'\gamma^2 \cos(2nt - n't - \omega + \omega') \\
&\quad + \tfrac{9}{16}ee'\gamma^2 \cos(n't - \omega - \omega') \pm \tfrac{9}{16}ee'\gamma^2 \cos(n't + \omega - \omega') \\
&\quad \pm \tfrac{117}{32}ee'\gamma^2 \cos(2nt - n't + \omega + \omega' - 2\Omega) \\
&\quad \pm \tfrac{117}{32}ee'\gamma^2 \cos(2nt + n't + \omega - \omega' - 2\Omega) \\
&\quad \mp \tfrac{45}{32}ee'\gamma^2 \cos(n't - \omega - \omega' + 2\Omega) - \tfrac{45}{32}ee'\gamma^2 \cos(n't + \omega - \omega' - 2\Omega) \\
&\quad \mp \tfrac{81}{32}ee'\gamma^2 \cos(4nt + n't - \omega - \omega' - 2\Omega) \\
&\quad \mp \tfrac{81}{32}ee'\gamma^2 \cos(4nt - n't - \omega + \omega' - 2\Omega) \\
&\quad + \tfrac{9}{32}ee'\gamma^2 \cos(2nt + n't - \omega - \omega' - 2\Omega) \\
&\quad + \tfrac{9}{32}ee'\gamma^2 \cos(2nt - n't - \omega + \omega' - 2\Omega) \\
&\quad \pm \tfrac{9}{8}\gamma^2 \{e^2 - \tfrac{1}{4}\gamma^2\} \cos(3nt - 2n't) - \tfrac{3}{8}\gamma^2 \{e^2 - \tfrac{1}{4}\gamma^2\} \cos(nt - 2n't) \\
&\quad \mp \tfrac{3}{8}\gamma^2 \{1 - 25e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(5nt - 2n't - 2\Omega) \\
&\quad \mp \tfrac{3}{8}\gamma^2 \{1 - \tfrac{1}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt + 2n't - 2\Omega) \\
&\quad + \tfrac{3}{8}\gamma^2 \{1 - \tfrac{15}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(3nt - 2n't - 2\Omega) \\
&\quad \pm \tfrac{3}{8}\gamma^2 \{1 - \tfrac{3}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - 2n't + 2\Omega) \\
&\quad \mp \tfrac{51}{32}e\gamma^2 \cos(6nt - 2n't - \omega - 2\Omega) + \tfrac{3}{16}e\gamma^2 \cos(2nt - 2n't - \omega) \\
&\quad \mp \tfrac{3}{16}e\gamma^2 \cos(2n't - \omega) \pm \tfrac{63}{32}e\gamma^2 \cos(4nt - 2n't + \omega - 2\Omega) \\
&\quad \mp \tfrac{3}{32}e\gamma^2 \cos(2nt + 2n't - \omega - 2\Omega) \pm \tfrac{3}{16}e\gamma^2 \cos(2nt - 2n't + \omega) \\
&\quad \mp \tfrac{3}{16}e\gamma^2 \cos(4nt - 2n't - \omega) + \tfrac{27}{32}e\gamma^2 \cos(4nt - 2n't - \omega - 2\Omega)
\end{aligned}
\qquad (270)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&-\tfrac{39}{32}e\gamma^2\cos(2nt-2n't+\omega-2\Omega)-\tfrac{21}{32}e\gamma^2\cos(2n't-\omega-2\Omega)\\
&\pm\tfrac{9}{32}e\gamma^2\cos(2nt-2n't-\omega+2\Omega)\pm\tfrac{15}{32}e\gamma^2\cos(2n't+\omega-2\Omega)\\
&\pm\tfrac{3}{16}e'\gamma^2\cos(5nt-n't-\omega'-2\Omega)\pm\tfrac{3}{16}e'\gamma^2\cos(nt+n't+\omega'-2\Omega)\\
&\mp\tfrac{21}{16}e'\gamma^2\cos(5nt-3n't+\omega'-2\Omega)\mp\tfrac{21}{16}e'\gamma^2\cos(nt+3n't-\omega'-2\Omega)\\
&-\tfrac{3}{16}e'\gamma^2\cos(3nt-n't-\omega'-2\Omega)\mp\tfrac{3}{16}e'\gamma^2\cos(nt-n't-\omega'+2\Omega)\\
&+\tfrac{21}{16}e'\gamma^2\cos(3nt-3n't+\omega'-2\Omega)\pm\tfrac{21}{16}e'\gamma^2\cos(nt-3n't+\omega'+2\Omega)\\
&\mp\tfrac{279}{64}e^2\gamma^2\cos(7nt-2n't-2\omega-2\Omega)\\
&+\tfrac{3}{16}\gamma^2\{e^2-\tfrac{3}{4}\gamma^2\}\cos(3nt-2n't-2\omega)\\
&+\{\tfrac{51}{64}\gamma^4-\tfrac{7}{32}e^2\gamma^2\}\cos(nt-2n't-2\omega+2\Omega)\\
&\mp\{\tfrac{19}{64}e^2\gamma^2+\tfrac{9}{16}\gamma^4\}\cos(5nt-2n't-2\omega)\\
&\mp\tfrac{3}{64}e^2\gamma^2\cos(3nt+2n't-2\omega-2\Omega)\\
&\mp\{\tfrac{9}{16}e^2\gamma^2-\tfrac{3}{64}\gamma^4\}\cos(nt-2n't+2\omega)\\
&+\tfrac{93}{64}e^2\gamma^2\cos(5nt-2n't-2\omega-2\Omega)\\
&\pm\tfrac{15}{64}e^2\gamma^2\cos(3nt-2n't-2\omega+2\Omega)\\
&\pm\{\tfrac{3}{16}e^2\gamma^2+\tfrac{3}{64}\gamma^4\}\cos(nt+2n't-2\omega)\\
&+\tfrac{9}{64}e^2\gamma^2\cos(nt+2n't-2\omega-2\Omega)\mp\tfrac{249}{64}e^2\gamma^2\cos(3nt-2n't+2\omega-2\Omega)\\
&+\tfrac{75}{64}e^2\gamma^2\cos(nt-2n't+2\omega-2\Omega)\pm\tfrac{3}{32}\gamma^4\cos(7nt-2n't-4\Omega)\\
&\mp\tfrac{3}{32}\gamma^4\cos(3nt+2n't-4\Omega)\mp\tfrac{15}{64}\gamma^4\cos(5nt-2n't+2\omega-4\Omega)\\
&\mp\tfrac{3}{64}\gamma^4\cos(nt+2n't+2\omega-4\Omega)+\tfrac{3}{16}\gamma^4\cos(nt+2n't-4\Omega)\\
&+\tfrac{9}{64}\gamma^4\cos(3nt-2n't+2\omega-4\Omega)\mp\tfrac{3}{64}\gamma^4\cos(nt-2n't-2\omega+4\Omega)\\
&\mp\tfrac{51}{16}e'^2\gamma^2\cos(5nt-4n't+2\omega'-2\Omega)\\
&\mp\tfrac{51}{16}e'^2\gamma^2\cos(nt+4n't-2\omega'-2\Omega)\\
&+\tfrac{51}{16}e'^2\gamma^2\cos(3nt-4n't+2\omega'-2\Omega)\pm\tfrac{51}{16}e'^2\gamma^2\cos(nt-4n't+2\omega'+2\Omega)\\
&\pm\tfrac{51}{64}ee'\gamma^2\cos(6nt-n't-\omega-\omega'-2\Omega)\\
&-\tfrac{3}{32}ee'\gamma^2\cos(2nt-n't-\omega-\omega')\pm\tfrac{105}{64}ee'\gamma^2\cos(3n't+\omega-\omega'-2\Omega)\\
&\pm\tfrac{441}{64}ee'\gamma^2\cos(4nt-3n't+\omega+\omega'-2\Omega)\\
&\mp\tfrac{21}{32}ee'\gamma^2\cos(3n't-\omega-\omega')\mp\tfrac{21}{64}ee'\gamma^2\cos(2nt+3n't-\omega-\omega'-2\Omega)\\
&\pm\tfrac{21}{32}ee'\gamma^2\cos(2nt-3n't+\omega+\omega')\\
&\mp\tfrac{357}{64}ee'\gamma^2\cos(6nt-3n't-\omega+\omega'-2\Omega)\\
&+\tfrac{21}{32}ee'\gamma^2\cos(2nt-3n't-\omega+\omega')\\
&\mp\tfrac{63}{64}ee'\gamma^2\cos(4nt-n't+\omega-\omega'-2\Omega)\pm\tfrac{3}{32}ee'\gamma^2\cos(n't-\omega+\omega')\\
&\pm\tfrac{3}{64}ee'\gamma^2\cos(2nt+n't-\omega+\omega'-2\Omega)\\
&\mp\tfrac{3}{32}ee'\gamma^2\cos(2nt-n't+\omega-\omega')\pm\tfrac{3}{32}ee'\gamma^2\cos(4nt-n't-\omega-\omega')\\
&-\tfrac{27}{64}ee'\gamma^2\cos(4nt-n't-\omega-\omega'-2\Omega)\\
&-\tfrac{273}{64}ee'\gamma^2\cos(2nt-3n't+\omega+\omega'-2\Omega)\\
&-\tfrac{147}{64}ee'\gamma^2\cos(3n't-\omega-\omega'-2\Omega)\mp\tfrac{21}{32}ee'\gamma^2\cos(4nt-3n't-\omega+\omega')
\end{aligned}
\right\}\ . \qquad (270)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\tfrac{189}{64}ee'\gamma^2\cos(4nt-3n't-\omega+\omega'-2\Omega)\\
&\mp\tfrac{15}{64}ee'\gamma^2\cos(n't+\omega+\omega'-2\Omega)+\tfrac{39}{64}ee'\gamma^2\cos(2nt-n't+\omega-\omega'-2\Omega)\\
&+\tfrac{21}{64}ee'\gamma^2\cos(n't-\omega+\omega'-2\Omega)\mp\tfrac{9}{64}ee'\gamma^2\cos(2nt-n't-\omega-\omega'+2\Omega)\\
&\left.\pm\tfrac{63}{64}ee'\gamma^2\cos(2nt-3n't-\omega+\omega'+2\Omega)\right\}\\
&+\overline{m}^2\frac{a}{a'}ndt\Big\{\mp\tfrac{33}{32}\gamma^2\cos(4nt-n't-2\Omega)\mp\tfrac{33}{32}\gamma^2\cos(2nt+n't-2\Omega)\\
&\mp\tfrac{15}{32}\gamma^2\cos(6nt-3n't-2\Omega)\mp\tfrac{15}{32}\gamma^2\cos(3n't-2\Omega)\\
&+\tfrac{33}{32}\gamma^2\cos(2nt-n't-2\Omega)+\tfrac{33}{32}\gamma^2\cos(n't-2\Omega)\\
&\left.+\tfrac{15}{32}\gamma^2\cos(4nt-3n't-2\Omega)\pm\tfrac{15}{32}\gamma^2\cos(2nt-3n't+2\Omega)\right\}
\end{aligned}
\right\}. \tag{270}
$$

This equation will give the value of $c_4 \cos\beta\left(\frac{dR}{d\theta}\right)$ by using the lower signs and changing *cos* to *sin* in the second member.

If we now add equations (268), (269), and (270) together, we shall obtain the following development:

$$c_2\cos\beta\left(\frac{dR}{dr}\right)+c_3\sin\beta\left(\frac{dR}{dv}\right)+c_4\sin\beta\left(\frac{dR}{d\theta}\right)=$$

$$
\left.
\begin{aligned}
&\overline{m}^2ndt\Big\{\pm\tfrac{1}{2}\{1-e^2+\tfrac{3}{2}e'^2-\tfrac{7}{4}\gamma^2+\tfrac{7}{64}e^4+\tfrac{15}{8}e'^4-\tfrac{9}{4}e^2e'^2\\
&\qquad\qquad+\tfrac{13}{4}e^2\gamma^2-\tfrac{21}{8}e'^2\gamma^2+\tfrac{95}{64}\gamma^4\}\cos nt\\
&\pm\tfrac{1}{4}e\{1-\tfrac{5}{4}e^2+\tfrac{3}{2}e'^2-\tfrac{13}{4}\gamma^2\}\cos(2nt-\omega)\\
&\pm\tfrac{3}{4}e'\{1-e^2+\tfrac{9}{8}e'^2-\tfrac{7}{4}\gamma^2\}\cos(nt+n't-\omega')\\
&\pm\tfrac{3}{4}e'\{1-e^2+\tfrac{9}{8}e'^2-\tfrac{7}{4}\gamma^2\}\cos(nt-n't+\omega')\\
&\pm\tfrac{3}{16}e^2\{1-\tfrac{3}{2}e^2+\tfrac{3}{2}e'^2-4\gamma^2+\frac{\gamma^4}{e^2}\}\cos(3nt-2\omega)\\
&+\tfrac{1}{16}e^2\{1+\tfrac{25}{2}e^2+\tfrac{3}{2}e'^2-\tfrac{31}{4}\gamma^2-2\frac{\gamma^4}{e^2}\}\cos(nt-2\omega)\\
&\pm\tfrac{9}{8}e'^2\{1-e^2+\tfrac{7}{9}e'^2-\tfrac{7}{4}\gamma^2\}\cos(nt+2n't-2\omega')\\
&\pm\tfrac{9}{8}e'^2\{1-e^2+\tfrac{7}{9}e'^2-\tfrac{7}{4}\gamma^2\}\cos(nt-2n't+2\omega')\\
&\pm\tfrac{3}{8}ee'\{1-\tfrac{5}{4}e^2+\tfrac{9}{8}e'^2-\tfrac{13}{4}\gamma^2\}\cos(2nt+n't-\omega-\omega')\\
&-\tfrac{9}{8}ee'\{1-\tfrac{1}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{9}{4}\gamma^2\}\cos(n't-\omega-\omega')\\
&\pm\tfrac{3}{8}ee'\{1-\tfrac{5}{4}e^2+\tfrac{9}{8}e'^2-\tfrac{13}{4}\gamma^2\}\cos(2nt-n't-\omega+\omega')\\
&\mp\tfrac{9}{8}ee'\{1-\tfrac{1}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{9}{4}\gamma^2\}\cos(n't+\omega-\omega')\\
&\mp\tfrac{3}{8}\gamma^2\{1-\tfrac{27}{16}e^2+\tfrac{3}{2}e'^2-\tfrac{17}{4}\gamma^2\}\cos(3nt-2\Omega)
\end{aligned}
\right\}. \tag{271}
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\gamma^2\left\{1-\tfrac{1}{2}e^2+\tfrac{3}{2}e'^2-\tfrac{9}{8}\gamma^2\right\}\cos(nt-2\Omega)\\
&\pm\tfrac{1}{16}\gamma^2\left\{1-\tfrac{27}{4}e^2+\tfrac{3}{2}e'^2-\tfrac{9}{4}\gamma^2\right\}\cos(nt+2\omega-2\Omega)\\
&\mp\tfrac{1}{16}\gamma^2\left\{1+9e^2+\tfrac{3}{2}e'^2-\tfrac{9}{4}\gamma^2\right\}\cos(nt-2\omega+2\Omega)\\
&\pm\tfrac{5}{3}e^3\cos(4nt-3\omega)+\tfrac{1}{48}e^3\cos(2nt-3\omega)\\
&\pm\tfrac{53}{32}e'^3\cos(nt+3n't-3\omega')\pm\tfrac{53}{32}e'^3\cos(nt-3n't+3\omega')\\
&\pm\tfrac{9}{16}ee'^2\cos(2nt+2n't-\omega-2\omega')-\tfrac{27}{16}ee'^2\cos(2n't-\omega-2\omega')\\
&\pm\tfrac{9}{16}ee'^2\cos(2nt-2n't-\omega+2\omega')\mp\tfrac{27}{16}ee'^2\cos(2n't+\omega-2\omega')\\
&\pm\tfrac{9}{32}e^2e'\cos(3nt+n't-2\omega-\omega')+\tfrac{3}{32}e^2e'\cos(nt+n't-2\omega-\omega')\\
&\pm\tfrac{9}{32}e^2e'\cos(3nt-n't-2\omega+\omega')+\tfrac{3}{32}e^2e'\cos(nt-n't-2\omega+\omega')\\
&\mp\tfrac{3}{4}e\gamma^2\cos(4nt-\omega-2\Omega)+\tfrac{5}{16}e\gamma^2\cos(2nt-\omega-2\Omega)\\
&\pm\tfrac{37}{32}e\gamma^2\cos(2nt+\omega-2\Omega)\mp\tfrac{1}{32}e\gamma^2\cos(2nt-3\omega+2\Omega)\\
&\mp\tfrac{9}{16}e'\gamma^2\cos(3nt+n't-\omega'-2\Omega)+\tfrac{3}{2}e'\gamma^2\cos(nt+n't-\omega'-2\Omega)\\
&\mp\tfrac{9}{16}e'\gamma^2\cos(3nt-n't+\omega'-2\Omega)+\tfrac{3}{2}e'\gamma^2\cos(nt-n't+\omega'-2\Omega)\\
&\mp\tfrac{3}{32}e'\gamma^2\cos(nt+n't-2\omega-\omega'+2\Omega)\\
&\mp\tfrac{3}{32}e'\gamma^2\cos(nt-n't-2\omega+\omega'+2\Omega)\\
&\pm\tfrac{3}{32}e'\gamma^2\cos(nt+n't+2\omega-\omega'-2\Omega)\\
&\pm\tfrac{3}{32}e'\gamma^2\cos(nt-n't+2\omega+\omega'-2\Omega)\\
&\pm\tfrac{53}{64}ee'^3\cos(2nt+3n't-\omega-3\omega')\pm\tfrac{53}{64}ee'^3\cos(2nt-3n't-\omega+3\omega')\\
&-\tfrac{159}{64}ee'^3\cos(3n't-\omega-3\omega')\mp\tfrac{159}{64}ee'^3\cos(3n't+\omega-3\omega')\\
&\pm\tfrac{1}{4}e^3e'\cos(4nt+n't-3\omega-\omega')+\tfrac{1}{32}e^3e'\cos(2nt+n't-3\omega-\omega')\\
&\pm\tfrac{1}{4}e^3e'\cos(4nt-n't-3\omega+\omega')+\tfrac{1}{32}e^3e'\cos(2nt-n't-3\omega+\omega')\\
&\pm\tfrac{27}{64}e^2e'^2\cos(3nt+2n't-2\omega-2\omega')\\
&+\tfrac{9}{64}e^2e'^2\cos(nt+2n't-2\omega-2\omega')\\
&\pm\tfrac{27}{64}e^2e'^2\cos(3nt-2n't-2\omega+2\omega')\\
&+\tfrac{9}{64}e^2e'^2\cos(nt-2n't-2\omega+2\omega')\\
&\mp\tfrac{27}{32}e'^2\gamma^2\cos(3nt+2n't-2\omega'-2\Omega)+\tfrac{9}{4}e'^2\gamma^2\cos(nt+2n't-2\omega'-2\Omega)\\
&\mp\tfrac{27}{32}e'^2\gamma^2\cos(3nt-2n't+2\omega'-2\Omega)\\
&+\tfrac{9}{4}e'^2\gamma^2\cos(nt-2n't+2\omega'-2\Omega)\\
&\mp\tfrac{9}{64}e'^2\gamma^2\cos(nt+2n't-2\omega-2\omega'+2\Omega)\\
&\mp\tfrac{9}{64}e'^2\gamma^2\cos(nt-2n't-2\omega+2\omega'+2\Omega)\\
&\pm\tfrac{9}{64}e'^2\gamma^2\cos(nt+2n't+2\omega-2\omega'-2\Omega)\\
&\pm\tfrac{9}{64}e'^2\gamma^2\cos(nt-2n't+2\omega+2\omega'-2\Omega)\\
&\pm\tfrac{125}{768}e^4\cos(5nt-4\omega)+\tfrac{3}{256}e^4\cos(3nt-4\omega)\\
&\pm\tfrac{77}{32}e'^4\cos(nt+4n't-4\omega')\pm\tfrac{77}{32}e'^4\cos(nt-4n't+4\omega')\\
&\mp\tfrac{75}{64}e^2\gamma^2\cos(5nt-2\omega-2\Omega)+\tfrac{3}{16}e^2\gamma^2\cos(3nt-2\omega-2\Omega)\\
&\pm\tfrac{3}{64}e^2\gamma^2\cos(3nt-4\omega+2\Omega)+\tfrac{1}{128}e^2\gamma^2\cos(nt-4\omega+2\Omega)
\end{aligned}
\right\}\quad .\ (271)
$$

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$$
\left.
\begin{aligned}
&\mp \tfrac{27}{32} ee'\gamma^2 \cos(4nt - n't - \omega + \omega' - 2\Omega) \\
&+ \tfrac{3}{4} ee'\gamma^2 \cos(2nt - n't - \omega + \omega' - 2\Omega) \\
&\pm \tfrac{129}{64} ee'\gamma^2 \cos(2nt + n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{99}{64} ee'\gamma^2 \cos(n't + \omega - \omega' - 2\Omega) \\
&\mp \tfrac{9}{8} ee'\gamma^2 \cos(4nt + n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{15}{32} ee'\gamma^2 \cos(2nt + n't - \omega - \omega' - 2\Omega) \\
&\pm \tfrac{111}{64} ee'\gamma^2 \cos(2nt - n't + \omega + \omega' - 2\Omega) \\
&\mp \tfrac{117}{64} ee'\gamma^2 \cos(n't - \omega - \omega' + 2\Omega) \\
&\mp \tfrac{3}{64} ee'\gamma^2 \cos(2nt + n't - 3\omega - \omega' + 2\Omega) \\
&- \tfrac{9}{64} ee'\gamma^2 \cos(n't - 3\omega - \omega' + 2\Omega) \\
&\mp \tfrac{3}{64} ee'\gamma^2 \cos(2nt - n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{9}{64} ee'\gamma^2 \cos(n't + 3\omega - \omega' - 2\Omega) \mp \tfrac{9}{64}\gamma^4 \cos(3nt + 2\omega - 4\Omega) \\
&+ \tfrac{1}{8}\gamma^4 \cos(nt + 2\omega - 4\Omega) + \tfrac{3}{32}\gamma^4 \cos(3nt - 4\Omega) \\
&\mp \tfrac{1}{256}\gamma^4 \cos(nt - 4\omega + 4\Omega) \pm \tfrac{3}{256}\gamma^4 \cos(nt + 4\omega - 4\Omega) \\
&\mp \tfrac{3}{4}\{1 - 6e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2 + \tfrac{615}{32}e^4 - \tfrac{9}{16}e'^4 + \tfrac{29}{64}\gamma^4 \\
&\qquad\qquad + \tfrac{63}{4}e^2e'^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{5}{8}e'^2\gamma^2\} \cos(3nt - 2n't) \\
&+ \tfrac{9}{4}\{1 - \tfrac{4}{3}e^2 - \tfrac{5}{2}e'^2 - \tfrac{7}{12}\gamma^2 + \tfrac{37}{192}e^4 + \tfrac{61}{48}e'^4 + \tfrac{41}{64}\gamma^4 \\
&\qquad\qquad + \tfrac{11}{6}e^2e'^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{85}{24}e'^2\gamma^2\} \cos(nt - 2n't) \\
&\mp \tfrac{3}{2}e\,\{1 - \tfrac{19}{4}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{16}\gamma^2\} \cos(4nt - 2n't - \omega) \\
&+ \tfrac{3}{2}e\,\{1 - \tfrac{11}{8}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{16}\gamma^2\} \cos(2nt - 2n't - \omega) \\
&\pm \tfrac{9}{4}e\,\{1 - \tfrac{11}{6}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{24}\gamma^2\} \cos(2nt - 2n't + \omega') \\
&\mp \tfrac{15}{4}e\,\{1 - \tfrac{1}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{43}{40}\gamma^2\} \cos(2n't - \omega) \\
&\pm \tfrac{3}{8}e'\,\{1 - 6e^2 - \tfrac{1}{8}e'^2 - \tfrac{1}{4}\gamma^2\} \cos(3nt - n't - \omega') \\
&- \tfrac{9}{8}e'\,\{1 - \tfrac{4}{3}e^2 - \tfrac{1}{8}e'^2 - \tfrac{7}{12}\gamma^2\} \cos(nt - n't - \omega') \\
&\mp \tfrac{21}{8}e'\,\{1 - 6e^2 - \tfrac{123}{56}e'^2 - \tfrac{1}{4}\gamma^2\} \cos(3nt - 3n't + \omega') \\
&+ \tfrac{63}{8}e'\,\{1 - \tfrac{4}{3}e^2 - \tfrac{123}{56}e'^2 - \tfrac{7}{12}\gamma^2\} \cos(nt - 3n't + \omega') \\
&\mp \tfrac{75}{32}e^2\,\{1 - \tfrac{31}{6}e^2 - \tfrac{5}{2}e'^2 - \tfrac{73}{300}\gamma^2 + \tfrac{592888}{1510725}\tfrac{\gamma^4}{e^2}\} \cos(5nt - 2n't - 2\omega) \\
&+ \tfrac{39}{32}e^2\,\{1 - \tfrac{41}{26}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2 - \tfrac{7}{52}\tfrac{\gamma^4}{e^2}\} \cos(3nt - 2n't - 2\omega) \\
&\pm \tfrac{21}{32}e^2\,\{1 + \tfrac{101}{42}e^2 - \tfrac{5}{2}e'^2 + \tfrac{1}{28}\gamma^2 + \tfrac{5}{128}\tfrac{\gamma^4}{e^2}\} \cos(nt + 2n't - 2\omega) \\
&\mp \tfrac{57}{32}e^2\,\{1 - \tfrac{14}{57}e^2 - \tfrac{5}{2}e'^2 - \tfrac{29}{228}\gamma^2 + \tfrac{3}{76}\tfrac{\gamma^4}{e^2}\} \cos(nt - 2n't + 2\omega) \\
&\mp \tfrac{51}{8}e'^2\,\{1 - 6e^2 - \tfrac{115}{51}e'^2 - \tfrac{1}{4}\gamma^2\} \cos(3nt - 4n't + 2\omega') \\
&+ \tfrac{153}{8}e'^2\,\{1 - \tfrac{4}{3}e^2 - \tfrac{115}{51}e'^2 - \tfrac{119}{204}\gamma^2\} \cos(nt - 4n't + 2\omega')
\end{aligned}
\right\} \quad . \quad (271)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\pm \tfrac{3}{16}\gamma^2 \{1 - \tfrac{43}{16}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2\} \cos (5nt - 2n't - 2\Omega)\\
&+ \tfrac{3}{32}\gamma^2 \{\tfrac{21}{8}e^2 - \gamma^2\} \cos (3nt - 2n't - 2\Omega)\\
&\pm \tfrac{3}{16}\gamma^2 \{1 - \tfrac{31}{16}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos (nt + 2n't - 2\Omega)\\
&\mp \tfrac{9}{64}\gamma^2 \{\tfrac{131}{8}e^2 + \gamma^2\} \cos (nt - 2n't + 2\Omega)\\
&\mp \tfrac{9}{32}\gamma^2 \{1 - \tfrac{245}{12}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos (3nt - 2n't + 2\omega - 2\Omega)\\
&+ \tfrac{9}{32}\gamma^2 \{1 - \tfrac{73}{3}e^2 - \tfrac{5}{2}e'^2 - \tfrac{13}{12}\gamma^2\} \cos (nt - 2n't + 2\omega - 2\Omega)\\
&\pm \tfrac{9}{32}\gamma^2 \{1 - \tfrac{401}{36}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos (3nt - 2n't - 2\omega + 2\Omega)\\
&- \tfrac{9}{32}\gamma^2 \{1 - \tfrac{833}{72}e^2 - \tfrac{5}{2}e'^2 - \tfrac{47}{24}\gamma^2\} \cos (nt - 2n't - 2\omega + 2\Omega)\\
&\pm \tfrac{3}{4}ee' \{1 - \tfrac{175}{32}e^2 - \tfrac{1}{8}e'^2 - \tfrac{23}{16}\gamma^2\} \cos (4nt - n't - \omega - \omega')\\
&- \tfrac{3}{4}ee' \{1 - \tfrac{11}{8}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{16}\gamma^2\} \cos (2nt - n't - \omega - \omega')\\
&\pm \tfrac{63}{8}ee' \{1 - \tfrac{328}{63}e^2 - \tfrac{123}{56}e'^2 - \tfrac{85}{168}\gamma^2\} \cos (2nt - 3n't + \omega + \omega')\\
&\mp \tfrac{105}{8}ee' \{1 - \tfrac{41}{84}e^2 - \tfrac{123}{56}e'^2 - \tfrac{31}{40}\gamma^2\} \cos (3n't - \omega - \omega')\\
&\mp \tfrac{21}{4}ee' \{1 - \tfrac{19}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{16}\gamma^2\} \cos (4nt - 3n't - \omega + \omega')\\
&+ \tfrac{21}{4}ee' \{1 - \tfrac{7}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{7}{80}\gamma^2\} \cos (2nt - 3n't - \omega + \omega')\\
&\mp \tfrac{9}{8}ee' \{1 - \tfrac{11}{6}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{24}\gamma^2\} \cos (2nt - n't + \omega - \omega')\\
&\pm \tfrac{15}{8}ee' \{1 - \tfrac{17}{80}e^2 - \tfrac{1}{8}e'^2 - \tfrac{19}{40}\gamma^2\} \cos (n't - \omega + \omega')\\
&\mp \tfrac{27}{8}e^3 \cos (6nt - 2n't - 3\omega) + 2e^3 \cos (4nt - 2n't - 3\omega)\\
&\pm \tfrac{5}{32}e^3 \cos (2nt + 2n't - 3\omega) - \tfrac{1}{32}e^3 \cos (2n't - 3\omega)\\
&\mp \tfrac{1}{64}e'^3 \cos (3nt + n't - 3\omega') + \tfrac{3}{64}e'^3 \cos (nt + n't - 3\omega')\\
&\mp \tfrac{845}{64}e'^3 \cos (3nt - 5n't + 3\omega') + \tfrac{2535}{64}e'^3 \cos (nt - 5n't + 3\omega')\\
&\mp \tfrac{99}{64}e\gamma^2 \cos (4nt - 2n't + \omega - 2\Omega) + \tfrac{21}{64}e\gamma^2 \cos (2nt - 2n't + \omega - 2\Omega)\\
&\mp \tfrac{33}{64}e\gamma^2 \cos (2nt - 2n't - \omega + 2\Omega) \pm \tfrac{9}{64}e\gamma^2 \cos (2n't + \omega - 2\Omega)\\
&\pm \tfrac{9}{16}e\gamma^2 \cos (4nt - 2n't - 3\omega + 2\Omega) - \tfrac{3}{16}e\gamma^2 \cos (2nt - 2n't - 3\omega + 2\Omega)\\
&\pm \tfrac{27}{32}e\gamma^2 \cos (2nt - 2n't + 3\omega - 2\Omega) \mp \tfrac{15}{32}e\gamma^2 \cos (2n't - 3\omega + 2\Omega)\\
&\pm \tfrac{9}{64}e\gamma^2 \cos (2nt + 2n't - \omega - 2\Omega) - \tfrac{3}{128}e\gamma^2 \cos (n't - \omega - 2\Omega)\\
&- \tfrac{9}{64}e\gamma^2 \cos (4nt - 2n't - \omega - 2\Omega) \pm \tfrac{51}{64}e\gamma^2 \cos (6nt - 2n't - \omega - 2\Omega)\\
&\pm \tfrac{21}{32}e'\gamma^2 \cos (nt + 3n't - \omega' - 2\Omega) \mp \tfrac{3}{32}e'\gamma^2 \cos (nt + n't + \omega' - 2\Omega)\\
&\mp \tfrac{9}{64}e'\gamma^2 \cos (3nt - n't - 2\omega - \omega' + 2\Omega)\\
&+ \tfrac{9}{64}e'\gamma^2 \cos (nt - n't - 2\omega - \omega' + 2\Omega)\\
&\mp \tfrac{63}{64}e'\gamma^2 \cos (3nt - 3n't + 2\omega + \omega' - 2\Omega)\\
&+ \tfrac{63}{64}e'\gamma^2 \cos (nt - 3n't + 2\omega + \omega' - 2\Omega)\\
&\pm \tfrac{9}{64}e'\gamma^2 \cos (3nt - n't + 2\omega - \omega' - 2\Omega)\\
&- \tfrac{9}{64}e'\gamma^2 \cos (nt - n't + 2\omega - \omega' - 2\Omega)\\
&\pm \tfrac{63}{64}e'\gamma^2 \cos (3nt - 3n't - 2\omega + \omega' + 2\Omega)\\
&- \tfrac{63}{64}e'\gamma^2 \cos (nt - 3n't - 2\omega + \omega' + 2\Omega)\\
&\mp \tfrac{3}{8}e'\gamma^2 \cos (5nt - n't - \omega' - 2\Omega) \pm \tfrac{21}{32}e'\gamma^2 \cos (5nt - 2n't + \omega' - 2\Omega)
\end{aligned}\right\} . \quad (271)
$$

(*Continued on the next page.*)

$$\left.\begin{aligned}
&\pm \tfrac{153}{8}ee'^2 \cos(2nt - 4n't + \omega + 2\omega') \mp \tfrac{255}{8}ee'^2 \cos(4n't - \omega - 2\omega') \\
&\mp \tfrac{51}{4}ee'^2 \cos(4nt - 4n't - \omega + 2\omega') + \tfrac{51}{4}ee'^2 \cos(2nt - 4n't - \omega + 2\omega') \\
&\pm \tfrac{75}{64}e^2e' \cos(5nt - n't - 2\omega - \omega') - \tfrac{39}{64}e^2e' \cos(3nt - n't - 2\omega - \omega') \\
&\pm \tfrac{147}{64}e^2e' \cos(nt + 3n't - 2\omega - \omega') \mp \tfrac{399}{64}e^2e' \cos(nt - 3n't + 2\omega + \omega') \\
&\mp \tfrac{525}{64}e^2e' \cos(5nt - 3n't - 2\omega + \omega') \\
&+ \tfrac{273}{64}e^2e' \cos(3nt - 3n't - 2\omega + \omega') \\
&\mp \tfrac{21}{64}e^2e' \cos(nt + n't - 2\omega + \omega') \pm \tfrac{57}{64}e^2e' \cos(nt - n't + 2\omega - \omega') \\
&\mp \tfrac{2401}{512}e^4 \cos(7nt - 2n't - 4\omega) + \tfrac{575}{512}e^4 \cos(5nt - 2n't - 4\omega) \\
&\pm \tfrac{35}{512}e^4 \cos(3nt + 2n't - 4\omega) + \tfrac{91}{512}e^4 \cos(nt + 2n't - 4\omega) \\
&\mp \tfrac{1}{32}e'^4 \cos(3nt + 2n't - 4\omega') + \tfrac{3}{32}e'^4 \cos(nt + 2n't - 4\omega') \\
&\mp \tfrac{1599}{64}e'^4 \cos(3nt - 6n't + 4\omega') + \tfrac{4797}{64}e'^4 \cos(nt - 6n't + 4\omega') \\
&\mp \tfrac{45}{512}\gamma^4 \cos(3nt - 2n't + 4\omega - 4\Omega) + \tfrac{39}{512}\gamma^4 \cos(nt - 2n't + 4\omega - 4\Omega) \\
&\mp \tfrac{15}{256}\gamma^4 \cos(3nt + 2n't - 4\Omega) + \tfrac{27}{256}\gamma^4 \cos(nt + 2n't - 4\Omega) \\
&\pm \tfrac{3}{128}\gamma^4 \cos(nt + 2n't + 2\omega - 4\Omega) - \tfrac{9}{512}\gamma^4 \cos(nt - 2n't - 4\omega + 4\Omega) \\
&\mp \tfrac{3}{128}\gamma^4 \cos(3nt - 2n't - 4\omega + 4\Omega) \mp \tfrac{21}{256}\gamma^4 \cos(7nt - 2n't - 4\Omega) \\
&+ \tfrac{9}{256}\gamma^4 \cos(5nt - 2n't - 4\Omega) \pm \tfrac{15}{128}\gamma^4 \cos(5n't - 2n't + 2\omega - 4\Omega) \\
&\pm \tfrac{63}{64}e^2\gamma^2 \cos(5nt - 2n't - 4\omega + 2\Omega) \\
&- \tfrac{39}{256}e^2\gamma^2 \cos(3nt - 2n't - 4\omega + 2\Omega) \\
&\pm \tfrac{3}{32}e^2\gamma^2 \cos(nt + 2n't - 4\omega + 2\Omega) \mp \tfrac{21}{32}e^2\gamma^2 \cos(nt - 2n't + 4\omega - 2\Omega) \\
&\pm \tfrac{15}{128}e^2\gamma^2 \cos(3nt + 2n't - 2\omega - 2\Omega) \\
&+ \tfrac{15}{64}e^2\gamma^2 \cos(nt + 2n't - 2\omega - 2\Omega) - \tfrac{27}{64}e^2\gamma^2 \cos(5nt - 2n't - 2\omega - 2\Omega) \\
&\pm \tfrac{279}{128}e^2\gamma^2 \cos(7nt - 2n't - 2\omega - 2\Omega) \mp \tfrac{1}{32}ee'^3 \cos(4nt + n't - \omega - 3\omega') \\
&+ \tfrac{1}{32}ee'^3 \cos(2nt + n't - \omega - 3\omega') \pm \tfrac{3}{64}ee'^3 \cos(2nt + n't + \omega - 3\omega') \\
&- \tfrac{5}{64}ee'^3 \cos(n't + \omega - 3\omega') \pm \tfrac{2535}{64}ee'^3 \cos(2nt - 5n't + \omega + 3\omega') \\
&\mp \tfrac{4225}{64}ee'^3 \cos(5n't - \omega - 3\omega') \mp \tfrac{1317}{32}ee'^3 \cos(4nt - 5n't - \omega + 3\omega') \\
&+ \tfrac{373}{32}ee'^3 \cos(2nt - 5n't - \omega + 3\omega') \\
&\pm \tfrac{21}{32}e^3e' \cos(2nt + 3n't - 3\omega - \omega') \mp \tfrac{3}{32}e^3e' \cos(2nt + n't - 3\omega + \omega') \\
&\pm \tfrac{27}{16}e^3e' \cos(6nt - n't - 3\omega - \omega') - \tfrac{9}{16}e^3e' \cos(4nt - n't - 3\omega - \omega') \\
&\pm \tfrac{37}{32}e^3e' \cos(6nt - 3n't - 3\omega + \omega') \\
&+ \tfrac{541}{32}e^3e' \cos(4nt - 3n't - 3\omega + \omega') \\
&\mp \tfrac{1275}{64}e^2e'^2 \cos(5nt - 4n't - 2\omega + 2\omega') \\
&+ \tfrac{653}{64}e^2e'^2 \cos(3nt - 4n't - 2\omega + 2\omega') \\
&\pm \tfrac{357}{64}e^2e'^2 \cos(nt + 4n't - 2\omega - 2\omega') \\
&\quad \tfrac{959}{64}e^2e'^2 \cos(nt - 4n't + 2\omega + 2\omega') \\
&\mp \tfrac{153}{64}e'^2\gamma^2 \cos(3nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&+ \tfrac{153}{64}e'^2\gamma^2 \cos(nt - 4n't + 2\omega + 2\omega' - 2\Omega)
\end{aligned}\right\} \quad . \quad (271)$$

(Continued on the next page)

$$
\left.\begin{aligned}
&\pm \tfrac{51}{32} e'^2\gamma^2 \cos(nt + 4n't - 2\omega' - 2\Omega) \\
&\pm \tfrac{153}{64} e'^2\gamma^2 \cos(3nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- \tfrac{153}{64} e'^2\gamma^2 \cos(nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&\pm \tfrac{51}{32} e'^2\gamma^2 \cos(5nt - 4n't + 2\omega' - 2\Omega) \\
&\mp \tfrac{693}{128} ee'\gamma^2 \cos(4nt - 3n't + \omega + \omega' - 2\Omega) \\
&+ \tfrac{147}{128} ee'\gamma^2 \cos(2nt - 3n't + \omega + \omega' - 2\Omega) \\
&\pm \tfrac{39}{128} ee'\gamma^2 \cos(2nt - n't - \omega - \omega' + 2\Omega) \\
&\mp \tfrac{9}{128} ee'\gamma^2 \cos(n't + \omega + \omega' - 2\Omega) \\
&\mp \tfrac{243}{128} ee'\gamma^2 \cos(6nt - n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{585}{128} ee'\gamma^2 \cos(4nt - n't - \omega - \omega' - 2\Omega) \\
&\mp \tfrac{309}{128} ee'\gamma^2 \cos(2nt - 3n't - \omega + \omega' + 2\Omega) \\
&\pm \tfrac{99}{128} ee'\gamma^2 \cos(3n't + \omega - \omega' - 2\Omega) \\
&\pm \tfrac{63}{32} ee'\gamma^2 \cos(4nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- \tfrac{21}{32} ee'\gamma^2 \cos(2nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{27}{64} ee'\gamma^2 \cos(2nt - n't + 3\omega - \omega' - 2\Omega) \\
&\pm \tfrac{15}{64} ee'\gamma^2 \cos(n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{9}{32} ee'\gamma^2 \cos(4nt - n't - 3\omega - \omega' + 2\Omega) \\
&+ \tfrac{3}{32} ee'\gamma^2 \cos(2nt - n't - 3\omega - \omega' + 2\Omega) \\
&\pm \tfrac{99}{128} ee'\gamma^2 \cos(4nt - n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{21}{128} ee'\gamma^2 \cos(2nt - n't + \omega - \omega' - 2\Omega) \\
&\pm \tfrac{207}{64} ee'\gamma^2 \cos(2nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&\mp \tfrac{123}{64} ee'\gamma^2 \cos(3n't - 3\omega - \omega' + 2\Omega) \\
&\mp \tfrac{9}{128} ee'\gamma^2 \cos(2nt + n't - \omega + \omega' - 2\Omega) \\
&+ \tfrac{15}{128} ee'\gamma^2 \cos(n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{513}{128} ee'\gamma^2 \cos(2nt + 3n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{87}{128} ee'\gamma^2 \cos(3n't - \omega - \omega' - 2\Omega) \\
&- \tfrac{63}{128} ee'\gamma^2 \cos(4nt - 3n't - \omega + \omega' - 2\Omega) \\
&\pm \tfrac{357}{128} ee'\gamma^2 \cos(6nt - 3n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{3}{4} e \{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\} \cos\omega \mp \tfrac{3}{32} e\gamma^2 \cos(3\omega - 2\Omega) \\
&- \tfrac{39}{32} e\gamma^2 \cos(\omega - 2\Omega) \mp \tfrac{1}{2}(e^2 - e_0^2)\cos nt \mp \tfrac{7}{8}(\gamma^2 - \gamma_0^2)\cos nt \\
&\pm \tfrac{1}{2} h' \{\cos(nt + \alpha't - \beta') + \cos(nt - \alpha't + \beta')\} \Big\} \\
&+ \overline{m}^2 \frac{a}{a'}\, ndt \Big\{ \pm \tfrac{3}{16}\{1 - 5e^2 + 2e'^2 + 18\gamma^2\} \cos(2nt - n't) \\
&\pm \tfrac{15}{16}\{1 + e^2 + 2e'^2 - \tfrac{9}{2}\gamma^2\} \cos n't \pm \tfrac{3}{16} e' \cos(2nt - \omega') \\
&\pm \tfrac{15}{16} e' \cos\omega' \pm \tfrac{9}{16} e' \cos(2nt - 2n't + \omega') \pm \tfrac{45}{16} e' \cos(2n't - \omega')
\end{aligned}\right\} \quad (271)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp \tfrac{33}{32}e \cos(nt + n't - \omega) \mp \tfrac{21}{32}e \cos(nt - n't + \omega) \\
&\pm \tfrac{9}{32}e \cos(3nt - n't - \omega) - \tfrac{27}{32}e \cos(nt - n't - \omega) \\
&\pm \tfrac{3}{4}e^2 \cos(4nt - n't - 2\omega) - \tfrac{45}{64}e^2 \cos(2nt - n't - 2\omega) \\
&\mp \tfrac{3}{16}e^2 \cos(2nt + n't - 2\omega) + \tfrac{45}{64}e^2 \cos(n't - 2\omega) \\
&\pm \tfrac{33}{128}e'^2 \cos(2nt + n't - 2\omega') + \tfrac{165}{128}e'^2 \cos(n't - 2\omega') \\
&\pm \tfrac{159}{128}e^2 \cos(2nt - 3n't + 2\omega') \pm \tfrac{795}{128}e'^2 \cos(3n't - 2\omega') \\
&\mp \tfrac{15}{32}ee' \cos(nt + \omega - \omega') \pm \tfrac{33}{32}ee' \cos(nt - \omega + \omega') \\
&\pm \tfrac{9}{32}ee' \cos(3nt - \omega - \omega') - \tfrac{27}{32}ee' \cos(nt - \omega - \omega') \\
&\mp \tfrac{99}{32}ee' \cos(nt + 2n't - \omega - \omega') \mp \tfrac{63}{32}ee' \cos(nt - 2n't + \omega + \omega') \\
&\pm \tfrac{27}{32}ee' \cos(3nt - 2n't - \omega + \omega') - \tfrac{81}{32}ee' \cos(nt - 2n't - \omega + \omega') \\
&\mp \tfrac{9}{64}\gamma^2 \cos(2nt - n't - 2\omega + 2\Omega) \pm \tfrac{3}{32}\gamma^2 \cos(n't + 2\omega - 2\Omega) \\
&\pm \tfrac{3}{64}\gamma^2 \cos(2nt - n't + 2\omega - 2\Omega) \mp \tfrac{45}{32}\gamma^2 \cos(4nt - n't - 2\Omega) \\
&+ \tfrac{177}{64}\gamma^2 \cos(2nt - n't - 2\Omega) \pm \tfrac{63}{64}\gamma^2 \cos(2nt + n't - 2\Omega) \\
&+ \tfrac{15}{32}\gamma^2 \cos(n't - 2\Omega) \mp \tfrac{3}{16}h'' \cos(nt + \varkappa''t - \beta'') \mp \tfrac{33}{16}h'' \cos(nt - \varkappa''t + \beta'') \\
&\mp \tfrac{15}{16}\{1 - 11e^2 - 6e'^2 - \gamma^2\} \cos(4nt - 3n't) \\
&+ \tfrac{45}{16}\{1 - \tfrac{11}{8}e^2 - 6e'^2 - \gamma^2\} \cos(2nt - 3n't) \\
&\mp \tfrac{75}{32}e \cos(5nt - 3n't - \omega) + \tfrac{105}{32}e \cos(3nt - 3n't - \omega) \\
&\pm \tfrac{135}{32}e \cos(3nt - 3n't + \omega) - \tfrac{285}{32}e \cos(nt - 3n't + \omega) \\
&\mp \tfrac{75}{16}e' \cos(4nt - 4n't + \omega') + \tfrac{225}{16}e' \cos(2nt - 4n't + \omega') \\
&\pm \tfrac{15}{16}e' \cos(4nt - 2n't - \omega') - \tfrac{45}{16}e' \cos(2nt - 2n't - \omega') \\
&\mp \tfrac{135}{32}e^2 \cos(6nt - 3n't - 2\omega) + \tfrac{225}{64}e^2 \cos(4nt - 3n't - 2\omega) \\
&\mp \tfrac{105}{16}e^2 \cos(2nt - 3n't + 2\omega) \pm \tfrac{525}{64}e^2 \cos(3n't - 2\omega) \\
&\mp \tfrac{15}{128}e'^2 \cos(4nt - n't - 2\omega') + \tfrac{45}{128}e'^2 \cos(2nt - n't - 2\omega') \\
&\mp \tfrac{1905}{128}e'^2 \cos(4nt - 5n't + 2\omega') + \tfrac{5715}{128}e'^2 \cos(2nt - 5n't + 2\omega') \\
&\mp \tfrac{375}{32}ee' \cos(5nt - 4n't - \omega + \omega') + \tfrac{525}{32}ee' \cos(3nt - 4n't - \omega + \omega') \\
&\mp \tfrac{135}{32}ee' \cos(3nt - 2n't + \omega - \omega') + \tfrac{285}{32}ee' \cos(nt - 2n't + \omega - \omega') \\
&\pm \tfrac{75}{32}ee' \cos(5nt - 2n't - \omega - \omega') - \tfrac{105}{32}ee' \cos(3nt - 2n't - \omega - \omega') \\
&\pm \tfrac{675}{32}ee' \cos(3nt - 4n't + \omega + \omega') - \tfrac{1425}{32}ee' \cos(nt - 4n't + \omega + \omega') \\
&\mp \tfrac{15}{32}\gamma^2 \cos(2nt - 3n't + 2\Omega) \pm \tfrac{75}{64}\gamma^2 \cos(3n't - 2\Omega) \\
&\mp \tfrac{15}{32}\gamma^2 \cos(4nt - 3n't + 2\omega - 2\Omega) + \tfrac{45}{64}\gamma^2 \cos(2nt - 3n't + 2\omega - 2\Omega) \\
&\pm \tfrac{15}{32}\gamma^2 \cos(4nt - 3n't - 2\omega + 2\Omega) - \tfrac{45}{64}\gamma^2 \cos(2nt - 3n't - 2\omega + 2\Omega) \\
&+ \tfrac{15}{64}\gamma^2 \cos(4nt - 3n't - 2\Omega) \Big\}
\end{aligned}
\right\} \quad . \quad (271)
$$

Equation (271) will give the value of $c_2 \sin\beta\left(\frac{dR}{dr}\right) + c_3 \cos\beta\left(\frac{dR}{dv}\right) + c_4 \cos\beta\left(\frac{dR}{d\theta}\right)$ by using the lower signs and changing *cos* to *sin* in the second member.

$$
\left.\begin{aligned}
&\pm \tfrac{3}{16}\gamma^2 \{1 - \tfrac{43}{16}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2\} \cos(5nt - 2n't - 2\Omega) \\
&+ \tfrac{3}{32}\gamma^2 \{\tfrac{21}{8}e^2 - \gamma^2\} \cos(3nt - 2n't - 2\Omega) \\
&\pm \tfrac{3}{16}\gamma^2 \{1 - \tfrac{31}{16}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\} \cos(nt + 2n't - 2\Omega) \\
&\mp \tfrac{9}{64}\gamma^2 \{\tfrac{131}{8}e^2 + \gamma^2\} \cos(nt - 2n't + 2\Omega) \\
&\mp \tfrac{9}{32}\gamma^2 \{1 - \tfrac{245}{12}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 2n't + 2\omega - 2\Omega) \\
&+ \tfrac{9}{32}\gamma^2 \{1 - \tfrac{73}{3}e^2 - \tfrac{5}{2}e'^2 - \tfrac{13}{12}\gamma^2\} \cos(nt - 2n't + 2\omega - 2\Omega) \\
&\pm \tfrac{9}{32}\gamma^2 \{1 - \tfrac{401}{36}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - 2n't - 2\omega + 2\Omega) \\
&- \tfrac{9}{32}\gamma^2 \{1 - \tfrac{833}{72}e^2 - \tfrac{5}{2}e'^2 - \tfrac{47}{24}\gamma^2\} \cos(nt - 2n't - 2\omega + 2\Omega) \\
&\pm \tfrac{3}{4}ee' \{1 - \tfrac{175}{32}e^2 - \tfrac{1}{8}e'^2 - \tfrac{23}{16}\gamma^2\} \cos(4nt - n't - \omega - \omega') \\
&- \tfrac{3}{4}ee' \{1 - \tfrac{11}{8}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{16}\gamma^2\} \cos(2nt - n't - \omega - \omega') \\
&\pm \tfrac{63}{8}ee' \{1 - \tfrac{328}{63}e^2 - \tfrac{123}{56}e'^2 - \tfrac{85}{168}\gamma^2\} \cos(2nt - 3n't + \omega + \omega') \\
&\mp \tfrac{105}{8}ee' \{1 - \tfrac{41}{84}e^2 - \tfrac{123}{56}e'^2 - \tfrac{31}{40}\gamma^2\} \cos(3n't - \omega - \omega') \\
&\mp \tfrac{21}{4}ee' \{1 - \tfrac{19}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{16}\gamma^2\} \cos(4nt - 3n't - \omega + \omega') \\
&+ \tfrac{21}{4}ee' \{1 - \tfrac{7}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{7}{80}\gamma^2\} \cos(2nt - 3n't - \omega + \omega') \\
&\mp \tfrac{9}{8}ee' \{1 - \tfrac{11}{6}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{24}\gamma^2\} \cos(2nt - n't + \omega - \omega') \\
&\pm \tfrac{15}{8}ee' \{1 - \tfrac{17}{80}e^2 - \tfrac{1}{8}e'^2 - \tfrac{19}{40}\gamma^2\} \cos(n't - \omega + \omega') \\
&\mp \tfrac{27}{8}e^3 \cos(6nt - 2n't - 3\omega) + 2e^3 \cos(4nt - 2n't - 3\omega) \\
&\pm \tfrac{5}{32}e^3 \cos(2nt + 2n't - 3\omega) - \tfrac{1}{32}e^3 \cos(2n't - 3\omega) \\
&\mp \tfrac{1}{64}e'^3 \cos(3nt + n't - 3\omega') + \tfrac{3}{64}e'^3 \cos(nt + n't - 3\omega') \\
&\mp \tfrac{845}{64}e'^3 \cos(3nt - 5n't + 3\omega') + \tfrac{2535}{64}e'^3 \cos(nt - 5n't + 3\omega') \\
&\mp \tfrac{99}{64}e\gamma^2 \cos(4nt - 2n't + \omega - 2\Omega) + \tfrac{21}{64}e\gamma^2 \cos(2nt - 2n't + \omega - 2\Omega) \\
&\mp \tfrac{33}{64}e\gamma^2 \cos(2nt - 2n't - \omega + 2\Omega) \pm \tfrac{9}{64}e\gamma^2 \cos(2n't + \omega - 2\Omega) \\
&\pm \tfrac{9}{16}e\gamma^2 \cos(4nt - 2n't - 3\omega + 2\Omega) - \tfrac{3}{16}e\gamma^2 \cos(2nt - 2n't - 3\omega + 2\Omega) \\
&\pm \tfrac{27}{32}e\gamma^2 \cos(2nt - 2n't + 3\omega - 2\Omega) \mp \tfrac{15}{32}e\gamma^2 \cos(2n't - 3\omega + 2\Omega) \\
&\pm \tfrac{9}{64}e\gamma^2 \cos(2nt + 2n't - \omega - 2\Omega) - \tfrac{3}{128}e\gamma^2 \cos(n't - \omega - 2\Omega) \\
&- \tfrac{9}{64}e\gamma^2 \cos(4nt - 2n't - \omega - 2\Omega) \pm \tfrac{51}{64}e\gamma^2 \cos(6nt - 2n't - \omega - 2\Omega) \\
&\pm \tfrac{21}{32}e'\gamma^2 \cos(nt + 3n't - \omega' - 2\Omega) \mp \tfrac{3}{32}e'\gamma^2 \cos(nt + n't + \omega' - 2\Omega) \\
&\mp \tfrac{9}{64}e'\gamma^2 \cos(3nt - n't - 2\omega - \omega' + 2\Omega) \\
&+ \tfrac{9}{64}e'\gamma^2 \cos(nt - n't - 2\omega - \omega' + 2\Omega) \\
&\mp \tfrac{63}{64}e'\gamma^2 \cos(3nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&+ \tfrac{63}{64}e'\gamma^2 \cos(nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&\pm \tfrac{9}{64}e'\gamma^2 \cos(3nt - n't + 2\omega - \omega' - 2\Omega) \\
&- \tfrac{9}{64}e'\gamma^2 \cos(nt - n't + 2\omega - \omega' - 2\Omega) \\
&\pm \tfrac{63}{64}e'\gamma^2 \cos(3nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&- \tfrac{63}{64}e'\gamma^2 \cos(nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&\mp \tfrac{3}{8}e'\gamma^2 \cos(5nt - n't - \omega' - 2\Omega) \pm \tfrac{21}{32}e'\gamma^2 \cos(5nt - 2n't + \omega' - 2\Omega)
\end{aligned}\right\} \quad . \quad (271)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&\pm \tfrac{153}{8}ee'^2 \cos(2nt - 4n't + \omega + 2\omega') \mp \tfrac{255}{8}ee'^2 \cos(4n't - \omega - 2\omega') \\
&\mp \tfrac{51}{4}ee'^2 \cos(4nt - 4n't - \omega + 2\omega') + \tfrac{51}{4}ee'^2 \cos(2nt - 4n't - \omega + 2\omega') \\
&\pm \tfrac{75}{64}e^2e' \cos(5nt - n't - 2\omega - \omega') - \tfrac{39}{64}e^2e' \cos(3nt - n't - 2\omega - \omega') \\
&\pm \tfrac{147}{64}e^2e' \cos(nt + 3n't - 2\omega - \omega') \mp \tfrac{399}{64}e^2e' \cos(nt - 3n't + 2\omega + \omega') \\
&\mp \tfrac{525}{64}e^2e' \cos(5nt - 3n't - 2\omega + \omega') \\
&+ \tfrac{273}{64}e^2e' \cos(3nt - 3n't - 2\omega + \omega') \\
&\mp \tfrac{21}{64}e^2e' \cos(nt + n't - 2\omega + \omega') \pm \tfrac{57}{64}e^2e' \cos(nt - n't + 2\omega - \omega') \\
&\mp \tfrac{2401}{512}e^4 \cos(7nt - 2n't - 4\omega) + \tfrac{575}{512}e^4 \cos(5nt - 2n't - 4\omega) \\
&\pm \tfrac{35}{512}e^4 \cos(3nt + 2n't - 4\omega) + \tfrac{91}{512}e^4 \cos(nt + 2n't - 4\omega) \\
&\mp \tfrac{1}{32}e'^4 \cos(3nt + 2n't - 4\omega') + \tfrac{3}{32}e'^4 \cos(nt + 2n't - 4\omega') \\
&\mp \tfrac{1599}{64}e'^4 \cos(3nt - 6n't + 4\omega') + \tfrac{4797}{64}e'^4 \cos(nt - 6n't + 4\omega') \\
&\mp \tfrac{45}{512}\gamma^4 \cos(3nt - 2n't + 4\omega - 4\Omega) + \tfrac{39}{512}\gamma^4 \cos(nt - 2n't + 4\omega - 4\Omega) \\
&\mp \tfrac{15}{256}\gamma^4 \cos(3nt + 2n't - 4\Omega) + \tfrac{27}{256}\gamma^4 \cos(nt + 2n't - 4\Omega) \\
&\pm \tfrac{3}{128}\gamma^4 \cos(nt + 2n't + 2\omega - 4\Omega) - \tfrac{9}{512}\gamma^4 \cos(nt - 2n't - 4\omega + 4\Omega) \\
&\mp \tfrac{3}{128}\gamma^4 \cos(3nt - 2n't - 4\omega + 4\Omega) \mp \tfrac{21}{256}\gamma^4 \cos(7nt - 2n't - 4\Omega) \\
&+ \tfrac{9}{256}\gamma^4 \cos(5nt - 2n't - 4\Omega) \pm \tfrac{15}{128}\gamma^4 \cos(5n't - 2n't + 2\omega - 4\Omega) \\
&\pm \tfrac{63}{64}e^2\gamma^2 \cos(5nt - 2n't - 4\omega + 2\Omega) \\
&- \tfrac{39}{256}e^2\gamma^2 \cos(3nt - 2n't - 4\omega + 2\Omega) \\
&\pm \tfrac{3}{32}e^2\gamma^3 \cos(nt + 2n't - 4\omega + 2\Omega) \mp \tfrac{21}{32}e^2\gamma^2 \cos(nt - 2n't + 4\omega - 2\Omega) \\
&\pm \tfrac{15}{128}e^2\gamma^2 \cos(3nt + 2n't - 2\omega - 2\Omega) \\
&+ \tfrac{15}{64}e^2\gamma^2 \cos(nt + 2n't - 2\omega - 2\Omega) - \tfrac{27}{64}e^2\gamma^2 \cos(5nt - 2n't - 2\omega - 2\Omega) \\
&\pm \tfrac{279}{128}e^2\gamma^2 \cos(7nt - 2n't - 2\omega - 2\Omega) \mp \tfrac{1}{32}ee'^3 \cos(4nt + n't - \omega - 3\omega') \\
&+ \tfrac{1}{32}ee'^3 \cos(2nt + n't - \omega - 3\omega') \pm \tfrac{3}{64}ee'^3 \cos(2nt + n't + \omega - 3\omega') \\
&- \tfrac{5}{64}ee'^3 \cos(n't + \omega - 3\omega') \pm \tfrac{2535}{64}ee'^3 \cos(2nt - 5n't + \omega + 3\omega') \\
&\mp \tfrac{4225}{64}ee'^3 \cos(5n't - \omega - 3\omega') \mp \tfrac{1317}{32}ee'^3 \cos(4nt - 5n't - \omega + 3\omega') \\
&+ \tfrac{373}{32}ee'^3 \cos(2nt - 5n't - \omega + 3\omega') \\
&\pm \tfrac{21}{32}e^3e' \cos(2nt + 3n't - 3\omega - \omega') \mp \tfrac{3}{32}e^3e' \cos(2nt + n't - 3\omega + \omega') \\
&\pm \tfrac{27}{16}e^3e' \cos(6nt - n't - 3\omega - \omega') - \tfrac{9}{16}e^3e' \cos(4nt - n't - 3\omega - \omega') \\
&\pm \tfrac{37}{32}e^3e' \cos(6nt - 3n't - 3\omega + \omega') \\
&+ \tfrac{541}{32}e^3e' \cos(4nt - 3n't - 3\omega + \omega') \\
&\mp \tfrac{1575}{64}e^2e'^2 \cos(5nt - 4n't - 2\omega + 2\omega') \\
&+ \tfrac{663}{64}e^2e'^2 \cos(3nt - 4n't - 2\omega + 2\omega') \\
&\pm \tfrac{357}{64}e^2e'^2 \cos(nt + 4n't - 2\omega - 2\omega') \\
&\mp \tfrac{969}{64}e^2e'^2 \cos(nt - 4n't + 2\omega + 2\omega') \\
&\mp \tfrac{153}{64}e'^2\gamma^2 \cos(3nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&+ \tfrac{153}{64}e'^2\gamma^2 \cos(nt - 4n't + 2\omega + 2\omega' - 2\Omega)
\end{aligned}\right\} \quad (271)$$

(*Continued on the next page*)

$$
\begin{aligned}
&\pm \tfrac{51}{32}e'^2\gamma^2 \cos(nt + 4n't - 2\omega' - 2\Omega) \\
&\pm \tfrac{153}{64}e'^2\gamma^2 \cos(3nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- \tfrac{153}{64}e'^2\gamma^2 \cos(nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&\pm \tfrac{51}{32}e'^2\gamma^2 \cos(5nt - 4n't + 2\omega' - 2\Omega) \\
&\mp \tfrac{693}{128}ee'\gamma^2 \cos(4nt - 3n't + \omega + \omega' - 2\Omega) \\
&+ \tfrac{147}{128}ee'\gamma^2 \cos(2nt - 3n't + \omega + \omega' - 2\Omega) \\
&\pm \tfrac{39}{128}ee'\gamma^2 \cos(2nt - n't - \omega - \omega' + 2\Omega) \\
&\mp \tfrac{9}{128}ee'\gamma^2 \cos(n't + \omega + \omega' - 2\Omega) \\
&\mp \tfrac{243}{128}ee'\gamma^2 \cos(6nt - n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{585}{128}ee'\gamma^2 \cos(4nt - n't - \omega - \omega' - 2\Omega) \\
&\mp \tfrac{309}{128}ee'\gamma^2 \cos(2nt - 3n't - \omega + \omega' + 2\Omega) \\
&\pm \tfrac{99}{128}ee'\gamma^2 \cos(3n't + \omega - \omega' - 2\Omega) \\
&\pm \tfrac{63}{32}ee'\gamma^2 \cos(4nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- \tfrac{21}{32}ee'\gamma^2 \cos(2nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{27}{64}ee'\gamma^2 \cos(2nt - n't + 3\omega - \omega' - 2\Omega) \\
&\pm \tfrac{15}{64}ee'\gamma^2 \cos(n't - 3\omega + \omega' + 2\Omega) \\
&\mp \tfrac{9}{32}ee'\gamma^2 \cos(4nt - n't - 3\omega - \omega' + 2\Omega) \\
&+ \tfrac{3}{32}ee'\gamma^2 \cos(2nt - n't - 3\omega - \omega' + 2\Omega) \\
&\pm \tfrac{99}{128}ee'\gamma^2 \cos(4nt - n't + \omega - \omega' - 2\Omega) \\
&- \tfrac{21}{128}ee'\gamma^2 \cos(2nt - n't + \omega - \omega' - 2\Omega) \\
&\pm \tfrac{207}{64}ee'\gamma^2 \cos(2nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&\mp \tfrac{123}{64}ee'\gamma^2 \cos(3n't - 3\omega - \omega' + 2\Omega) \\
&\mp \tfrac{9}{128}ee'\gamma^2 \cos(2nt + n't - \omega + \omega' - 2\Omega) \\
&+ \tfrac{15}{128}ee'\gamma^2 \cos(n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{513}{128}ee'\gamma^2 \cos(2nt + 3n't - \omega - \omega' - 2\Omega) \\
&+ \tfrac{87}{128}ee'\gamma^2 \cos(3n't - \omega - \omega' - 2\Omega) \\
&- \tfrac{63}{128}ee'\gamma^2 \cos(4nt - 3n't - \omega + \omega' - 2\Omega) \\
&\pm \tfrac{357}{128}ee'\gamma^2 \cos(6nt - 3n't - \omega + \omega' - 2\Omega) \\
&\mp \tfrac{3}{4}e\left\{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\right\}\cos\omega \mp \tfrac{3}{32}e\gamma^2\cos(3\omega - 2\Omega) \\
&- \tfrac{39}{32}e\gamma^2\cos(\omega - 2\Omega) \mp \tfrac{1}{2}(e^2 - e_0^2)\cos nt \mp \tfrac{7}{8}(\gamma^2 - \gamma_0^2)\cos nt \\
&\left.\pm \tfrac{1}{2}h'\{\cos(nt + \alpha't - \beta') + \cos(nt - \alpha't + \beta')\}\right\} \\
&+ \overline{m}^2 \frac{a}{a'}\,ndt\left\{\pm \tfrac{3}{16}\{1 - 5e^2 + 2e'^2 + 18\gamma^2\}\cos(2nt - n't)\right. \\
&\pm \tfrac{15}{16}\{1 + e^2 + 2e'^2 - \tfrac{9}{2}\gamma^2\}\cos n't \pm \tfrac{3}{16}e'\cos(2nt - \omega') \\
&\pm \tfrac{15}{16}e'\cos\omega' \pm \tfrac{9}{16}e'\cos(2nt - 2n't + \omega') \pm \tfrac{45}{16}e'\cos(2n't - \omega')
\end{aligned}
\qquad (271)
$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
\mp \frac{33}{32}e \cos(nt + n't - \omega) \mp \frac{21}{32}e \cos(nt - n't + \omega) \\
\pm \frac{9}{32}e \cos(3nt - n't - \omega) - \frac{27}{32}e \cos(nt - n't - \omega) \\
\pm \frac{3}{4}e^2 \cos(4nt - n't - 2\omega) - \frac{45}{64}e^2 \cos(2nt - n't - 2\omega) \\
\mp \frac{3}{16}e^2 \cos(2nt + n't - 2\omega) + \frac{45}{64}e^2 \cos(n't - 2\omega) \\
\pm \frac{33}{128}e'^2 \cos(2nt + n't - 2\omega') + \frac{165}{128}e'^2 \cos(n't - 2\omega') \\
\pm \frac{159}{128}e^2 \cos(2nt - 3n't + 2\omega') \pm \frac{795}{128}e'^2 \cos(3n't - 2\omega') \\
\mp \frac{15}{32}ee' \cos(nt + \omega - \omega') \pm \frac{33}{32}ee' \cos(nt - \omega + \omega') \\
\pm \frac{9}{32}ee' \cos(3nt - \omega - \omega') - \frac{27}{32}ee' \cos(nt - \omega - \omega') \\
\mp \frac{99}{32}ee' \cos(nt + 2n't - \omega - \omega') \mp \frac{63}{32}ee' \cos(nt - 2n't + \omega + \omega') \\
\pm \frac{27}{32}ee' \cos(3nt - 2n't - \omega + \omega') - \frac{81}{32}ee' \cos(nt - 2n't - \omega + \omega') \\
\mp \frac{9}{64}\gamma^2 \cos(2nt - n't - 2\omega + 2\Omega) \pm \frac{3}{32}\gamma^2 \cos(n't + 2\omega - 2\Omega) \\
\pm \frac{3}{64}\gamma^2 \cos(2nt - n't + 2\omega - 2\Omega) \mp \frac{45}{32}\gamma^2 \cos(4nt - n't - 2\Omega) \\
+ \frac{177}{64}\gamma^2 \cos(2nt - n't - 2\Omega) \pm \frac{63}{64}\gamma^2 \cos(2nt + n't - 2\Omega) \\
+ \frac{15}{32}\gamma^2 \cos(n't - 2\Omega) \mp \frac{3}{16}h'' \cos(nt + \alpha''t - \beta'') \mp \frac{33}{16}h'' \cos(nt - \alpha''t + \beta'') \\
\mp \frac{15}{16}\{1 - 11e^2 - 6e'^2 - \gamma^2\} \cos(4nt - 3n't) \\
+ \frac{45}{16}\{1 - \frac{11}{3}e^2 - 6e'^2 - \gamma^2\} \cos(2nt - 3n't) \\
\mp \frac{75}{32}e \cos(5nt - 3n't - \omega) + \frac{105}{32}e \cos(3nt - 3n't - \omega) \\
\pm \frac{185}{32}e \cos(3nt - 3n't + \omega) - \frac{285}{32}e \cos(nt - 3n't + \omega) \\
\mp \frac{75}{16}e' \cos(4nt - 4n't + \omega') + \frac{225}{16}e' \cos(2nt - 4n't + \omega') \\
\pm \frac{15}{16}e' \cos(4nt - 2n't - \omega') - \frac{45}{16}e' \cos(2nt - 2n't - \omega') \\
\mp \frac{185}{32}e^2 \cos(6nt - 3n't - 2\omega) + \frac{225}{64}e^2 \cos(4nt - 3n't - 2\omega) \\
\mp \frac{105}{16}e^2 \cos(2nt - 3n't + 2\omega) \pm \frac{525}{64}e^2 \cos(3n't - 2\omega) \\
\mp \frac{15}{128}e'^2 \cos(4nt - n't - 2\omega') + \frac{45}{128}e'^2 \cos(2nt - n't - 2\omega') \\
\mp \frac{1905}{128}e'^2 \cos(4nt - 5n't + 2\omega') + \frac{5715}{128}e'^2 \cos(2nt - 5n't + 2\omega') \\
\mp \frac{375}{32}ee' \cos(5nt - 4n't - \omega + \omega') + \frac{525}{32}ee' \cos(3nt - 4n't - \omega + \omega') \\
\mp \frac{185}{32}ee' \cos(3nt - 2n't + \omega - \omega') + \frac{285}{32}ee' \cos(nt - 2n't + \omega - \omega') \\
\pm \frac{75}{32}ee' \cos(5nt - 2n't - \omega - \omega') - \frac{105}{32}ee' \cos(3nt - 2n't - \omega - \omega') \\
\pm \frac{675}{32}ee' \cos(3nt - 4n't + \omega + \omega') - \frac{1425}{32}ee' \cos(nt - 4n't + \omega + \omega') \\
\mp \frac{15}{32}\gamma^2 \cos(2nt - 3n't + 2\Omega) \pm \frac{75}{64}\gamma^2 \cos(3n't - 2\Omega) \\
\mp \frac{15}{32}\gamma^2 \cos(4nt - 3n't + 2\omega - 2\Omega) + \frac{45}{64}\gamma^2 \cos(2nt - 3n't + 2\omega - 2\Omega) \\
\pm \frac{15}{32}\gamma^2 \cos(4nt - 3n't - 2\omega + 2\Omega) - \frac{45}{64}\gamma^2 \cos(2nt - 3n't - 2\omega + 2\Omega) \\
+ \frac{15}{64}\gamma^2 \cos(4nt - 3n't - 2\Omega) \Big\}
\end{array}\right\} \quad . \quad (271)
$$

Equation (271) will give the value of $c_2 \sin\beta\left(\frac{dR}{dr}\right) + c_3 \cos\beta\left(\frac{dR}{dv}\right) + c_4 \cos\beta\left(\frac{dR}{d\theta}\right)$ by using the lower signs and changing *cos* to *sin* in the second member.

Equation (271) will give, by integration,

$$\int\left\{ c_2 \cos\beta\left(\frac{dR}{dr}\right) + c_3 \sin\beta\left(\frac{dR}{dv}\right) + c_4 \sin\beta\left(\frac{dR}{d\theta}\right)\right\} =$$

$$\left.\begin{aligned}
\bar{m}^2 \Big\{ &+ \tfrac{1}{2}\{1 - e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2 + \tfrac{7}{64}e^4 + \tfrac{15}{8}e'^4 - \tfrac{9}{4}e^2e'^2 \\
&\qquad\qquad + \tfrac{13}{4}e^2\gamma^2 - \tfrac{21}{8}e'^2\gamma^2 + \tfrac{95}{64}\gamma^4\} \sin nt \\
&+ \tfrac{1}{8}e\,\{1 - \tfrac{5}{4}e^2 + \tfrac{3}{2}e'^2 - \tfrac{13}{4}\gamma^2\} \sin(2nt - \omega) \\
&+ e'\,\{1 - e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\}(0.6978034) \sin(nt + n't - \omega') \\
&+ e'\,\{1 - e^2 + \tfrac{9}{8}e'^2 - \tfrac{7}{4}\gamma^2\}(0.8106367) \sin(nt - n't + \omega') \\
&+ \tfrac{1}{16}e^2\,\{1 - \tfrac{3}{2}e^2 + \tfrac{3}{2}e'^2 - 4\gamma^2 + \frac{\gamma^4}{e^2}\} \sin(3nt - 2\omega) \\
&\pm \tfrac{1}{16}e^2\,\{1 + \tfrac{25}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{81}{4}\gamma^2 - 2\frac{\gamma^4}{e^2}\} \sin(nt - 2\omega) \\
&+ e'^2\,\{1 - e^2 + \tfrac{7}{9}e'^2 - \tfrac{7}{4}\gamma^2\}(0.9785989) \sin(nt + 2n't - 2\omega') \\
&+ e'^2\,\{1 - e^2 + \tfrac{7}{9}e'^2 - \tfrac{7}{4}\gamma^2\}(1.322911) \sin(nt - 2n't + 2\omega') \\
&+ ee'\,\{1 - \tfrac{5}{4}e^2 + \tfrac{9}{8}e'^2 - \tfrac{13}{4}\gamma^2\}(0.1807402) \sin(2nt + n't - \omega - \omega') \\
&\mp ee'\,\{1 - \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{9}{4}\gamma^2\}(15.03985) \sin(n't - \omega - \omega') \\
&+ ee'\,\{1 - \tfrac{5}{4}e^2 + \tfrac{9}{8}e'^2 - \tfrac{13}{4}\gamma^2\}(0.1946955) \sin(2nt - n't - \omega + \omega') \\
&- ee'\,\{1 - \tfrac{1}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{9}{4}\gamma^2\}(15.03985) \sin(n't + \omega - \omega') \\
&- \tfrac{1}{8}\gamma^2\,\{1 - \tfrac{97}{16}e^2 + \tfrac{3}{2}e'^2 - \tfrac{17}{4}\gamma^2\} \sin(3nt - 2\Omega) \\
&\pm \gamma^2\,\{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{8}\gamma^2\} \sin(nt - 2\Omega) \\
&+ \tfrac{1}{16}\gamma^2\,\{1 - \tfrac{97}{4}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\} \sin(nt + 2\omega - 2\Omega) \\
&- \tfrac{1}{16}\gamma^2\,\{1 + 9e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\} \sin(nt - 2\omega + 2\Omega) \\
&+ \tfrac{1}{24}e^3 \sin(4nt - 3\omega) \pm \tfrac{1}{96}e^3 \sin(2nt - 3\omega) \\
&+ e'^3(1.352699) \sin(nt + 3n't - 3\omega') + e'^3(2.135455) \sin(nt - 3n't + 3\omega') \\
&+ ee'^2(0.2616762) \sin(2nt + 2n't - \omega - 2\omega') \\
&\mp ee'^2(11.27989) \sin(2n't - \omega - 2\omega') \\
&+ ee'^2(0.3039888) \sin(2nt - 2n't - \omega + 2\omega') \\
&- ee'^2(11.27989) \sin(2n't + \omega - 2\omega') \\
&+ e^2e'\,(0.09146913) \sin(3nt + n't - 2\omega - \omega') \\
&\pm e^2e'\,(0.08722542) \sin(nt + n't - 2\omega + \omega') \\
&+ e^2e'\,(0.0961473) \sin(3nt - n't - 2\omega + \omega') \\
&\pm e^2e'\,(0.1013296) \sin(nt - n't - 2\omega + \omega') \\
&- \tfrac{3}{16}e\gamma^2 \sin(4nt - \omega - 2\Omega) \pm \tfrac{5}{32}e\gamma^2 \sin(2nt - \omega - 2\Omega) \\
&+ \tfrac{37}{64}e\gamma^2 \sin(2nt + \omega - 2\Omega) - \tfrac{1}{64}e\gamma^2 \sin(2nt - 3\omega + 2\Omega) \\
&- e'\gamma^2(0.1829386) \sin(3nt + n't - \omega' - 2\Omega)
\end{aligned}\right\}. \quad (272)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\pm e'\gamma^2\,(1.395607)\sin(nt+n't-\omega'-2\Omega)\\
&-e'\gamma^2\,(0.1922946)\sin(3nt-n't+\omega'-2\Omega)\\
&\pm e'\gamma^2\,(1.621274)\sin(nt-n't+\omega'-2\Omega)\\
&-e'\gamma^2\,(0.08722542)\sin(nt+n't-2\omega-\omega'+2\Omega)\\
&-e'\gamma^2\,(0.1013296)\sin(nt-n't-2\omega+\omega'+2\Omega)\\
&+e'\gamma^2\,(0.08722542)\sin(nt+n't+2\omega-\omega'-2\Omega)\\
&+e'\gamma^2\,(0.1013296)\sin(nt-n't+2\omega+\omega'-2\Omega)\\
&+ee'^3\,(0.3722907)\sin(2nt+3n't-\omega-3\omega')\\
&\mp ee'^3\,(11.07100)\sin(3n't-\omega-3\omega')\\
&+ee'^3\,(0.4663928)\sin(2nt-3n't-\omega+3\omega')\\
&-ee'^3\,(11.07100)\sin(3n't+\omega-3\omega')\\
&+e^3e'\,(0.06135270)\sin(4nt+n't-3\omega-\omega')\\
&\pm e^3e'\,(0.01506169)\sin(2nt+n't-3\omega-\omega')\\
&+e^3e'\,(0.06369103)\sin(4nt-n't-3\omega+\omega')\\
&\pm e^3e'\,(0.01623209)\sin(2nt-n't-3\omega+\omega')\\
&+e^2e'^2\,(0.1339455)\sin(3nt+2n't-2\omega-2\omega')\\
&\pm e^2e'^2\,(0.1223248)\sin(nt+2n't-2\omega-2\omega')\\
&+e^2e'^2\,(0.1480057)\sin(3nt-2n't-2\omega+2\omega')\\
&\pm e^2e'^2\,(0.1653638)\sin(nt-2n't-2\omega+2\omega')\\
&-e'^2\gamma^2\,(0.2678909)\sin(3nt+2n't-2\omega'-2\Omega)\\
&\pm e'^2\gamma^2\,(1.957198)\sin(nt+2n't-2\omega'-2\Omega)\\
&-e'^2\gamma^2\,(0.2960114)\sin(3nt-2n't+2\omega'-2\Omega)\\
&\pm e'^2\gamma^2\,(2.645821)\sin(nt-2n't+2\omega'-2\Omega)\\
&-e'^2\gamma^2\,(0.1223248)\sin(nt+2n't-2\omega-2\omega'+2\Omega)\\
&-e'^2\gamma^2\,(0.1653638)\sin(nt-2n't-2\omega+2\omega'+2\Omega)\\
&+e'^2\gamma^2\,(0.1223248)\sin(nt+2n't+2\omega-2\omega'-2\Omega)\\
&+e'^2\gamma^2\,(0.1653638)\sin(nt-2n't+2\omega+2\omega'-2\Omega)\\
&+\tfrac{25}{768}e^4\sin(5nt-4\omega)\pm\tfrac{1}{256}e^4\sin(3nt-4\omega)\\
&+e'^4\,(1.852094)\sin(nt+4n't-4\omega')\\
&+e'^4\,(3.433601)\sin(nt-4n't+4\omega')\\
&-\tfrac{15}{64}e^2\gamma^2\sin(5nt-2\omega-2\Omega)\pm\tfrac{1}{16}e^2\gamma^2\sin(3nt-2\omega-2\Omega)\\
&+\tfrac{1}{64}e^2\gamma^2\sin(3nt-4\omega+2\Omega)\pm\tfrac{1}{128}e^2\gamma^2\sin(nt-4\omega+2\Omega)\\
&-ee'\gamma^2\,(0.2149572)\sin(4nt-n't-\omega+\omega'-2\Omega)\\
&\pm ee'\gamma^2\,(0.3895702)\sin(2nt-n't-\omega+\omega'-2\Omega)\\
&+ee'\gamma^2\,(0.9714787)\sin(2nt+n't+\omega-\omega'-2\Omega)\\
&\mp ee'\gamma^2\,(20.67979)\sin(n't+\omega-\omega'-2\Omega)
\end{aligned}\right\}\quad .\quad (272)
$$

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$$
\left.\begin{aligned}
&- ee'\gamma^2 (0.2760871) \sin (4nt + n't - \omega - \omega' - 2\Omega) \\
&\pm ee'\gamma^2 (0.2259253) \sin (2nt + n't - \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.9008810) \sin (2nt - n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (24.43976) \sin (n't - \omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.02259253) \sin (2nt + n't - 3\omega - \omega' + 2\Omega) \\
&\mp ee'\gamma^2 (1.879981) \sin (n't - 3\omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.02434813) \sin (2nt - n't - 3\omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (1.879981) \sin (n't + 3\omega - \omega' - 2\Omega) \\
&- \tfrac{3}{64}\gamma^4 \sin (3nt + 2\omega - 4\Omega) + \tfrac{1}{8}\gamma^4 \sin (nt + 2\omega - 4\Omega) \pm \tfrac{1}{32}\gamma^4 \sin (3nt - 4\Omega) \\
&+ \tfrac{3}{256}\gamma^4 \sin (nt + 4\omega - 4\Omega) - \tfrac{1}{256}\gamma^4 \sin (nt - 4\omega + 4\Omega) \\
&- \{1 - 6e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2 + \tfrac{615}{32}e^4 - \tfrac{9}{16}e'^4 \\
&\qquad + \tfrac{29}{64}\gamma^4 + \tfrac{63}{4}e^2e'^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{5}{8}e'^2\gamma^2\} (0.2631212) \sin (3nt - 2n't) \\
&\pm \{1 - \tfrac{4}{3}e^2 - \tfrac{5}{2}e'^2 - \tfrac{7}{12}\gamma^2 + \tfrac{37}{192}e^4 + \tfrac{61}{48}e'^4 \\
&\qquad + \tfrac{41}{64}\gamma^4 + \tfrac{11}{6}e^2e'^2 + \tfrac{1}{4}e^2\gamma^2 + \tfrac{35}{24}e'^2\gamma^2\} (2.645821) \sin (nt - 2n't) \\
&- e \{1 - \tfrac{19}{4}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{16}\gamma^2\} (0.3895703) \sin (4nt - 2n't - \omega) \\
&\pm e \{1 - \tfrac{11}{8}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{16}\gamma^2\} (0.8106367) \sin (2nt - 2n't - \omega) \\
&+ e \{1 - \tfrac{11}{6}e^2 - \tfrac{5}{2}e'^2 - \tfrac{5}{24}\gamma^2\} (1.215955) \sin (2nt - 2n't + \omega) \\
&- e \{1 - \tfrac{1}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{43}{40}\gamma^2\} (25.06642) \sin (2n't - \omega) \\
&+ e' \{1 - 6e^2 - \tfrac{1}{8}e'^2 - \tfrac{1}{4}\gamma^2\} (0.1281964) \sin (3nt - n't - \omega') \\
&\mp e' \{1 - \tfrac{4}{3}e^2 - \tfrac{1}{8}e'^2 - \tfrac{7}{12}\gamma^2\} (1.215955) \sin (nt - n't - \omega') \\
&- e' \{1 - 6e^2 - \tfrac{123}{56}e'^2 - \tfrac{1}{4}\gamma^2\} (0.9457427) \sin (3nt - 3n't + \omega') \\
&\pm e' \{1 - \tfrac{4}{3}e^2 - \tfrac{123}{56}e'^2 - \tfrac{7}{12}\gamma^2\} (10.15348) \sin (nt - 3n't + \omega') \\
&- e^2 \{1 - \tfrac{31}{6}e^2 - \tfrac{5}{2}e'^2 - \tfrac{73}{300}\gamma^2 + \tfrac{592888}{1510725}\frac{\gamma^4}{e^2}\} (0.4832079) \sin (5nt - 2n't - 2\omega) \\
&\pm e^2 \{1 - \tfrac{41}{26}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2 - \tfrac{7}{52}\frac{\gamma^4}{e^2}\} (0.4275720) \sin (3nt - 2n't - 2\omega) \\
&+ e^2 \{1 + \tfrac{101}{42}e^2 - \tfrac{5}{2}e'^2 + \tfrac{1}{28}\gamma^2 + \tfrac{5}{128}\frac{\gamma^4}{e^2}\} (0.5708494) \sin (nt + 2n't - 2\omega) \\
&- e^2 \{1 - \tfrac{14}{57}e^2 - \tfrac{5}{2}e'^2 - \tfrac{29}{228}\gamma^2 + \tfrac{3}{76}\frac{\gamma^4}{e^2}\} (2.094609) \sin (nt - 2n't + 2\omega) \\
&- e'^2 \{1 - 6e^2 - \tfrac{115}{51}e'^2 - \tfrac{1}{4}\gamma^4\} (2.360416) \sin (3nt - 4n't + 2\omega') \\
&\pm e'^2 \{1 - \tfrac{4}{3}e^2 - \tfrac{115}{51}e'^2 - \tfrac{119}{204}\gamma^2\} (27.29044) \sin (nt - 4n't + 2\omega') \\
&+ ee' \{1 - \tfrac{175}{32}e^2 - \tfrac{1}{8}e'^2 - \tfrac{23}{16}\gamma^2\} (0.1910731) \sin (4nt - n't - \omega - \omega') \\
&\mp ee' \{1 - \tfrac{11}{8}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{16}\gamma^2\} (0.3895702) \sin (2nt - n't - \omega - \omega') \\
&+ ee' \{1 - \tfrac{328}{63}e^2 - \tfrac{123}{56}e'^2 - \tfrac{35}{168}\gamma^2\} (4.435132) \sin (2nt - 3n't + \omega + \omega') \\
&- ee' \{1 - \tfrac{41}{84}e^2 - \tfrac{123}{56}e'^2 - \tfrac{31}{40}\gamma^2\} (58.48828) \sin (3n't - \omega - \omega')
\end{aligned}\right\} \quad . \quad (272)
$$

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$$
\left.\begin{aligned}
&- ee' \{1 - \tfrac{19}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{3}{16}\gamma^2\}(1.390509) \sin(4nt - 3n't - \omega + \omega') \\
&\pm ee' \{1 - \tfrac{7}{4}e^2 - \tfrac{123}{56}e'^2 - \tfrac{7}{80}\gamma^2\}(2.956754) \sin(2nt - 3n't - \omega + \omega') \\
&- ee' \{1 - \tfrac{11}{6}e^2 - \tfrac{1}{8}e'^2 - \tfrac{5}{24}\gamma^2\}(0.5843553) \sin(2nt - n't + \omega - \omega') \\
&+ ee' \{1 - \tfrac{17}{80}e^2 - \tfrac{1}{8}e'^2 - \tfrac{19}{40}\gamma^2\}(25.06642) \sin(n't - \omega + \omega') \\
&+ \gamma^2 \{1 - \tfrac{43}{16}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2\}(0.03865663) \sin(5nt - 2n't - 2\Omega) \\
&+ \gamma^2 \{1 - \tfrac{31}{16}e^2 - \tfrac{5}{2}e'^2 - \gamma^2\}(0.1630998) \sin(nt + 2n't - 2\Omega) \\
&- \gamma^2 \{1 - \tfrac{245}{12}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\}(0.09867045) \sin(3nt - 2n't + 2\omega - 2\Omega) \\
&\pm \gamma^2 \{1 - \tfrac{73}{8}e^2 - \tfrac{5}{2}e'^2 - \tfrac{13}{12}\gamma^2\}(0.3307276) \sin(nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2 \{1 - \tfrac{401}{36}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{4}\gamma^2\}(0.09867045) \sin(3nt - 2n't - 2\omega + 2\Omega) \\
&\mp \gamma^2 \{1 - \tfrac{833}{72}e^2 - \tfrac{5}{2}e'^2 - \tfrac{47}{24}\gamma^2\}(0.3307276) \sin(nt - 2n't - 2\omega + 2\Omega) \\
&- e^3 (0.5768839) \sin(6nt - 2n't - 3\omega) \\
&\pm e^3 (0.5194270) \sin(4nt - 2n't - 3\omega) \\
&+ e^3 (0.07185580) \sin(2nt + 2n't - 3\omega) \\
&\mp e^3 (0.2088867) \sin(2n't - 3\omega) - e'^3 (0.005081628) \sin(3nt + n't - 3\omega') \\
&\pm e'^3 (0.04361271) \sin(nt + n't - 3\omega') \\
&- e'^3 (5.027860) \sin(3nt - 5n't + 3\omega') \\
&\pm e'^3 (63.27443) \sin(nt - 5n't + 3\omega') \\
&- e\gamma^2 (0.4017443) \sin(4nt - 2n't + \omega - 2\Omega) \\
&\pm e\gamma^2 (0.1773251) \sin(2nt - 2n't + \omega - 2\Omega) \\
&- e\gamma^2 (0.2786564) \sin(2nt - 2n't - \omega + 2\Omega) \\
&+ e\gamma^2 (0.9399904) \sin(2n't + \omega - 2\Omega) \\
&+ e\gamma^2 (0.1460888) \sin(4nt - 2n't - 3\omega + 2\Omega) \\
&\mp e\gamma^2 (0.1013320) \sin(2nt - 2n't - 3\omega + 2\Omega) \\
&+ e\gamma^2 (0.4559831) \sin(2nt - 2n't + 3\omega - 2\Omega) \\
&- e\gamma^2 (3.133301) \sin(2n't - 3\omega + 2\Omega) \\
&+ e\gamma^2 (0.06541906) \sin(2nt + 2n't - \omega - 2\Omega) \\
&\mp e\gamma^2 (1.566651) \sin(2n't - \omega - 2\Omega) \\
&\mp e\gamma^2 (0.03652220) \sin(4nt - 2n't - \omega - 2\Omega) \\
&+ e\gamma^2 (0.1362087) \sin(6nt - 2n't - \omega - 2\Omega) \\
&+ e'\gamma^2 (0.5359750) \sin(nt + 3n't - \omega' - 2\Omega) \\
&- e'\gamma^2 (0.08722543) \sin(nt + n't + \omega' - 2\Omega) \\
&- e'\gamma^2 (0.04807366) \sin(3nt - n't - 2\omega - \omega' + 2\Omega) \\
&\pm e'\gamma^2 (0.1519944) \sin(nt - n't - 2\omega - \omega' + 2\Omega) \\
&- e'\gamma^2 (0.3546534) \sin(3nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&\pm e'\gamma^2 (1.269185) \sin(nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&+ e'\gamma^2 (0.04807366) \sin(3nt - n't + 2\omega - \omega' - 2\Omega)
\end{aligned}\right\} . \quad (272)
$$

(*Continued on the next page.*)

$$
\left.\begin{aligned}
&\mp e'\gamma^2 (0.1519944) \sin (nt - n't + 2\omega - \omega' - 2\Omega) \\
&+ e'\gamma^2 (0.3546534) \sin (3nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&\mp e'\gamma^2 (1.269185) \sin (nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&- e'\gamma^2 (0.01903476) \sin (5nt - n't - \omega' - 2\Omega) \\
&+ e'\gamma^2 (0.1374174) \sin (5nt - 3n't + \omega' - 2\Omega) \\
&+ ee'^2 (11.24474) \sin (2nt - 4n't + \omega + 2\omega') \\
&- ee'^2 (106.5322) \sin (4n't - \omega - 2\omega') \\
&- ee'^2 (3.445206) \sin (4nt - 4n't - \omega + 2\omega') \\
&\pm ee'^2 (7.496496) \sin (2nt - 4n't - \omega + 2\omega') \\
&+ e^2e' (0.2379346) \sin (5nt - n't - 2\omega - \omega') \\
&\mp e^2e' (0.2083192) \sin (3nt - n't - 2\omega - \omega') \\
&+ e^2e' (1.875913) \sin (nt + 3n't - 2\omega - \omega') \\
&- e^2e' (8.038172) \sin (nt - 3n't + 2\omega + \omega') \\
&- e^2e' (1.717718) \sin (5nt - 3n't - 2\omega + \omega') \\
&\pm e^2e' (1.536831) \sin (3nt - 3n't - 2\omega + \omega') \\
&- e^2e' (0.3052889) \sin (nt + n't - 2\omega + \omega') \\
&+ e^2e' (0.9626311) \sin (nt - n't + 2\omega - \omega') \\
&- e^4 (0.6845520) \sin (7nt - 2n't - 4\omega) \\
&\pm e^4 (0.2304732) \sin (5nt - 2n't - 4\omega) \\
&+ e^4 (0.02165420) \sin (3nt + 2n't - 4\omega) \\
&\pm e^4 (0.1546050) \sin (nt + 2n't - 4\omega) \\
&- e'^4 (0.009921885) \sin (3nt + 2n't - 4\omega') \\
&\pm e'^4 (0.08154990) \sin (nt + 2n't - 4\omega') \\
&- e'^4 (9.793216) \sin (3nt - 6n't + 4\omega') \\
&\pm e'^4 (135.9836) \sin (nt - 6n't + 4\omega') \\
&- \gamma^4 (0.03090560) \sin (3nt - 2n't + 4\omega - 4\Omega) \\
&\pm \gamma^4 (0.08957206) \sin (nt - 2n't + 4\omega - 4\Omega) \\
&- \gamma^4 (0.01860354) \sin (3nt + 2n't - 4\Omega) \\
&\pm \gamma^4 (0.09174364) \sin (nt + 2n't - 4\Omega) \\
&+ \gamma^4 (0.02038748) \sin (nt + 2n't + 2\omega - 4\Omega) \\
&\mp \gamma^4 (0.02067047) \sin (nt - 2n't - 4\omega + 4\Omega) \\
&- \gamma^4 (0.008222538) \sin (3nt - 2n't - 4\omega + 4\Omega) \\
&- \gamma^4 (0.01197467) \sin (7nt - 2n't - 4\Omega) \\
&\pm \gamma^4 (0.007248115) \sin (5nt - 2n't - 4\Omega) \\
&+ \gamma^4 (0.02416039) \sin (5nt - 2n't + 2\omega - 4\Omega) \\
&\pm \gamma^2 \{\tfrac{21}{8}e^2 - \gamma^2\} (0.03289015) \sin (3nt - 2n't - 2\Omega)
\end{aligned}\right\} . \quad (272)
$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
-\gamma^2\left\{\frac{131}{8}e^2+\gamma^2\right\}(0.1653638)\sin(nt-2n't+2\Omega)\\
+e^2\gamma^2(0.2029472)\sin(5nt-2n't-4\omega+2\Omega)\\
\mp e^2\gamma^2(0.05344650)\sin(3nt-2n't-4\omega+2\Omega)\\
+e^2\gamma^2(0.08154990)\sin(nt+2n't-4\omega+2\Omega)\\
-e^2\gamma^2(0.7716977)\sin(nt-2n't+4\omega-2\Omega)\\
+e^2\gamma^2(0.03720707)\sin(3nt+2n't-2\omega-2\Omega)\\
\pm e^2\gamma^2(0.2038748)\sin(nt+2n't-2\omega-2\Omega)\\
\mp e^2\gamma^2(0.08697740)\sin(5nt-2n't-2\omega-2\Omega)\\
+e^2\gamma^2(0.3181841)\sin(7nt-2n't-2\omega-2\Omega)\\
-ee'^3(0.007669086)\sin(4nt+n't-\omega-3\omega')\\
\pm ee'^3(0.01506168)\sin(2nt+n't-\omega-3\omega')\\
+ee'^3(0.007530844)\sin(2nt+n't+\omega-3\omega')\\
\mp ee'^3(1.044433)\sin(n't+\omega-3\omega')\\
+ee'^3(24.36010)\sin(2nt-5n't+\omega+3\omega')\\
-ee'^3(176.5093)\sin(5n't-\omega-3\omega')\\
-ee'^3(11.35034)\sin(4nt-5n't-\omega+3\omega')\\
\pm ee'^3(7.168685)\sin(2nt-5n't-\omega+3\omega')\\
+e^3e'(0.2950228)\sin(2nt+3n't-3\omega+\omega')\\
+e^3e'(0.04518506)\sin(2nt+n't-3\omega+\omega')\\
+e^3e'(0.2848006)\sin(6nt-n't-3\omega-\omega')\\
\mp e^3e'(0.1433048)\sin(4nt-n't-3\omega-\omega')\\
+e^3e'(0.2001957)\sin(6nt-3n't-3\omega+\omega')\\
\pm e^3e'(4.477770)\sin(4nt-3n't-3\omega+\omega')\\
-e^2e'^2(4.237980)\sin(5nt-4n't-2\omega+2\omega')\\
\pm e^2e'^2(3.835676)\sin(3nt-4n't-2\omega+2\omega')\\
+e^2e'^2(4.293490)\sin(nt+4n't-2\omega-2\omega')\\
-e^2e'^2(21.60493)\sin(nt-4n't+2\omega+2\omega')\\
-e'^2\gamma^2(0.8851560)\sin(3nt-4n't+2\omega+2\omega'-2\Omega)\\
\pm e'^2\gamma^2(3.411305)\sin(nt-4n't+2\omega+2\omega'-2\Omega)\\
+e'^2\gamma^2(1.226711)\sin(nt+4n't-2\omega'-2\Omega)\\
+e'^2\gamma^2(0.8851560)\sin(3nt-4n't-2\omega+2\omega'+2\Omega)\\
\mp e'^2\gamma^2(3.411305)\sin(nt-4n't-2\omega+2\omega'+2\Omega)\\
+e'^2\gamma^2(0.3390384)\sin(5nt-4n't+2\omega'-2\Omega)\\
-ee'\gamma^2(1.433962)\sin(4nt-3n't+\omega+\omega'-2\Omega)\\
\pm ee'\gamma^2(0.6467899)\sin(2nt-3n't+\omega+\omega'-2\Omega)\\
+ee'\gamma^2(0.1582629)\sin(2nt-n't-\omega-\omega'+2\Omega)
\end{array}
\right\} . \quad (272)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- ee'\gamma^2 (0.9399904) \sin (n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.3204007) \sin (6nt - n't - \omega - \omega' - 2\Omega) \\
&\pm ee'\gamma^2 (1.164354) \sin (4nt - n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (1.355179) \sin (2nt - 3n't - \omega + \omega' + 2\Omega) \\
&+ ee'\gamma^2 (3.446631) \sin (3n't + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.5214408) \sin (4nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&\mp ee'\gamma^2 (0.3695943) \sin (2nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.2191333) \sin (2nt - n't + 3\omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (3.133301) \sin (n't - 3\omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.07165241) \sin (4nt - n't - 3\omega - \omega' + 2\Omega) \\
&\pm ee'\gamma^2 (0.04869627) \sin (2nt - n't - 3\omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 (0.1970441) \sin (4nt - n't + \omega - \omega' - 2\Omega) \\
&\mp ee'\gamma^2 (0.08521846) \sin (2nt - n't + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (1.821571) \sin (2nt - 3n't - 3\omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (8.564355) \sin (3n't - 3\omega' - \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.03389113) \sin (2nt + n't - \omega + \omega' - 2\Omega) \\
&\pm ee'\gamma^2 (1.566650) \sin (n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (1.801746) \sin (2nt + 3n't - \omega - \omega' - 2\Omega) \\
&\pm ee'\gamma^2 (3.028858) \sin (3n't - \omega - \omega' - 2\Omega) \\
&\mp ee'\gamma^2 (0.1303602) \sin (4nt - 3n't - \omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.4829047) \sin (6nt - 3n't - \omega + \omega' - 2\Omega) \\
&\left. + \tfrac{1}{2} \frac{h'n}{n + \alpha'} \sin (nt + \alpha't - \beta') + \tfrac{1}{2} \frac{h'n}{n - \alpha'} \sin (nt - \alpha't + \beta') \right\} \\
&+ m^2 n \int \{\mp \tfrac{1}{2} (e^2 - e_0^2) \cos nt \mp \tfrac{7}{8} (\gamma^2 - \gamma_0^2) \cos nt\} \, dt \\
&+ \tfrac{3}{4} \overline{m}^2 n \int \left\{ \{\mp e \{1 - \tfrac{1}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{9}{4}\gamma^2\} \cos \omega \right. \\
&\qquad\qquad \left. - \tfrac{13}{8} e\gamma^2 \cos (\omega - 2\Omega) \mp \tfrac{1}{8} e\gamma^2 \cos (3\omega - 3\Omega) \right\} dt \\
+ \overline{m}^2 \frac{a}{a'} &\left\{ + \{1 - 5e + 2e'^2 + 18\gamma^2\} (0.09739252) \sin (2nt - n't) \right. \\
&+ \{1 + e^2 + 2e'^2 - \tfrac{9}{2}\gamma^2\} (12.53321) \sin n't + e' (0.09375000) \sin (2nt - \omega') \\
&+ e' (0.3039888) \sin (2nt - 2n't + \omega') + e' (18.79981) \sin (2n't - \omega') \\
&- e (0.9594796) \sin (nt + n't - \omega) - e (0.7093070) \sin (nt - n't + \omega) \\
&+ e (0.09614730) \sin (3nt - n't - \omega) \mp e (0.9140687) \sin (nt - n't - \omega) \\
&+ e^2 (0.1910731) \sin (4nt - n't - 2\omega) \mp e^2 (0.3652220) \sin (2nt - n't - 2\omega) \\
&- e^2 (0.09037013) \sin (2nt + n't - 2\omega) \pm e^2 (9.399904) \sin (n't - 2\omega)
\end{aligned}\right\} \quad (272)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ e'^2 (0.1242589) \sin (2nt + n't - 2\omega) \pm e'^2 (17.23316) \sin (n't - 2\omega') \\
&+ e'^2 (0.6995890) \sin (2nt - 3n't + 2\omega') + e'^2 (27.67749) \sin (3n't - 2\omega') \\
&- ee' (0.4687500) \sin (nt + \omega - \omega') + ee' (1.031250) \sin (nt - \omega + \omega') \\
&+ ee' (0.937500) \sin (3nt - \omega - \omega') \mp ee' (0.843750) \sin (nt - \omega - \omega') \\
&- ee' (2.6911470) \sin (nt + 2n't - \omega - \omega') \\
&- ee' (2.315094) \sin (nt - 2n't + \omega + \omega') \\
&+ ee' (0.2960114) \sin (3nt - 2n't - \omega + \omega') \\
&\mp ee' (2.976549) \sin (nt - 2n't - \omega + \omega') \\
&- \gamma^2 (0.07304440) \sin (2nt - n't - 2\omega + 2\Omega) \\
&+ \gamma^2 (1.253321) \sin (n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (0.02434816) \sin (2nt - n't + 2\omega - 2\Omega) \\
&- \gamma^2 (0.3582622) \sin (4nt - n't - 2\Omega) \pm \gamma^2 (1.436540) \sin (2nt - n't - 2\Omega) \\
&+ \gamma^2 (0.4744431) \sin (2nt + n't - 2\Omega) \pm \gamma^2 (6.266603) \sin (n't - 2\Omega) \\
&- \tfrac{3}{16} h'' \frac{n}{n + \alpha''} \sin (nt + \alpha'' t - \beta'') - \tfrac{33}{16} h'' \frac{n}{n - \alpha''} \sin (nt - \alpha'' t + \beta'') \\
&- \{1 - 11e^2 - 6e'^2 - \gamma^2\} (0.2483052) \sin (4nt - 3n't) \\
&\pm \{1 - \tfrac{11}{8}e^2 - 6e'^2 - \gamma^2\} (1.583975) \sin (2nt - 3n't) \\
&- e (0.4907765) \sin (5nt - 3n't - \omega) \pm e (1.182178) \sin (3nt - 3n't - \omega) \\
&+ e (1.519944) \sin (3nt - 3n't + \omega) \mp e (11.48310) \sin (nt - 3n't + \omega) \\
&- e' (1.266620) \sin (4nt - 4n't + \omega') \pm e' (8.268192) \sin (2nt - 4n't + \omega') \\
&+ e' (0.2434814) \sin (4nt - 2n't - \omega') \\
&\mp e' (1.519944) \sin (2nt - 2n't - \omega') \\
&- e^2 (0.7772970) \sin (6nt - 3n't - 2\omega) \\
&\pm e^2 (0.9311443) \sin (4nt - 3n't - 2\omega) \\
&- e^2 (3.695943) \sin (2nt - 3n't + 2\omega) + e^2 (36.55518) \sin (3n't - 2\omega) \\
&- e'^2 (0.02985517) \sin (4nt - n't - 2\omega') \\
&\pm e'^2 (0.1826110) \sin (2nt - n't - 2\omega') \\
&- e'^2 (4.104478) \sin (4nt - 5n't + 2\omega') \\
&\pm e'^2 (27.45917) \sin (2nt - 5n't + 2\omega') \\
&- ee' (2.492930) \sin (5nt - 4n't - \omega + \omega') \\
&\pm ee' (6.074600) \sin (3nt - 4n't - \omega + \omega') \\
&- ee' (1.480057) \sin (3nt - 2n't + \omega - \omega') \\
&\pm ee' (10.47304) \sin (nt - 2n't + \omega - \omega') \\
&+ ee' (0.4832077) \sin (5nt - 2n't - \omega - \omega') \\
&\mp ee' (1.151155) \sin (3nt - 2n't - \omega - \omega') \\
&+ ee' (7.810200) \sin (3nt - 4n't + \omega + \omega')
\end{aligned}\right\} \quad . \quad (272)
$$

(Continued on the next page.)

$$\left.\begin{array}{l}\mp ee'\,(63.54393)\sin(nt-4n't+\omega+\omega') \\ -\gamma^2\,(0.2639958)\sin(2nt-3n't+2\Omega)+\gamma^2\,(5.222168)\sin(3n't-2\Omega) \\ -\gamma^2\,(0.1241526)\sin(4nt-3n't+2\omega-2\Omega) \\ \pm\gamma^2\,(0.3959938)\sin(2nt-3n't+2\omega-2\Omega) \\ +\gamma^2\,(0.1241526)\sin(4nt-3n't-2\omega+2\Omega) \\ \mp\gamma^2\,(0.3959938)\sin(2nt-3n't-2\omega+2\Omega) \\ \pm\gamma^2\,(0.06207630)\sin(4nt-3n't-2\Omega)\pm\frac{15}{8}n\int e'\cos\omega'\,dt\ \Big\} \end{array}\right\}. \quad (272)$$

This equation will give the value of $\int\left\{c_2\sin\beta\left(\frac{dR}{dr}\right)+c_3\cos\beta\left(\frac{dR}{dv}\right)\right.$ $\left.+\,c_4\cos\beta\left(\frac{dR}{d\theta}\right)\right\}$ by using the lower signs and changing *sin* to *cos* in the second member.

We shall designate

$$\int\left\{c_2\sin\beta\left(\frac{dR}{dr}\right)+c_3\cos\beta\left(\frac{dR}{dv}\right)+c_4\cos\beta\left(\frac{dR}{d\theta}\right)\right\}\text{ as equation} \qquad (273)$$

19. If we neglect the square of the disturbing force, we may suppose e, ω, γ, and Ω to be constant. The integration of the terms under the sign of integration, which depend on these quantities, would in this case seem to introduce the time t into the preceding equation without the signs of *sin* and *cos*, or under the form of arcs of a circle; but it is important to observe that such is not the case, and that in the case of constant elements the integral of the terms under consideration becomes identically equal to nothing. For, if we suppose ω to be a function of the time, and that we have

$$\omega=\omega_0+\alpha t,$$

the term $e\cos\omega$ will become

$$e\cos\omega=e\cos(\omega_0+\alpha t);$$

and this, by multiplying by dt and taking the integral, gives

$$\int e\cos\omega\,dt=\int e\cos(\omega_0+\alpha t)\,dt=\frac{e}{\alpha}\sin(\omega_0+\alpha t)+c.$$

This integral is evidently equal to nothing when $t=0$; consequently,

$$c=-\frac{e}{\alpha}\sin\omega_0,$$

and the complete integral is

$$\int e\cos\omega\,dt=\frac{e}{\alpha}\sin(\omega_0+\alpha t)-\frac{e}{\alpha}\sin\omega_0.$$

But this integral vanishes at the epoch when $t=0$, whether the elements are constant or variable; and we must therefore omit all the terms of equation (272) which remain under the sign of integration for all powers of the disturbing force.

20. We have given the values of $c_1 \cos\beta$ and $c_1 \sin\beta$ in equation (240). If we now multiply equation (272) by $c_1\cos\beta$, and equation (273) by $c_1 \sin\beta$, and substitute the products in equation (259), we shall get the following development of the value of $\frac{d\delta_1 r}{dt}$:

$$
\left.
\begin{aligned}
\frac{d\delta_1 r}{dt} = a\frac{\overline{m}^2}{\mu}n\Big\{ & -e\{0.875-0.765625e^2+1.3125e'^2-1.125\gamma^2\}\sin(nt-\omega) \\
& - e'\{0.1128333-30.2349947e^2+0.1269375e'^2-0.16925\gamma^2\}\sin(n't-\omega') \\
& - e^2\{0.625-3.859375e^2+0.9375e'^2-0.609375\gamma^2 \\
& \qquad\qquad +0.234375\frac{\gamma^4}{e^2}\}\sin 2(nt-\omega) \\
& - e'^2\{0.343312-23.033394e^2+0.2677982e'^2-0.516469\gamma^2\}\sin 2(n't-\omega') \\
& - ee'\{16.36755-18.15625e^2+18.41350e'^2 \\
& \qquad\qquad -31.80013\gamma^2\}\sin(nt+n't-\omega-\omega') \\
& + ee'\{13.72610-15.84258e^2+15.44136e'^2 \\
& \qquad\qquad -28.40113\gamma^2\}\sin(nt-n't-\omega+\omega') \\
& + \gamma^2\{1-1.304687e^2+\tfrac{3}{2}e'^2-\tfrac{7}{4}\gamma^2\}\sin(2nt-\Omega) \\
& - e^3(0.6718400)\sin 3(nt-\omega)-e'^3(0.782756)\sin 3(n't-\omega') \\
& - ee'^2(13.31972)\sin(nt+2n't-\omega-2\omega') \\
& + ee'^2(9.28237)\sin(nt-2n't-\omega+2\omega') \\
& - e^2e'(16.04110)\sin(2nt+n't-2\omega-\omega') \\
& + e^2e'(14.15627)\sin(2nt-n't-2\omega+\omega') \\
& + \tfrac{5}{4}e\gamma^2\sin(3nt-\omega-2\Omega)-\tfrac{11}{16}e\gamma^2\sin(nt+\omega-2\Omega) \\
& + e'\gamma^2(1.387119)\sin(2nt+n't-\omega'-2\Omega) \\
& + e'\gamma^2(1.631638)\sin(2nt-n't+\omega'-2\Omega) \\
& - ee'^3(14.18686)\sin(nt+3n't-\omega-3\omega') \\
& + ee'^3(8.04924)\sin(nt-3n't-\omega+3\omega') \\
& - e^3e'(18.00923)\sin(3nt+n't-3\omega-\omega') \\
& + e^3e'(15.98262)\sin(3nt-n't-3\omega+\omega') \\
& - e^2e'^2(12.89589)\sin(2nt+2n't-2\omega-2\omega') \\
& + e^2e'^2(10.02299)\sin(2nt-2n't-2\omega+2\omega') \\
& + e'^2\gamma^2(1.933957)\sin(2nt+2n't-2\omega'-2\Omega)
\end{aligned}
\right\} . \quad (274)
$$

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$$\left.\begin{aligned}
&+ e'^2\gamma^2 (2.680538) \sin (2nt - 2n't + 2\omega' - 2\Omega) \\
&- e^4 (0.8645832) \sin 4 (nt - \omega) - e'^4 (1.581507) 4 (n't - \omega') \\
&- \tfrac{9}{16} e^2\gamma^2 \sin 2 (\omega - \Omega) + \tfrac{3}{2} e^2\gamma^2 \sin (4nt - 2\omega - 2\Omega) \\
&+ e^2\gamma^2 (0.0234370) \sin (2nt - 4\omega + 2\Omega) + \tfrac{1}{4}\gamma^4 \sin (2nt + 2\omega - 4\Omega) \\
&+ ee'\gamma^2 (1.748020) \sin (3nt + n't - \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (2.2395147) \sin (3nt - n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (27.12385) \sin (nt + n't + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (28.94002) \sin (nt - n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.014105) \sin (nt + n't - 3\omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 (0.014105) \sin (nt - n't - 3\omega + \omega' + 2\Omega) \\
&+ \{2.382700 + 19.50988 e^2 - 5.95675 e'^2 - 0.881940\gamma^2 - 31.446945 e^4 \\
&\qquad + 3.51041 e'^4 + 0.912851\gamma^4 - 52.94075 e^2 e'^2 + 2.204852 e'^2\gamma^2 \\
&\qquad - 20.154758 e^2\gamma^2\} \sin 2 (nt - n't) \\
&+ e\{3.330008 + 20.70296 e^2 - 8.325021 e'^2 - 0.9569532\gamma^2\} \sin (3nt - 2n't - \omega) \\
&+ e\{23.37343 - 27.12579 e^2 - 58.43359 e'^2 - 19.848521\gamma^2\} \sin (nt - 2n't + \omega) \\
&- e'\{1.087759 + 22.50545 e^2 - 0.135969 e'^2 - 0.405318\gamma^2\} \sin (2nt - n't - \omega') \\
&+ e'\{9.20774 + 37.23850 e^2 - 20.22414 e'^2 - 3.384493\gamma^2\} \sin (2nt - 3n't + \omega') \\
&+ e^2\{4.154010 + 23.56346 e^2 - 11.385025 e'^2 - 1.021721\gamma^2 \\
&\qquad\qquad - 0.2094615 \frac{\gamma^4}{e^2}\} \sin (4nt - 2n't - 2\omega) \\
&+ e^2\{27.14267 + 1.131696 e^2 - 68.85665 e'^2 - 20.420373\gamma^2 \\
&\qquad\qquad + 0.1253682 \frac{\gamma^4}{e^2}\} \sin 2 (n't - \omega) \\
&+ e'^2\{24.93002 + 49.6560 e^2 - 56.21175 e'^2 \\
&\qquad\qquad - 9.096810\gamma^2\} \sin (2nt - 4n't + 2\omega') \\
&- ee'\{1.5426485 + 24.85213 e^2 - 0.192831 e'^2 \\
&\qquad\qquad + 0.202551\gamma^2\} \sin (3nt - n't - \omega - \omega') \\
&+ ee'\{51.82419 - 64.02707 e^2 - 113.8282 e'^2 \\
&\qquad\qquad - 27.137057\gamma^2\} \sin (nt - 3n't + \omega + \omega') \\
&+ ee'\{12.66547 + 36.28318 e^2 - 27.81881 e'^2 \\
&\qquad\qquad - 3.390928\gamma^2\} \sin (3nt - 3n't - \omega + \omega') \\
&- ee'\{24.30663 - 16.951009 e^2 - 3.038327 e'^2 \\
&\qquad\qquad - 5.210278\gamma^2\} \sin (nt - n't + \omega - \omega') \\
&- \gamma^2\{0.0271238 - 2.462713 e^2 - 0.0678085 e'^2 \\
&\qquad\qquad + 0.0096641 6\gamma^2\} \sin (4nt - 2n't - 2\Omega)
\end{aligned}\right\} . \quad (274)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&-\gamma^2\{0.4983562-7.306115e^2-1.245890e'^2\\
&\qquad\qquad\qquad-0.5942523\gamma^2\}\sin 2(n't-\Omega)\\
&+\gamma^2\{0.595675-2.414350e^2-1.489189e'^2\\
&\qquad\qquad\qquad-0.5083229\gamma^2\}\sin(2nt-2n't+2\omega-2\Omega)\\
&-\gamma^2\{0.595675+0.34540e^2-1.489189e'^2\\
&\qquad\qquad\qquad-0.807709\gamma^2\}\sin(2nt-2n't-2\omega+2\Omega)\\
&+e^3(5.363674)\sin(5nt-2n't-3\omega)+e^3(3.723395)\sin(nt+2n't-3\omega)\\
&+e'^3(0.03853108)\sin(2nt+n't-3\omega')\\
&+e'^3(58.24657)\sin(2nt-5n't+3\omega')\\
&+e\gamma^2(1.061268)\sin(3nt-2n't+\omega-2\Omega)\\
&-e\gamma^2(4.094615)\sin(nt-2n't-\omega+2\Omega)\\
&-e\gamma^2(0.8325049)\sin(3nt-2n't-3\omega+2\Omega)\\
&+e\gamma^2(5.843356)\sin(nt-2n't+3\omega-2\Omega)\\
&-e\gamma^2(8.592393)\sin(nt+2n't-\omega-2\Omega)\\
&-e\gamma^2(0.1021431)\sin(5nt-2n't-\omega-2\Omega)\\
&+e'\gamma^2(2.301935)\sin(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-e'\gamma^2(2.301935)\sin(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+e'\gamma^2(0.2719396)\sin(2nt-n't-2\omega-\omega'+2\Omega)\\
&-e'\gamma^2(0.2719396)\sin(2nt-n't+2\omega-\omega'-2\Omega)\\
&-e'\gamma^2(2.002395)\sin(3n't-\omega'-2\Omega)+e'\gamma^2(0.2167634)\sin(n't+\omega'-2\Omega)\\
&+e'\gamma^2(0.01301434)\sin(4nt-n't-\omega'-2\Omega)\\
&-e'\gamma^2(0.0990182)\sin(4nt-3n't+\omega'-2\Omega)\\
&+ee'^2(88.1261)\sin(nt-4n't+\omega+2\omega')\\
&+ee'^2(33.70215)\sin(3nt-4n't-\omega+2\omega')\\
&-e^2e'(1.935002)\sin(4nt-n't-2\omega-\omega')\\
&+e^2e'(66.30037)\sin(3n't-2\omega-\omega')-e^2e'(26.04620)\sin(n't-2\omega+\omega')\\
&+e^2e'(15.70726)\sin(4nt-3n't-2\omega+\omega')\\
&+e^4(6.561815)\sin(6nt-2n't-4\omega)+e^4(3.262481)\sin(2nt+2n't-4\omega)\\
&+e'^4(0.07162802)\sin(2nt+2n't-4\omega')\\
&+e'^4(126.1904)\sin(2nt-6n't+4\omega')\\
&+\gamma^4(0.1764083)\sin(2nt-2n't+4\omega-4\Omega)\\
&-\gamma^4(0.00205562)\sin(2nt-2n't-4\omega+4\Omega)\\
&+\gamma^4(0.1139151)\sin(2nt+2n't-4\Omega)\\
&-\gamma^4(0.01356185)\sin(4nt-2n't+2\omega-4\Omega)\\
&+\gamma^4(0.00493759)\sin(6nt-2n't-4\Omega)\\
&-e^2\gamma^2(1.0167605)\sin(4nt-2n't-4\omega+2\Omega)
\end{aligned}
\right\} \quad (274)
$$

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$$
\left.\begin{aligned}
&+ e^2\gamma^2 (6.699394) \sin (2n't - 4\omega + 2\Omega) \\
&- e^2\gamma^2 (8.279404) \sin (2nt + 2n't - 2\omega - 2\Omega) \\
&- e^2\gamma^2 (0.2385537) \sin (6nt - 2n't - 2\omega - 2\Omega) \\
&+ ee'^3 (0.05608693) \sin (3nt + n't - \omega - 3\omega') \\
&- ee'^3 (1.085597) \sin (nt + n't + \omega - 3\omega') \\
&+ ee'^3 (64.12063) \sin (3nt - 5n't - \omega + 3\omega') \\
&+ ee'^3 (132.5671) \sin (nt - 5n't + \omega + 3\omega') \\
&+ e^3e' (10.885907) \sin (nt + 3n't - 3\omega - \omega') \\
&- e^3e' (3.360318) \sin (nt + n't - 3\omega + \omega') \\
&- e^3e' (2.398864) \sin (5nt - n't - 3\omega - \omega') \\
&+ e^3e' (25.04946) \sin (5nt - 3n't - 3\omega + \omega') \\
&+ e^2e'^2 (41.53619) \sin (4nt - 4n't - 2\omega + 2\omega') \\
&+ e^2e'^2 (127.2527) \sin (4n't - 2\omega - 2\omega') \\
&+ e'^2\gamma^2 (6.232506) \sin (2nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&- e'^2\gamma^2 (6.232506) \sin (2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- e'^2\gamma^2 (5.595899) \sin (4n't - 2\omega' - 2\Omega) \\
&- e'^2\gamma^2 (0.251066) \sin (4nt - 4n't + 2\omega' - 2\Omega) \\
&+ ee'\gamma^2 (3.993677) \sin (3nt - 3n't + \omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (4.116294) \sin (nt - n't - \omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.4939654) \sin (3nt - n't + \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (11.118806) \sin (nt - 3n't - \omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (3.166367) \sin (3nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (6.076655) \sin (nt - n't + 3\omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.3856623) \sin (3nt - n't - 3\omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 (14.367764) \sin (nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (8.190578) \sin (nt + n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (16.46930) \sin (nt + 3n't - \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.942805) \sin (5nt - n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.3689357) \sin (5nt - 3n't - \omega + \omega' - 2\Omega) \\
&- h'\kappa' \frac{n}{n^2 - \kappa'^2} \sin (\kappa't - \beta') \Big\} \\
+ a\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{ &- \{12.43582 + 4.123261e^2 + 24.87164e'^2 \\
&\qquad - 55.04355\gamma^2\} \sin (nt - n't) \\
&+ \{1.335670 - 19.84814e^2 - 8.01402e'^2 - 1.001752\gamma^2\} \sin 3(nt - n't) \\
&- e(13.44852) \sin (2nt - n't - \omega) - e(12.88077) \sin (n't - \omega)
\end{aligned}\right\} . \quad (274)
$$

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$$\left.\begin{aligned}
&+ e\,(2.523682)\sin(4nt-3n't-\omega) - e\,(11.79544)\sin(2nt-3n't+\omega)\\
&- e'\,(18.49582)\sin(nt-2n't+\omega') + e'\,(0.093750)\sin(nt-\omega')\\
&+ e'\,(7.001572)\sin(3nt-4n't+\omega')\\
&- e'\,(1.276463)\sin(3nt-2n't-\omega')\\
&- e^2\,(15.29640)\sin(3nt-n't-2\omega) + e^2\,(9.302104)\sin(nt+n't-2\omega)\\
&+ e^2(3.637311)\sin(5nt-3n't-2\omega)\\
&- e^2\,(28.72292)\sin(nt-3n't+2\omega)\\
&+ e'^2\,(17.35742)\sin(nt+n't-2\omega')\\
&- e'^2\,(26.97790)\sin(nt-3n't+2\omega')\\
&+ e'^2\,(0.1527558)\sin(3nt-n't-2\omega')\\
&+ e'^2\,(23.35469)\sin(3nt-5n't+2\omega')\\
&- ee'\,(1.406250)\sin(\omega-\omega')\\
&- ee'\,(19.47985)\sin(2n't-\omega-\omega')\\
&- ee'\,(21.78434)\sin(2nt-2n't-\omega+\omega')\\
&+ ee'\,(13.116482)\sin(4nt-4n't-\omega+\omega')\\
&+ ee'\,(10.75640)\sin(2nt-2n't+\omega-\omega')\\
&- ee'\,(2.431372)\sin(4nt-2n't-\omega-\omega')\\
&- ee'\,(65.26854)\sin(2nt-4n't+\omega+\omega')\\
&- \gamma^2\,(1.554476)\sin(nt-n't+2\omega-2\Omega)\\
&+ \gamma^2\,(0.252459)\sin(nt-n't-2\omega+2\Omega)\\
&+ \gamma^2\,(1.102626)\sin(3nt-n't-2\Omega)\\
&+ \gamma^2\,(9.874348)\sin(nt+n't-2\Omega)\\
&- \gamma^2\,(5.095170)\sin(nt-3n't+2\Omega)\\
&+ \gamma^2\,(0.498376)\sin(3nt-3n't+2\omega-2\Omega)\\
&+ \gamma^2\,(0.498376)\sin(3nt-3nt-2\omega+2\Omega)\\
&\left.+ \tfrac{3}{16}h''n\,\frac{10n+12\alpha''}{n^2-\alpha''^2}\sin(\alpha''t-\beta'')\right\}\\
&+ \tfrac{15}{16}\,a\,\frac{\overline{m}^2}{\mu}\,\frac{a}{a'}\,n\cos nt\int e'\cos\omega'\,dt + \tfrac{15}{16}\,a\,\frac{\overline{m}^2}{\mu}\,\frac{a}{a'}\,n\sin nt\int e'\sin\omega'\,dt
\end{aligned}\right\} \quad . \quad (274)$$

This equation completes the perturbations of the radius vector of the moon's orbit, in so far as it depends on the sun's direct action; and we shall now determine the perturbations of the longitude and latitude depending on the same cause.

21. The perturbations of the moon's longitude arising from the sun's direct action is given by equation (261). We have given the value of $\int\left(\frac{dR}{dv}\right)dt$ in equation (265); and the value of $\frac{1}{r_1^2\cos^2\theta_1}$ is given by the following equation:

$$\left.\begin{aligned}\frac{a^2}{r_1^2\cos^2\theta_1} &= 1+\tfrac{1}{2}e^2+\tfrac{1}{2}\gamma^2+\tfrac{3}{8}e^4-\tfrac{1}{8}\gamma^4+\tfrac{1}{4}e^2\gamma^2+2e\,\{1+\tfrac{3}{8}e^2+\tfrac{1}{2}\gamma^2\}\cos(nt-\omega)\\ &+e^2\,\{\tfrac{5}{2}+\tfrac{1}{3}e^2+\tfrac{5}{4}\gamma^2+\tfrac{1}{8}\frac{\gamma^4}{e^2}\}\cos 2(nt-\omega)+\tfrac{13}{4}e^3\cos 3(nt-\omega)\\ &+\tfrac{103}{24}e^4\cos 4(nt-\omega)-\tfrac{1}{2}\gamma^2\,\{1-\tfrac{7}{2}e^2\}\cos 2(nt-\Omega)\\ &-\tfrac{3}{2}e\gamma^2\cos(3nt-\omega-2\Omega)+\tfrac{1}{2}e\gamma^2\cos(nt+\omega-2\Omega)\\ &-\tfrac{13}{4}e^2\gamma^2\cos(4nt-2\omega-2\Omega)-\tfrac{1}{8}\gamma^4\cos(2nt+2\omega-4\Omega)\\ &+\tfrac{1}{8}\gamma^4\cos 4(nt-\Omega)\end{aligned}\right\} \quad (275)$$

If we now multiply equations (265) and (275) together, we shall obtain the following development of

$$\left.\begin{aligned}\frac{d\delta_0 v}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{ &+\{0.8106367-6.386674e^2-2.026592e'^2+0.2026592\gamma^2\\ &\quad-30.56173e^4+0.6586422e'^4+0.8128361\gamma^4+15.96668e^2e'^2\\ &\quad-1.596668e^2\gamma^2-0.5066469e'^2\gamma^2\}\cos 2(nt-n't)\\ &+e\,\{1.3368791-8.935187e^2-3.342198e'^2\\ &\qquad\qquad+0.3342198\gamma^2\}\cos(3nt-2n't-\omega)\\ &-e\,\{4.481006+25.91073e^2-11.20252e'^2\\ &\qquad\qquad+1.120252\gamma^2\}\cos(nt-2n't+\omega)\\ &-e'\,\{0.3895702-2.952866e^2-0.0485843e'^2\\ &\qquad\qquad+0.0971686\gamma^2\}\cos(2nt-n't-\omega')\\ &+e'\,\{2.956754-24.32890e^2-6.494299e'^2\\ &\qquad\qquad+0.7391885\gamma^2\}\cos(2nt-3n't+\omega')\\ &+e^2\,\{1.928213-12.505748e^2-4.82053e'^2+0.4820532\gamma^2\\ &\qquad\qquad+0.101330\frac{\gamma^4}{e^2}\}\cos(4nt-2n't-2\omega)\\ &-e^2\,\{29.34475+12.81368e^2-73.36189e'^2+7.336189\gamma^2\\ &\qquad\qquad+0.626660\frac{\gamma^4}{e^2}\}\cos 2(n't-\omega)\\ &e'^2\,\{7.496496-64.85304e^2-16.90386e'^2\\ &\qquad\qquad+1.874124\gamma^2\}\cos(2nt-4n't+2\omega')\end{aligned}\right\} \quad (276)$$

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$$
\left.\begin{aligned}
&-\gamma^2\{0.2513555-6.693206e^2-0.628389e'^2\\
&\qquad\qquad\qquad-0.0628389\gamma^2\}\cos(4nt-2n't-2\Omega)\\
&-\gamma^2\{3.962620+0.857528e^2-9.90655e'^2-0.990655\gamma^2\}\cos 2(n't-\Omega)\\
&+\gamma^2\{0.2026592+0.397738e^2-0.506648e'^2\\
&\qquad\qquad\qquad-0.0506648\gamma^2\}\cos(2nt-2n't+2\omega-2\Omega)\\
&-\gamma^2\{0.2026592+2.389549e^2-0.506648e'^2\\
&\qquad\qquad\qquad-0.0506648\gamma^2\}\cos(2nt-2n't-2\omega+2\Omega)\\
&-ee'\{0.6450671-4.1554938e^2-0.0806334e'^2\\
&\qquad\qquad\qquad+0.1612668\gamma^2\}\cos(3nt-n't-\omega-\omega')\\
&+ee'\{2.043236+25.47053e^2-0.2554044e'^2\\
&\qquad\qquad\qquad+0.5108089\gamma^2\}\cos(nt-n't+\omega-\omega')\\
&-ee'\{17.35021+61.56078e^2-37.56858e'^2\\
&\qquad\qquad\qquad+4.337755\gamma^2\}\cos(nt-3n't+\omega+\omega')\\
&+ee'\{4.848327-33.82299e^2-10.648905e'^2\\
&\qquad\qquad\qquad+1.2120081\gamma^2\}\cos(3nt-3n't-\omega+\omega')\\
&+e^3(2.6859003)\cos(5nt-2n't-3\omega)\\
&-e^3(29.98311)\cos(nt+2n't-3\omega)\\
&+e'^3(0.01506168)\cos(2nt+n't-3\omega')\\
&+e'^3(16.24007)\cos(2nt-5n't+3\omega')\\
&-e^2e'(0.933309)\cos(4nt-n't-2\omega-\omega')\\
&-e^2e'(75.09929)\cos(3n't-2\omega-\omega')\\
&+e^2e'(6.978024)\cos(4nt-3n't-2\omega+\omega')\\
&+e^2e'(27.01249)\cos(n't-2\omega+\omega')\\
&+ee'^2(12.217329)\cos(3nt-4n't-\omega+2\omega')\\
&-ee'^2(47.08438)\cos(nt-4n't+\omega+2\omega')\\
&+e\gamma^2(2.151995)\cos(3nt-2n't+\omega-2\Omega)\\
&-e\gamma^2(3.230067)\cos(nt-2n't-\omega+2\Omega)\\
&-e\gamma^2(0.9042042)\cos(5nt-2n't-\omega-2\Omega)\\
&-e\gamma^2(2.555728)\cos(nt+2n't-\omega-2\Omega)\\
&-e\gamma^2(0.3342198)\cos(3nt-2n't-3\omega+2\Omega)\\
&-e\gamma^2(1.120252)\cos(nt-2n't+3\omega-2\Omega)\\
&+e'\gamma^2(0.1210527)\cos(4nt-n't-\omega'-2\Omega)\\
&-e'\gamma^2(9.512430)\cos(3n't-\omega'-2\Omega)\\
&-e'\gamma^2(0.9130021)\cos(4nt-3n't+\omega'-2\Omega)\\
&+e'\gamma^2(3.857130)\cos(n't+\omega'-2\Omega)\\
&+e'\gamma^2(0.09739255)\cos(2nt-n't-2\omega-\omega'+2\Omega)
\end{aligned}\right\}\;.\quad(276)
$$

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$$\left.\begin{aligned}
&+ e'\gamma^2 (0.7391886) \cos (2nt - 3n't + 2\omega + \omega' - 2\Omega)\\
&- e'\gamma^2 (0.09739255) \cos (2nt - n't + 2\omega - \omega' - 2\Omega)\\
&- e'\gamma^2 (0.7391886) \cos (2nt - 3n't - 2\omega + \omega' + 2\Omega)\\
&+ e^4 (3.691058) \cos (6nt - 2n't - 4\omega) - e^4 (37.76826) \cos (2nt + 2n't - 4\omega)\\
&+ e'^4 (0.02907514) \cos (2nt + 2n't - 4\omega')\\
&+ e'^4 (32.21313) \cos (2nt - 6n't + 4\omega')\\
&+ \gamma^4 (0.06884808) \cos (6nt - 2n't - 4\Omega)\\
&+ \gamma^4 (0.9742973) \cos (2nt + 2n't - 4\Omega)\\
&- \gamma^4 (0.1256777) \cos (4nt - 2n't + 2\omega - 4\Omega)\\
&+ \gamma^4 (0.0506648) \cos (2nt - 2n't + 4\omega - 4\Omega)\\
&- e^2\gamma^2 (0.482267) \cos (4nt - 2n't - 4\omega + 2\Omega)\\
&- e^2\gamma^2 (7.336190) \cos (2n't - 4\omega + 2\Omega)\\
&- e^2\gamma^2 (2.186283) \cos (6nt - 2n't - 2\omega - 2\Omega)\\
&+ e^2\gamma^2 (4.762816) \cos (2nt + 2n't - 2\omega - 2\Omega)\\
&- e'^2\gamma^2 (17.85396) \cos (4n't - 2\omega' - 2\Omega)\\
&- e'^2\gamma^2 (2.304775) \cos (4nt - 4n't + 2\omega' - 2\Omega)\\
&+ e'^2\gamma^2 (1.874124) \cos (2nt - 4n't + 2\omega + 2\omega' - 2\Omega)\\
&- e'^2\gamma^2 (1.874124) \cos (2nt - 4n't - 2\omega + 2\omega' + 2\Omega)\\
&+ e^2e'^2 (17.53666) \cos (4nt - 4n't - 2\omega + 2\omega')\\
&- e^2e'^2 (151.7428) \cos (4n't - 2\omega - 2\omega')\\
&+ ee'^3 (0.02522494) \cos (3nt + n't - \omega - 3\omega')\\
&- ee'^3 (0.08571925) \cos (nt + n't + \omega - 3\omega')\\
&- ee'^3 (110.3088) \cos (nt - 5n't + \omega + 3\omega')\\
&+ ee'^3 (26.29579) \cos (3nt - 5n't - \omega + 3\omega')\\
&- e^3e' (77.81664) \cos (nt + 3n't - 3\omega - \omega')\\
&+ e^3e' (27.27118) \cos (nt + n't - 3\omega + \omega')\\
&- e^3e' (1.301783) \cos (5nt - n't - 3\omega - \omega')\\
&+ e^3e' (9.704845) \cos (5nt - 3n't - 3\omega + \omega')\\
&- ee'\gamma^2 (3.276525) \cos (5nt - 3n't - \omega + \omega' - 2\Omega)\\
&+ ee'\gamma^2 (8.037391) \cos (3nt - 3n't + \omega + \omega' - 2\Omega)\\
&+ ee'\gamma^2 (3.520293) \cos (nt - n't - \omega - \omega' + 2\Omega)\\
&+ ee'\gamma^2 (1.0457048) \cos (5nt - n't - \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2 (1.855970) \cos (nt + 3n't - \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2 (1.0029993) \cos (3nt - n't + \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2 (6.345140) \cos (nt - 3n't - \omega + \omega' + 2\Omega)\\
&- ee'\gamma^2 (1.2120599) \cos (3nt - 3n't - 3\omega + \omega' + 2\Omega)
\end{aligned}\right\} . \quad (276)$$

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$$
\left.
\begin{aligned}
&+ ee'\gamma^2 (0.5105851) \cos (nt - n't + 3\omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.16149075) \cos (3nt - n't - 3\omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (4.700177) \cos (nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (3.181814) \cos (nt + n't - \omega + \omega' - 2\Omega) \Big\} \\
&+\frac{\overline{m}^2}{\mu}\frac{a}{a'} n \Big\{ + \{0.4053184 + 13.44911e^2 + 0.81063 68e'^2 \\
&\qquad\qquad - 2.4319104\gamma^2\} \cos (nt - n't) \\
&+ \{0.6755306 - 7.722429e^2 - 4.0531836e'^2 - 0.1688826\gamma^2\} \cos 3 (nt - n't) \\
&+ e (0.3079259) \cos (2nt - n't - \omega) + e (12.93853) \cos (n't - \omega) \\
&+ e (1.420446) \cos (4nt - 3n't - \omega) - e (4.076396) \cos (2nt - 3n't + \omega) \\
&+ \tfrac{3}{8}e' \cos (nt - \omega') + e' (1.322911) \cos (nt - 2n't + \omega') \\
&+ e' (3.471200) \cos (3nt - 4n't + \omega') - e' (0.657803) \cos (3nt - 2n't - \omega') \\
&+ e^2 (0.216961) \cos (3nt - n't - 2\omega) + e^2 (12.560118) \cos (nt + n't - 2\omega) \\
&+ e^2 (2.3254937) \cos (5nt - 3n't - 2\omega) \\
&+ e^2 (13.31713) \cos (nt - 3n't + 2\omega) + e'^2 (0.479740) \cos (nt + n't - 2\omega') \\
&+ e'^2 (3.203182) \cos (nt - 3n't + 2\omega') \\
&+ e'^2 (0.08012273) \cos (3nt - n't - 2\omega') \\
&+ e'^2 (11.33500) \cos (3nt - 5n't + 2\omega') \\
&+ ee' (0.281250) \cos (2nt - \omega - \omega') + ee' (0.375) \cos (\omega - \omega') \\
&- \tfrac{15}{16} h'' \frac{n}{\alpha''} \cos (\alpha'' t - \beta'') + ee' (1.018922) \cos (2nt - 2n't - \omega + \omega') \\
&+ ee' (20.12272) \cos (2n't - \omega - \omega') \\
&+ ee' (7.271060) \cos (4nt - 4n't - \omega + \omega') \\
&- ee' (21.33339) \cos (2nt - 4n't + \omega + \omega') \\
&- ee' (1.3882472) \cos (4nt - 2n't - \omega - \omega') \\
&+ ee' (3.902028) \cos (2nt - 2n't + \omega - \omega') \\
&- \gamma^2 (1.177543) \cos (nt + n't - 2\Omega) + \gamma^2 (0.4006139) \cos (3nt - n't - 2\Omega) \\
&+ \gamma^2 (0.0506648) \cos (nt - n't - 2\omega + 2\Omega) \\
&+ \gamma^2 (0.0506648) \cos (nt - n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (1.813121) \cos (nt - 3n't + 2\Omega) \\
&+ \gamma^2 (0.2533240) \cos (3nt - 3n't + 2\omega - 2\Omega) \\
&- \gamma^2 (0.2533240) \cos (3nt - 3n't - 2\omega + 2\Omega) \Big\}
\end{aligned}
\right\} . \quad (276)
$$

22. We shall now develop equation (263), which gives the part of the perturbations in latitude arising from the sun's direct action. If we multiply equations (219) and (241) together, we shall obtain

$$\tan\theta_1 \sin v_1 \left(\frac{dR}{dv}\right) =$$

$$\frac{\overline{m}^2}{a}\left\{\begin{aligned}
&+\gamma\{\tfrac{3}{8}-\tfrac{15}{16}e^2-\tfrac{15}{16}e'^2-\tfrac{21}{64}\gamma^2\}\sin(2nt-2n't+\Omega)\\
&\pm\gamma\{\tfrac{3}{8}-\tfrac{15}{16}e^2-\tfrac{15}{16}e'^2-\tfrac{9}{64}\gamma^2\}\sin(2nt-2n't-\Omega)\\
&-\gamma\{\tfrac{3}{8}-\tfrac{87}{16}e^2-\tfrac{15}{16}e'^2-\tfrac{3}{64}\gamma^2\}\sin(4nt-2n't-\Omega)\\
&+\gamma\{\tfrac{3}{8}+\tfrac{9}{16}e^2-\tfrac{15}{16}e'^2-\tfrac{15}{64}\gamma^2\}\sin(2n't-\Omega)\\
&+e\gamma\{\tfrac{3}{8}-\tfrac{57}{64}e^2-\tfrac{15}{16}e'^2-\tfrac{81}{64}\gamma^2\}\sin(3nt-2n't-\omega+\Omega)\\
&\pm e\gamma\{\tfrac{3}{8}-\tfrac{57}{64}e^2-\tfrac{15}{16}e'^2-\tfrac{9}{64}\gamma^2\}\sin(3nt-2n't-\omega-\Omega)\\
&-e\gamma\{\tfrac{9}{8}-\tfrac{39}{64}e^2-\tfrac{9}{32}e'^2-\tfrac{63}{64}\gamma^2\}\sin(nt-2n't+\omega+\Omega)\\
&\mp e\gamma\{\tfrac{9}{8}-\tfrac{39}{64}e^2-\tfrac{9}{32}e'^2-\tfrac{27}{64}\gamma^2\}\sin(nt-2n't+\omega-\Omega)\\
&-e\gamma\{\tfrac{9}{8}-\tfrac{781}{64}e^2-\tfrac{45}{16}e'^2-\tfrac{9}{64}\gamma^2\}\sin(5nt-2n't-\omega-\Omega)\\
&\pm e\gamma\{\tfrac{3}{8}-\tfrac{3}{64}\gamma^2-\tfrac{15}{16}e'^2+\tfrac{3}{64}\gamma^2\}\sin(nt-2n't-\omega+\Omega)\\
&+e\gamma\{\tfrac{15}{8}-\tfrac{627}{64}e^2-\tfrac{75}{16}e'^2-\tfrac{9}{64}\gamma^2\}\sin(3nt-2n't+\omega-\Omega)\\
&-e\gamma\{\tfrac{3}{8}-\tfrac{3}{64}e^2-\tfrac{15}{16}e'^2-\tfrac{15}{64}\gamma^2\}\sin(nt+2n't-\omega-\Omega)\\
&-e'\gamma\{\tfrac{3}{16}-\tfrac{15}{32}e^2-\tfrac{3}{128}e'^2-\tfrac{21}{128}\gamma^2\}\sin(2nt-n't-\omega'+\Omega)\\
&\mp e'\gamma\{\tfrac{3}{16}-\tfrac{15}{32}e^2-\tfrac{3}{128}e'^2-\tfrac{9}{128}\gamma^2\}\sin(2nt-n't-\omega'-\Omega)\\
&+e'\gamma\{\tfrac{21}{16}-\tfrac{105}{32}e^2-\tfrac{369}{128}e'^2-\tfrac{147}{128}\gamma^2\}\sin(2nt-3n't+\omega'+\Omega)\\
&\pm e'\gamma\{\tfrac{21}{16}-\tfrac{105}{32}e^2-\tfrac{369}{128}e'^2-\tfrac{63}{128}\gamma^2\}\sin(2nt-3n't+\omega'-\Omega)\\
&+e'\gamma\{\tfrac{3}{16}-\tfrac{87}{32}e^2-\tfrac{3}{128}e'^2-\tfrac{3}{128}\gamma^2\}\sin(4nt-n't-\omega'-\Omega)\\
&-e'\gamma\{\tfrac{3}{16}+\tfrac{3}{32}e^2-\tfrac{3}{128}e'^2-\tfrac{15}{128}\gamma^2\}\sin(n't+\omega'-\Omega)\\
&-e'\gamma\{\tfrac{21}{16}-\tfrac{609}{32}e^2-\tfrac{369}{128}e'^2-\tfrac{21}{128}\gamma^2\}\sin(4nt-3n't+\omega'-\Omega)\\
&+e'\gamma\{\tfrac{21}{16}+\tfrac{63}{32}e^2-\tfrac{369}{128}e'^2-\tfrac{105}{128}\gamma^2\}\sin(3n't-\omega'-\Omega)\\
&+\gamma\{\tfrac{3}{8}e^2+\tfrac{3}{16}\gamma^2\}\sin(4nt-2n't-2\omega+\Omega)\\
&\pm\tfrac{3}{8}e^2\gamma\sin(4nt-2n't-2\omega-\Omega)-\tfrac{15}{16}e^2\gamma\sin(2n't-2\omega+\Omega)\\
&\mp\tfrac{15}{16}e^2\gamma\sin(2n't-2\omega-\Omega)-\tfrac{75}{32}e^2\gamma\sin(6nt-2n't-2\omega-\Omega)\\
&\pm\tfrac{3}{32}\gamma\{e^2-\gamma^2\}\sin(2nt-2n't-2\omega+\Omega)\\
&-\tfrac{3}{32}e^2\gamma\sin(2nt+2n't-2\omega-\Omega)\\
&-\gamma\{\tfrac{111}{32}e^2-\tfrac{3}{32}\gamma^2\}\sin(2nt-2n't+2\omega-\Omega)\\
&+\tfrac{51}{16}e'^2\gamma\sin(2nt-4n't+2\omega'+\Omega)\pm\tfrac{51}{16}e'^2\gamma\sin(2nt-4n't+2\omega'-\Omega)\\
&-\tfrac{51}{16}e'^2\gamma\sin(4nt-4n't+2\omega'-\Omega)+\tfrac{51}{16}e'^2\gamma\sin(4n't-2\omega'-\Omega)\\
&\mp\tfrac{3}{64}\gamma^3\sin(4nt-2n't-3\Omega)\mp\tfrac{9}{64}\gamma^3(2n't-3\Omega)\\
&\pm\tfrac{3}{32}\gamma^3\sin(2nt-2n't+2\omega-3\Omega)-\tfrac{3}{32}\gamma^3\sin(2nt-2n't-2\omega+3\Omega)
\end{aligned}\right\} \quad . \tag{277}$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+\tfrac{9}{64}\gamma^3\sin(6nt-2n't-3\Omega)+\tfrac{3}{64}\gamma^3\sin(2nt+2n't-3\Omega)\\
&-\tfrac{3}{16}\gamma^3\sin(4nt-2n't+2\omega-3\Omega)-\tfrac{3}{16}ee'\gamma\sin(3nt-n't-\omega-\omega'+\Omega)\\
&\mp\tfrac{3}{16}ee'\gamma\sin(3nt-n't-\omega-\omega'-\Omega)\\
&-\tfrac{63}{16}ee'\gamma\sin(nt-3n't+\omega+\omega'+\Omega)\\
&\mp\tfrac{63}{16}ee'\gamma\sin(nt-3n't+\omega+\omega'-\Omega)\\
&+\tfrac{21}{16}ee'\gamma\sin(3nt-3n't-\omega+\omega'+\Omega)\\
&\pm\tfrac{21}{16}ee'\gamma\sin(3nt-3n't-\omega+\omega'-\Omega)\\
&+\tfrac{9}{16}ee'\gamma\sin(nt-n't+\omega-\omega'+\Omega)\\
&\pm\tfrac{9}{16}ee'\gamma\sin(nt-n't+\omega-\omega'-\Omega)\\
&+\tfrac{9}{16}ee'\gamma\sin(5nt-n't-\omega-\omega'-\Omega)\\
&\mp\tfrac{3}{16}ee'\gamma\sin(nt-n't-\omega-\omega'+\Omega)\\
&+\tfrac{105}{16}ee'\gamma\sin(3nt-3n't+\omega+\omega'-\Omega)\\
&-\tfrac{21}{16}ee'\gamma\sin(nt+3n't-\omega-\omega'-\Omega)\\
&-\tfrac{63}{16}ee'\gamma\sin(5nt-3n't-\omega+\omega'-\Omega)\\
&\pm\tfrac{21}{16}ee'\gamma\sin(nt-3n't-\omega+\omega'+\Omega)\\
&-\tfrac{15}{16}ee'\gamma\sin(3nt-n't+\omega-\omega'-\Omega)\\
&+\tfrac{3}{16}ee'\gamma\sin(nt+n't-\omega+\omega'-\Omega)\\
&+\tfrac{9}{64}e\gamma^3\sin(nt-2n't-\omega+3\Omega)\mp\tfrac{9}{64}e\gamma^3\sin(5nt-2n't-\omega-3\Omega)\\
&-\tfrac{3}{32}e\gamma^3\sin(3nt-2n't-3\omega+3\Omega)\\
&\pm\tfrac{3}{64}e\gamma\{e^2-2\gamma^2\}\sin(3nt-2n't-3\omega+\Omega)\\
&+e\gamma\{\tfrac{179}{64}e^2-\tfrac{9}{32}\gamma^2\}\sin(nt-2n't+3\omega-\Omega)\\
&\mp\tfrac{9}{32}e\gamma^3\sin(nt-2n't+3\omega-3\Omega)\pm\tfrac{21}{64}e\gamma^3\sin(3nt-2n't+\omega-3\Omega)\\
&\pm\tfrac{9}{64}e\gamma^3\sin(nt+2n't-\omega-3\Omega)-\tfrac{9}{64}e\gamma^3\sin(nt+2n't+\omega-3\Omega)\\
&+\tfrac{45}{64}e\gamma^3\sin(7nt-2n't-\omega-3\Omega)\\
&+e\gamma\{\tfrac{9}{16}\gamma^2+\tfrac{25}{64}e^2\}\sin(5nt-2n't-3\omega+\Omega)\\
&+\tfrac{15}{16}e\gamma^3\sin(3nt-2n't+3\omega-3\Omega)+\tfrac{7}{64}e^3\gamma\sin(nt+2n't-3\omega+\Omega)\\
&-\tfrac{99}{64}e\gamma^3\sin(5nt-2n't+\omega-3\Omega)+\tfrac{3}{64}e\gamma^3\sin(3nt+2n't-\omega-3\Omega)\\
&\pm\tfrac{25}{64}e^3\gamma\sin(5nt-2n't-3\omega-\Omega)\pm\tfrac{7}{64}e^3\gamma\sin(nt+2n't-3\omega-\Omega)\\
&-\tfrac{269}{64}e^3\gamma\sin(7nt-2n't-3\omega-\Omega)-\tfrac{3}{64}e^3\gamma\sin(3nt+2n't-3\omega-\Omega)\\
&\pm\tfrac{3}{128}e'\gamma^3\sin(4nt-n't-\omega'-3\Omega)\mp\tfrac{63}{128}e'\gamma^3\sin(3n't-\omega'-3\Omega)\\
&\mp\tfrac{21}{128}e'\gamma^3\sin(4nt-3n't+\omega'-3\Omega)\pm\tfrac{9}{128}e'\gamma^3\sin(n't+\omega'-3\Omega)\\
&+\tfrac{3}{64}e'\gamma^3\sin(2nt-n't-2\omega-\omega'+3\Omega)\\
&\pm\tfrac{21}{64}e'\gamma^3\sin(2nt-3n't+2\omega+\omega'-3\Omega)\\
&\mp\tfrac{3}{64}e'\gamma^3\sin(2nt-n't+2\omega-\omega'-3\Omega)\\
&-\tfrac{21}{64}e'\gamma^3\sin(2nt-3n't-2\omega+\omega'+3\Omega)\\
&-\tfrac{9}{128}e'\gamma^3\sin(6nt-n't-\omega'-3\Omega)+\tfrac{21}{128}e'\gamma^3\sin(2nt+3n't-\omega'-3\Omega)
\end{aligned}\right\}.\quad(277)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+ \tfrac{63}{128}e'\gamma^3 \sin(6nt - 3n't + \omega' - 3\Omega) - \tfrac{3}{128}e'\gamma^3 \sin(2nt + n't + \omega' - 3\Omega) \\
&- \tfrac{3}{16}e'\gamma\,\{e^2 + \tfrac{1}{2}\gamma^2\} \sin(4nt - n't - 2\omega - \omega' + \Omega) \\
&- \tfrac{21}{32}e'\gamma^3 \sin(4nt - 3n't + 2\omega + \omega' - 3\Omega) \\
&+ \tfrac{3}{32}e'\gamma^3 \sin(4nt - n't + 2\omega - \omega' - 3\Omega) \\
&+ \tfrac{15}{32}e^2e'\gamma \sin(n't - 2\omega + \omega' + \Omega) \\
&\mp \tfrac{3}{16}e^2e'\gamma \sin(4nt - n't - 2\omega - \omega' - \Omega) \\
&+ \tfrac{21}{16}e'\gamma\,\{e^2 + \tfrac{1}{2}\gamma^2\} \sin(4nt - 3n't - 2\omega + \omega' + \Omega) \\
&- \tfrac{105}{32}e^2e'\gamma \sin(3n't - 2\omega - \omega' + \Omega) \mp \tfrac{105}{32}e^2e'\gamma \sin(3n't - 2\omega - \omega' - \Omega) \\
&\pm \tfrac{21}{16}e^2e'\gamma \sin(4nt - 3n't - 2\omega + \omega' - \Omega) \\
&\pm \tfrac{15}{32}e^2e'\gamma \sin(n't - 2\omega + \omega' - \Omega) \\
&+ \tfrac{75}{64}e^2e'\gamma \sin(6nt - n't - 2\omega - \omega' - \Omega) \\
&\pm \tfrac{3}{64}e'\gamma\,\{\gamma^2 - e^2\} \sin(2nt - n't - 2\omega - \omega' + \Omega) \\
&- \tfrac{21}{64}e^2e'\gamma \sin(2nt + 3n't - 2\omega - \omega' - \Omega) \\
&- \tfrac{21}{64}e'\gamma\,\{37e^2 - \gamma^2\} \sin(2nt - 3n't + 2\omega + \omega' - \Omega) \\
&- \tfrac{525}{64}e^2e'\gamma \sin(6nt - 3n't - 2\omega + \omega' - \Omega) \\
&\pm \tfrac{21}{64}e'\gamma\,\{e^2 - \gamma^2\} \sin(2nt - 3n't - 2\omega + \omega' + \Omega) \\
&+ \tfrac{3}{64}e^2e'\gamma \sin(2nt + n't - 2\omega + \omega' - \Omega) \\
&+ \tfrac{3}{64}e'\gamma\,\{37e^2 - \gamma^2\} \sin(2nt - n't + 2\omega - \omega' - \Omega) \\
&- \tfrac{153}{16}ee'^2\gamma \sin(nt - 4n't + \omega + 2\omega' + \Omega) \\
&\mp \tfrac{153}{16}ee'^2\gamma \sin(nt - 4n't + \omega + 2\omega' - \Omega) \\
&+ \tfrac{51}{16}ee'^2\gamma \sin(3nt - 4n't - \omega + 2\omega' + \Omega) \\
&\pm \tfrac{51}{16}ee'^2\gamma \sin(3nt - 4n't - \omega + 2\omega' - \Omega) \\
&+ \tfrac{255}{16}ee'^2\gamma \sin(3nt - 4n't + \omega + 2\omega' - \Omega) \\
&- \tfrac{51}{16}ee'^2\gamma \sin(nt + 4n't - \omega - 2\omega' - \Omega) \\
&- \tfrac{153}{16}ee'^3\gamma \sin(5nt - 4n't - \omega + 2\omega' - \Omega) \\
&\pm \tfrac{51}{16}ee'^2\gamma \sin(nt - 4n't - \omega + 2\omega' + \Omega) \\
&+ \tfrac{1}{128}e'^3\gamma \sin(2nt + n't - 3\omega' + \Omega) \pm \tfrac{1}{128}e'^3\gamma \sin(2nt + n't - 3\omega' - \Omega) \\
&+ \tfrac{845}{128}e'^3\gamma \sin(2nt - 5n't + 3\omega' + \Omega) \\
&\pm \tfrac{845}{128}e'^3\gamma \sin(2nt - 5n't + 3\omega' - \Omega) \\
&- \tfrac{1}{128}e'^3\gamma \sin(4nt + n't - 3\omega' - \Omega) \mp \tfrac{1}{128}e'^3\gamma \sin(n't - 3\omega' + \Omega) \\
&- \tfrac{845}{128}e'^3\gamma \sin(4nt - 5n't + 3\omega' - \Omega) + \tfrac{845}{128}e'^3\gamma \sin(5nt - 3\omega' - \Omega) \Big\} \\
&+ \frac{\overline{m}^2}{a'}\Big\{ + \tfrac{3}{32}\gamma \sin(nt - n't + \Omega) \pm \tfrac{3}{32}\gamma \sin(nt - n't - \Omega) \\
&\quad - \tfrac{3}{32}\gamma \sin(3nt - n't - \Omega) + \tfrac{3}{32}\gamma \sin(nt + n't - \Omega) \\
&\quad - \tfrac{3}{64}e\gamma \sin(2nt - n't - \omega + \Omega) \mp \tfrac{3}{64}e\gamma \sin(2nt - n't - \omega - \Omega)
\end{aligned}
\right\} \quad . \quad (277)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\tfrac{15}{64}e\gamma\sin(n't-\omega+\Omega)\pm\tfrac{15}{64}e\gamma\sin(n't-\omega-\Omega)\\
&-\tfrac{9}{64}e\gamma\sin(4nt-n't-\omega-\Omega)-\tfrac{15}{64}e\gamma\sin(n't+\omega-\Omega)\\
&-\tfrac{3}{64}e\gamma\sin(2nt+n't-\omega-\Omega)+\tfrac{27}{64}e\gamma\sin(2nt-n't+\omega-\Omega)\\
&+\tfrac{3}{32}e'\gamma\sin(nt-\omega'+\Omega)\pm\tfrac{3}{32}e'\gamma\sin(nt-\omega'-\Omega)\\
&+\tfrac{9}{32}e'\gamma\sin(nt-2n't+\omega'+\Omega)\pm\tfrac{9}{32}e'\gamma\sin(nt-2n't+\omega'-\Omega)\\
&-\tfrac{3}{32}e'\gamma\sin(3nt-\omega'-\Omega)+\tfrac{3}{32}e'\gamma\sin(nt+\omega'-\Omega)\\
&-\tfrac{9}{32}e'\gamma\sin(3nt-2n't+\omega'-\Omega)+\tfrac{9}{32}e'\gamma\sin(nt+2n't-\omega'-\Omega)\\
&+\tfrac{15}{32}\gamma\sin(3nt-3n't+\Omega)\pm\tfrac{15}{32}\gamma\sin(3nt-3n't-\Omega)\\
&-\tfrac{15}{32}\gamma\sin(5nt-3n't-\Omega)\mp\tfrac{15}{32}\gamma\sin(nt-3n't+\Omega)\\
&+\tfrac{45}{64}e\gamma\sin(4nt-3n't-\omega+\Omega)\pm\tfrac{45}{64}e\gamma\sin(4nt-3n't-\omega-\Omega)\\
&-\tfrac{135}{64}e\gamma\sin(2nt-3n't+\omega+\Omega)\mp\tfrac{135}{64}e\gamma\sin(2nt-3n't+\omega-\Omega)\\
&-\tfrac{105}{64}e\gamma\sin(6nt-3n't-\omega-\Omega)\pm\tfrac{15}{64}e\gamma\sin(2nt-3n't-\omega+\Omega)\\
&+\tfrac{195}{64}e\gamma\sin(4nt-3n't+\omega-\Omega)-\tfrac{75}{64}e\gamma\sin(3n't-\omega-\Omega)\\
&+\tfrac{75}{32}e'\gamma\sin(3nt-4n't+\omega'+\Omega)\pm\tfrac{75}{32}e'\gamma\sin(3nt-4n't+\omega'-\Omega)\\
&-\tfrac{15}{32}e'\gamma\sin(3nt-2n't-\omega'+\Omega)\mp\tfrac{15}{32}e'\gamma\sin(3nt-2n't-\omega'-\Omega)\\
&-\tfrac{75}{32}e'\gamma\sin(5nt-4n't+\omega'-\Omega)\mp\tfrac{75}{32}e'\gamma\sin(nt-4n't+\omega'+\Omega)\\
&+\tfrac{15}{32}e'\gamma\sin(5nt-2n't-\omega'-\Omega)\pm\tfrac{15}{32}e'\gamma\sin(nt-2n't-\omega'+\Omega)\Big\}
\end{aligned}
\right\} \quad (277)
$$

In like manner equations (220) and (221) will give

$$\cos v_1\left(\frac{dR}{d\theta}\right)=$$

$$
\left.
\begin{aligned}
\frac{\overline{m}^2}{a}\Big\{&\pm\tfrac{3}{4}\gamma\{1-\tfrac{5}{2}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(2nt-\Omega)\\
&\mp\tfrac{3}{4}\gamma\{1+\tfrac{3}{2}e^2+\tfrac{3}{2}e'^2-\gamma^2\}\sin\Omega\mp\tfrac{3}{4}e\gamma\{1-\tfrac{1}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(nt+\omega-\Omega)\\
&\mp\tfrac{3}{4}e\gamma\{1-\tfrac{11}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{5}{4}\gamma^2\}\sin(nt-\omega+\Omega)\\
&\pm\tfrac{9}{8}e'\gamma\{1-\tfrac{5}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{3}{4}\gamma^2\}\sin(2nt+n't-\omega'-\Omega)\\
&+\tfrac{9}{8}e'\gamma\{1+\tfrac{3}{2}e^2+\tfrac{9}{8}e'^2-\gamma^2\}\sin(n't-\omega'-\Omega)\\
&\pm\tfrac{9}{8}e'\gamma\{1-\tfrac{5}{2}e^2+\tfrac{9}{8}e'^2-\tfrac{3}{4}\gamma^2\}\sin(2nt-n't+\omega'-\Omega)\\
&\mp\tfrac{9}{8}e'\gamma\{1+\tfrac{3}{2}e^2+\tfrac{9}{8}e'^2-\gamma^2\}\sin(n't-\omega'+\Omega)\\
&\pm\tfrac{3}{4}e\gamma\{1-\tfrac{19}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(3nt-\omega-\Omega)\\
&-\tfrac{3}{4}e\gamma\{1-\tfrac{1}{4}e^2+\tfrac{3}{2}e'^2-\gamma^2\}\sin(nt-\omega-\Omega)\\
&\pm\tfrac{3}{4}e^2\gamma\sin(4nt-2\omega-\Omega)-\tfrac{3}{16}e^2\gamma\sin(2nt-2\omega-\Omega)\\
&\pm\tfrac{3}{16}\gamma^3\sin(2nt+2\omega-3\Omega)+\tfrac{3}{16}\gamma^3\sin(2nt-3\Omega)\\
&\pm\tfrac{27}{16}e'^2\gamma\sin(2nt-2n't+2\omega'-\Omega)\mp\tfrac{27}{16}e'^2\gamma\sin(2n't-2\omega'+\Omega)\\
&\pm\tfrac{27}{16}e'^2\gamma\sin(2nt+2n't-2\omega'-\Omega)+\tfrac{27}{16}e'^2\gamma\sin(2n't-2\omega'-\Omega)
\end{aligned}
\right\} \quad (278)
$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
\pm \frac{15}{8}e^2\gamma \sin(2\omega - \Omega) \mp \frac{3}{16}\gamma\{e^2+\gamma^2\} \sin(2nt - 2\omega + \Omega) \\
\pm \frac{9}{8}ee'\gamma \sin(nt + n't - \omega - \omega' + \Omega) \mp \frac{27}{16}ee'\gamma \sin(nt - n't + \omega + \omega' - \Omega) \\
\mp \frac{27}{8}ee'\gamma \sin(nt + n't + \omega - \omega' - \Omega) \\
\pm \frac{9}{8}ee'\gamma \sin(nt - n't - \omega + \omega' + \Omega) \\
\pm \frac{9}{8}ee'\gamma \sin(3nt + n't - \omega - \omega' - \Omega) \\
\pm \frac{9}{8}ee'\gamma \sin(3nt - n't - \omega + \omega' - \Omega) \\
- \frac{9}{8}ee'\gamma \sin(nt + n't - \omega - \omega' - \Omega) - \frac{9}{8}ee'\gamma \sin(nt - n't - \omega + \omega' - \Omega) \\
\pm \frac{3}{16}e\gamma^3 \sin(3nt + \omega - 3\Omega) - \frac{9}{16}e\gamma^3 \sin(nt + \omega - 3\Omega) \\
\pm \frac{25}{32}e^3\gamma \sin(5nt - 3\omega - \Omega) - \frac{3}{32}e^3\gamma \sin(3nt - 3\omega - \Omega) \\
\mp \frac{3}{32}e\gamma\{2\gamma^2 - e^2\} \sin(3nt - 3\omega + \Omega) + \frac{7}{32}e^3\gamma \sin(nt - 3\omega + \Omega) \\
\pm \frac{159}{64}e'^3\gamma \sin(2nt + 3n't - 3\omega' - \Omega) + \frac{159}{64}e'^3\gamma \sin(3n't - 3\omega' - \Omega) \\
\pm \frac{159}{64}e'^3\gamma \sin(2nt - 3n't + 3\omega' - \Omega) \mp \frac{159}{64}e'^3\gamma \sin(3n't - 3\omega' + \Omega) \\
\pm \frac{27}{16}ee'^2\gamma \sin(nt + 2n't - \omega - 2\omega' + \Omega) \\
\mp \frac{81}{16}ee'^2\gamma \sin(nt - 2n't + \omega + 2\omega' - \Omega) \\
\mp \frac{81}{16}ee'^2\gamma \sin(nt + 2n't + \omega - 2\omega' - \Omega) \\
\pm \frac{27}{16}ee'^2\gamma \sin(nt - 2n't - \omega + 2\omega' + \Omega) \\
\pm \frac{27}{16}ee'^2\gamma \sin(3nt + 2n't - \omega - 2\omega' - \Omega) \\
\pm \frac{27}{16}ee'^2\gamma \sin(3nt - 2n't - \omega + 2\omega' - \Omega) \\
- \frac{27}{16}ee'^2\gamma \sin(nt + 2n't - \omega - 2\omega' - \Omega) \mp \frac{3}{2}h_{,} \sin(nt + \kappa_{,}t - \beta_{,}) \\
- \frac{27}{16}ee'^2\gamma \sin(nt - 2n't - \omega + 2\omega' - \Omega) \pm \frac{3}{2}h_{,} \sin(nt - \kappa_{,}t + \beta_{,}) \\
\pm \frac{9}{8}e^2e'\gamma \sin(4nt + n't - 2\omega - \omega' - \Omega) \\
- \frac{9}{32}e^2e'\gamma \sin(2nt + n't - 2\omega - \omega' - \Omega) \\
\pm \frac{9}{8}e^2e'\gamma \sin(4nt - n't - 2\omega + \omega' - \Omega) \\
- \frac{9}{32}e^2e'\gamma \sin(2nt - n't - 2\omega + \omega' - \Omega) \\
\mp \frac{9}{32}e'\gamma\{e^2+\gamma^2\} \sin(2nt - n't - 2\omega + \omega' + \Omega) \\
\mp \frac{9}{32}e'\gamma\{e^2+\gamma^2\} \sin(2nt + n't - 2\omega - \omega' + \Omega) \\
- \frac{45}{16}e^2e'\gamma \sin(n't - 2\omega - \omega' + \Omega) \pm \frac{45}{16}e^2e'\gamma \sin(n't + 2\omega - \omega' - \Omega) \\
+ \frac{3}{16}e\gamma^3 \sin(3nt - \omega - 3\Omega) \mp \frac{9}{16}e\gamma^3 \sin(nt + 3\omega - 3\Omega) \\
+ \frac{9}{32}e'\gamma^3 \sin(2nt + n't - \omega' - 3\Omega) + \frac{9}{32}e'\gamma^3 \sin(2nt - n't + \omega' - 3\Omega) \\
\pm \frac{9}{32}e'\gamma^3 \sin(2nt + n't + 2\omega - \omega' - 3\Omega) \\
\pm \frac{9}{32}e'\gamma^3 \sin(2nt - n't + 2\omega + \omega' - 3\Omega) \\
\pm \frac{3}{8}\gamma\{1 - \frac{29}{2}e^2 - \frac{5}{2}e'^2 - \frac{19}{8}\gamma^2\} \sin(4nt - 2n't - \Omega) \\
+ \frac{3}{8}\gamma\{1 - \frac{5}{2}e^2 - \frac{5}{2}e'^2 - \frac{21}{8}\gamma^2\} \sin(2nt - 2n't - \Omega) \\
\mp \frac{3}{8}\gamma\{1 - \frac{5}{2}e^2 - \frac{5}{2}e'^2 - \frac{5}{4}\gamma^2\} \sin(2nt - 2n't + \Omega) \\
\pm \frac{3}{8}\gamma\{1 + \frac{3}{2}e^2 - \frac{5}{2}e'^2 - \gamma^2\} \sin(2n't - \Omega) \\
\pm \frac{9}{8}e\gamma\{1 - \frac{239}{24}e^2 - \frac{5}{2}e'^2 - \frac{1}{2}\gamma^2\} \sin(5nt - 2n't - \omega - \Omega)
\end{array}
\right\} \quad . \quad (278)
$$

(*Continued on the next page.*)

$$
\left.
\begin{aligned}
&+\tfrac{3}{8}e\gamma\{1-\tfrac{19}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(3nt-2n't-\omega-\Omega)\\
&\mp\tfrac{15}{8}e\gamma\{1-\tfrac{207}{40}e^2-\tfrac{5}{2}e'^2-\tfrac{19}{40}\gamma^2\}\sin(3nt-2n't+\omega-\Omega)\\
&-\tfrac{9}{8}e\gamma\{1-\tfrac{13}{24}e^2-\tfrac{5}{2}e'^2-\tfrac{3}{4}\gamma^2\}\sin(nt-2n't+\omega-\Omega)\\
&\mp\tfrac{3}{8}e\gamma\{1-\tfrac{1}{8}e^2-\tfrac{3}{2}e'^2-\gamma^2\}\sin(nt+2n't-\omega-\Omega)\\
&\pm\tfrac{9}{8}e\gamma\{1-\tfrac{13}{24}e^2-\tfrac{5}{2}e'^2-\tfrac{5}{4}\gamma^2\}\sin(nt-2n't+\omega+\Omega)\\
&\mp\tfrac{3}{8}e\gamma\{1-\tfrac{19}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{15}{4}\gamma^2\}\sin(3nt-2n't-\omega+\Omega)\\
&+\tfrac{3}{8}e\gamma\{1+\tfrac{3}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{4}\gamma^2\}\sin(nt-2n't-\omega+\Omega)\\
&\mp\tfrac{3}{16}e'\gamma\{1-\tfrac{29}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{1}{2}\gamma^2\}\sin(4nt-n't-\omega'-\Omega)\\
&-\tfrac{3}{16}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{3}{4}\gamma^2\}\sin(2nt-n't-\omega'-\Omega)\\
&\pm\tfrac{3}{16}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{5}{4}\gamma^2\}\sin(2nt-n't-\omega'+\Omega)\\
&\mp\tfrac{3}{16}e'\gamma\{1+\tfrac{3}{2}e^2-\tfrac{1}{8}e'^2-\gamma^2\}\sin(n't-\omega'-\Omega)\\
&\pm\tfrac{21}{16}e'\gamma\{1-\tfrac{29}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{1}{2}\gamma^2\}\sin(4nt-3n't+\omega'-\Omega)\\
&+\tfrac{21}{16}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{3}{4}\gamma^2\}\sin(2nt-3n't+\omega'-\Omega)\\
&\mp\tfrac{21}{16}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{5}{4}\gamma^2\}\sin(2nt-3n't+\omega'+\Omega)\\
&\pm\tfrac{21}{16}e'\gamma\{1+\tfrac{3}{2}e^2-\tfrac{123}{56}e'^2-\gamma^2\}\sin(3n't-\omega'-\Omega)\\
&\pm\tfrac{75}{32}e^2\gamma\sin(6nt-2n't-2\omega-\Omega)+\tfrac{3}{8}e^2\gamma\sin(4nt-2n't-2\omega-\Omega)\\
&\mp\tfrac{3}{8}\gamma\{e^2+\tfrac{1}{2}\gamma^2\}\sin(4nt-2n't-2\omega+\Omega)\\
&-\tfrac{3}{32}\gamma\{\gamma^2-e^2\}\sin(2nt-2n't-2\omega+\Omega)\\
&\mp\tfrac{3}{32}e^2\gamma\sin(2nt+2n't-2\omega-\Omega)+\tfrac{15}{16}e^2\gamma\sin(2n't-2\omega-\Omega)\\
&\pm\tfrac{3}{32}\gamma\{37e^2-\gamma^2\}\sin(2nt-2n't+2\omega-\Omega)\\
&\mp\tfrac{15}{16}e^2\gamma\sin(2n't-2\omega+\Omega)\pm\tfrac{51}{16}e'^2\gamma\sin(4nt-4n't+2\omega'-\Omega)\\
&+\tfrac{51}{16}e'^2\gamma\sin(2nt-4n't+2\omega'-\Omega)\mp\tfrac{51}{16}e'^2\gamma\sin(2nt-4n't+2\omega'+\Omega)\\
&\pm\tfrac{51}{16}e'^2\gamma\sin(4n't-2\omega'-\Omega)\mp\tfrac{3}{32}\gamma^3\sin(6nt-2n't-3\Omega)\\
&\pm\tfrac{3}{32}\gamma^3\sin(2nt+2n't-3\Omega)+\tfrac{3}{16}\gamma^3\sin(2n't-3\Omega)\\
&\pm\tfrac{3}{16}\gamma^3\sin(4nt-2n't+2\omega-3\Omega)+\tfrac{3}{32}\gamma^3\sin(2nt-2n't+2\omega-3\Omega)\\
&\pm\tfrac{3}{32}\gamma^3\sin(2nt-2n't-2\omega+3\Omega)\\
&\mp\tfrac{9}{16}ee'\gamma\sin(5nt-n't-\omega-\omega'-\Omega)\\
&-\tfrac{3}{16}ee'\gamma\sin(3nt-n't-\omega-\omega'-\Omega)\\
&\mp\tfrac{105}{16}ee'\gamma\sin(3nt-3n't+\omega+\omega'-\Omega)\\
&-\tfrac{63}{16}ee'\gamma\sin(nt-3n't+\omega+\omega'-\Omega)\\
&\mp\tfrac{21}{16}ee'\gamma\sin(nt+3n't-\omega-\omega'-\Omega)\\
&\pm\tfrac{63}{16}ee'\gamma\sin(nt-3n't+\omega+\omega'+\Omega)\\
&\pm\tfrac{63}{16}ee'\gamma\sin(5nt-3n't-\omega+\omega'-\Omega)\\
&+\tfrac{21}{16}ee'\gamma\sin(3nt-3n't-\omega+\omega'-\Omega)\\
&\pm\tfrac{15}{16}ee'\gamma\sin(3nt-n't+\omega-\omega'-\Omega)\\
&+\tfrac{9}{16}ee'\gamma\sin(nt-n't+\omega-\omega'-\Omega)\pm\tfrac{3}{16}ee'\gamma\sin(nt+n't-\omega+\omega'-\Omega)
\end{aligned}
\right\}\quad(278)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp \tfrac{9}{16}ee'\gamma \sin(nt-n't+\omega-\omega'+\Omega) \pm \tfrac{3}{16}ee'\gamma \sin(3nt-n't-\omega-\omega'+\Omega) \\
&\mp \tfrac{21}{16}ee'\gamma \sin(3nt-3n't-\omega+\omega'+\Omega) \\
&- \tfrac{3}{16}ee'\gamma \sin(nt-n't-\omega-\omega'+\Omega) \\
&+ \tfrac{21}{16}ee'\gamma \sin(nt-3n't-\omega+\omega'+\Omega) \\
&\pm \tfrac{269}{64}e^3\gamma \sin(7nt-2n't-3\omega-\Omega) + \tfrac{25}{64}e^3\gamma \sin(5nt-2n't-3\omega-\Omega) \\
&\mp \tfrac{9}{16}e\gamma\{\tfrac{25}{36}e^2+\gamma^2\} \sin(5nt-2n't-3\omega+\Omega) \\
&+ \tfrac{3}{32}e\gamma\{\tfrac{1}{2}e^2-\gamma^2\} \sin(3nt-2n't-3\omega+\Omega) \\
&\mp \tfrac{3}{64}e^3\gamma \sin(3nt+2n't-3\omega-\Omega) - \tfrac{7}{64}e^3\gamma \sin(nt+2n't-3\omega-\Omega) \\
&\pm \tfrac{7}{64}e^3\gamma \sin(nt+2n't-3\omega+\Omega) \\
&\mp \tfrac{179}{64}e\gamma\{e^2-\tfrac{18}{179}\gamma^2\} \sin(nt-2n't+3\omega-\Omega) \\
&\pm \tfrac{1}{128}e'^3\gamma \sin(4nt+n't-3\omega'-\Omega) + \tfrac{1}{128}e'^3\gamma \sin(2nt+n't-3\omega'-\Omega) \\
&\mp \tfrac{1}{128}e'^3\gamma \sin(2nt+n't-3\omega'+\Omega) - \tfrac{1}{128}e'^3\gamma \sin(n't-3\omega'+\Omega) \\
&\pm \tfrac{845}{128}e'^3\gamma \sin(4nt-5n't+3\omega'-\Omega) + \tfrac{845}{128}e'^3\gamma \sin(2nt-5n't+3\omega'-\Omega) \\
&\mp \tfrac{845}{128}e'^3\gamma \sin(2nt-5n't+3\omega'+\Omega) \\
&\pm \tfrac{845}{128}e'^3\gamma \sin(5n't-3\omega'-\Omega) \pm \tfrac{9}{32}e\gamma^3 \sin(5nt-2n't+\omega-3\Omega) \\
&+ \tfrac{3}{32}e\gamma^3 \sin(3nt-2n't+\omega-3\Omega) \mp \tfrac{9}{32}e\gamma^3 \sin(nt+2n't+\omega-3\Omega) \\
&\mp \tfrac{3}{32}e\gamma^3 \sin(nt-2n't-\omega+3\Omega) \mp \tfrac{15}{32}e\gamma^3 \sin(7nt-2n't-\omega-3\Omega) \\
&\mp \tfrac{15}{16}e\gamma^3 \sin(3nt-2n't+3\omega-3\Omega) - \tfrac{9}{32}e\gamma^3 \sin(nt-2n't+3\omega-3\Omega) \\
&\pm \tfrac{3}{32}e\gamma^3 \sin(3nt-2n't-3\omega+3\Omega) \pm \tfrac{3}{32}e\gamma^3 \sin(3nt+2n't-\omega-3\Omega) \\
&- \tfrac{3}{16}e\gamma^3 \sin(nt+2n't-\omega-3\Omega) \pm \tfrac{3}{64}e'\gamma^3 \sin(6nt-n't-\omega'-3\Omega) \\
&\pm \tfrac{21}{64}e'\gamma^3 \sin(2nt+3n't-\omega'-3\Omega) + \tfrac{21}{32}e'\gamma^3 \sin(3n't-\omega'-3\Omega) \\
&\mp \tfrac{21}{64}e'\gamma^3 \sin(6nt-3n't+\omega'-3\Omega) \mp \tfrac{3}{64}e'\gamma^3 \sin(2nt+n't+\omega'-3\Omega) \\
&- \tfrac{3}{32}e'\gamma^3 \sin(n't+\omega'-3\Omega) \mp \tfrac{3}{64}e'\gamma^3 \sin(2nt-n't-2\omega-\omega'+3\Omega) \\
&\pm \tfrac{21}{32}e'\gamma^3 \sin(4nt-3n't+2\omega+\omega'-3\Omega) \\
&+ \tfrac{21}{64}e'\gamma^3 \sin(2nt-3n't+2\omega+\omega'-3\Omega) \\
&\mp \tfrac{3}{32}e'\gamma^3 \sin(4nt-n't+2\omega-\omega'-3\Omega) \\
&- \tfrac{3}{64}e'\gamma^3 \sin(2nt-n't+2\omega-\omega'-3\Omega) \\
&\pm \tfrac{21}{64}e'\gamma^3 \sin(2nt-3n't-2\omega+\omega'+3\Omega) \\
&\pm \tfrac{3}{32}e'\gamma\{\gamma^2-2e^2\} \sin(4nt-n't-2\omega-\omega'+\Omega) \\
&+ \tfrac{3}{64}e'\gamma\{\gamma^2-e^2\} \sin(2nt-n't-2\omega-\omega'+\Omega) \\
&\pm \tfrac{21}{64}e'\gamma\{37e^2-\gamma^2\} \sin(2nt-3n't+2\omega+\omega'-\Omega) \\
&\mp \tfrac{3}{64}e'\gamma\{37e^2-\gamma^2\} \sin(2nt-n't+2\omega-\omega'-\Omega) \\
&\mp \tfrac{105}{32}e^2e'\gamma \sin(3n't-2\omega-\omega'+\Omega) \pm \tfrac{15}{32}e^2e'\gamma \sin(n't-2\omega+\omega'+\Omega) \\
&\mp \tfrac{21}{16}e'\gamma\{e^2+\tfrac{1}{2}\gamma^2\} \sin(4nt-3n't-2\omega+\omega'+\Omega) \\
&+ \tfrac{21}{64}e'\gamma\{e^2-\gamma^2\} \sin(2nt-3n't-2\omega+\omega'+\Omega) \\
&\mp \tfrac{75}{64}e^2e'\gamma \sin(6nt-n't-2\omega-\omega'-\Omega)
\end{aligned}
\right\} . \quad (278)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- \tfrac{3}{16}e^2e'\gamma \sin(4nt - n't - 2\omega - \omega' - \Omega) \\
&\mp \tfrac{21}{64}e^2e'\gamma \sin(2nt + 3n't - 2\omega - \omega' - \Omega) \\
&+ \tfrac{105}{32}e^2e'\gamma \sin(3n't - 2\omega - \omega' - \Omega) \\
&\pm \tfrac{525}{64}e^2e'\gamma \sin(6nt - 3n't - 2\omega + \omega' - \Omega) \\
&+ \tfrac{21}{16}e^2e'\gamma \sin(4nt - 3n't - 2\omega + \omega' - \Omega) \\
&\pm \tfrac{3}{64}e^2e'\gamma \sin(2nt + n't - 2\omega + \omega' - \Omega) \\
&- \tfrac{15}{32}e^2e'\gamma \sin(n't - 2\omega + \omega' - \Omega) \mp \tfrac{255}{16}ee'^2\gamma \sin(3nt - 4n't + \omega + 2\omega' - \Omega) \\
&- \tfrac{153}{16}ee^{2\prime}\gamma \sin(nt - 4n't + \omega + 2\omega' - \Omega) \\
&\mp \tfrac{51}{16}ee'^2\gamma \sin(nt + 4n't - \omega - 2\omega' - \Omega) \\
&\pm \tfrac{153}{16}ee'^2\gamma \sin(nt - 4n't + \omega + 2\omega' + \Omega) \\
&\pm \tfrac{153}{16}ee'^2\gamma \sin(5nt - 4n't - \omega + 2\omega' - \Omega) \\
&+ \tfrac{51}{16}ee'^2\gamma \sin(3nt - 4n't - \omega + 2\omega' - \Omega) \\
&\mp \tfrac{51}{16}ee'^2\gamma \sin(3nt - 4n't - \omega + 2\omega' + \Omega) \\
&+ \tfrac{51}{16}ee'^2\gamma \sin(nt - 4n't - \omega + 2\omega' + \Omega) \Big\} \\
&+ \frac{\overline{m}^2}{a'}\Big\{ \pm \tfrac{33}{32}\gamma \sin(3nt - n't - \Omega) + \tfrac{33}{32}\gamma \sin(nt - n't - \Omega) \\
&+ \tfrac{33}{32}\gamma \sin(nt + n't - \Omega) \mp \tfrac{33}{32}\gamma \sin(nt - n't + \Omega) \\
&\pm \tfrac{99}{64}e\gamma \sin(4nt - n't - \omega - \Omega) - \tfrac{33}{64}e\gamma \sin(2nt - n't - \omega - \Omega) \\
&\pm \tfrac{33}{64}e\gamma \sin(2nt - n't - \omega + \Omega) \mp \tfrac{165}{64}e\gamma \sin(n't + \omega - \Omega) \\
&\mp \tfrac{33}{64}e\gamma \sin(2nt + n't - \omega - \Omega) - \tfrac{165}{64}e\gamma \sin(n't - \omega - \Omega) \\
&\mp \tfrac{297}{64}e\gamma \sin(2nt - n't + \omega - \Omega) \pm \tfrac{165}{64}e\gamma \sin(n't - \omega + \Omega) \\
&\pm \tfrac{33}{32}h_{,,} \sin(nt + \alpha_{,,}t - \beta_{,,}) \mp \tfrac{33}{32}h_{,,} \sin(nt - \alpha_{,,}t + \beta_{,,}) \\
&\pm \tfrac{99}{32}e'\gamma \sin(3nt - 2n't + \omega' - \Omega) + \tfrac{99}{32}e'\gamma \sin(nt - 2n't + \omega' - \Omega) \\
&\pm \tfrac{99}{32}e'\gamma \sin(nt + 2n't - \omega' - \Omega) \mp \tfrac{99}{32}e'\gamma \sin(nt - 2n't + \omega' + \Omega) \\
&\pm \tfrac{33}{32}e'\gamma \sin(3nt - \omega' - \Omega) + \tfrac{33}{32}e'\gamma \sin(nt - \omega' - \Omega) \\
&\pm \tfrac{15}{32}\gamma \sin(5nt - 3n't - \Omega) + \tfrac{15}{32}\gamma \sin(3nt - 3n't - \Omega) \\
&\mp \tfrac{15}{32}\gamma \sin(3nt - 3n't + \Omega) - \tfrac{15}{32}\gamma \sin(nt - 3n't + \Omega) \\
&\pm \tfrac{105}{64}e\gamma \sin(6nt - 3n't - \omega - \Omega) + \tfrac{45}{64}e\gamma \sin(4nt - 3n't - \omega - \Omega) \\
&\mp \tfrac{45}{64}e\gamma \sin(4nt - 3n't - \omega + \Omega) + \tfrac{15}{64}e\gamma \sin(2nt - 3n't - \omega + \Omega) \\
&\mp \tfrac{195}{64}e\gamma \sin(4nt - 3n't + \omega - \Omega) - \tfrac{135}{64}e\gamma \sin(2nt - 3n't + \omega - \Omega) \\
&\pm \tfrac{135}{64}e\gamma \sin(2nt - 3n't + \omega + \Omega) \mp \tfrac{75}{64}e\gamma \sin(3n't - \omega - \Omega) \\
&\pm \tfrac{75}{32}e'\gamma \sin(5nt - 4n't + \omega' - \Omega) + \tfrac{75}{32}e'\gamma \sin(3nt - 4n't + \omega' - \Omega) \\
&\mp \tfrac{75}{32}e'\gamma \sin(3nt - 4n't + \omega' + \Omega) - \tfrac{75}{32}e'\gamma \sin(nt - 4n't + \omega' + \Omega) \\
&\mp \tfrac{15}{32}e'\gamma \sin(5nt - 2n't - \omega' - \Omega) - \tfrac{15}{32}e'\gamma \sin(3nt - 2n't - \omega' - \Omega) \\
&\pm \tfrac{15}{32}e'\gamma \sin(3nt - 2n't - \omega' + \Omega) + \tfrac{15}{32}e'\gamma \sin(nt - 2n't - \omega' + \Omega) \Big\}
\end{aligned}\right\} \quad . \tag{278}
$$

If we now add equations (277) and (278) together, we shall obtain

$$
\tan\theta_1 \sin v_1 \left(\frac{dR}{dv}\right) + \cos v_1 \left(\frac{dR}{d\theta}\right) =
$$

$$
\left.\begin{aligned}
\frac{\overline{m}^2}{a}\Big\{ &+ \tfrac{3}{4}\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \sin(2nt - \Omega) \\
&- \tfrac{3}{4}e\gamma\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \sin(nt + \omega - \Omega) \\
&- \tfrac{3}{4}e\gamma\{1 - \tfrac{11}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \sin(nt - \omega + \Omega) \\
&+ \tfrac{3}{4}e\gamma\{1 - \tfrac{19}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \sin(3nt - \omega - \Omega) \\
&\mp \tfrac{3}{4}e\gamma\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2 - \gamma^2\} \sin(nt - \omega - \Omega) \\
&+ \tfrac{9}{8}e'\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \sin(2nt + n't - \omega' - \Omega) \\
&\pm \tfrac{9}{8}e'\gamma\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2 - \gamma^2\} \sin(n't - \omega' - \Omega) \\
&+ \tfrac{9}{8}e'\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{4}\gamma^2\} \sin(2nt - n't + \omega' - \Omega) \\
&- \tfrac{9}{8}e'\gamma\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2 - \gamma^2\} \sin(n't - \omega' + \Omega) \\
&+ \tfrac{3}{4}e^2\gamma \sin(4nt - 2\omega - \Omega) \mp \tfrac{3}{16}e^2\gamma \sin(2nt - 2\omega - \Omega) \\
&- \tfrac{3}{16}\gamma\{e^2 + \gamma^2\} \sin(2nt - 2\omega + \Omega) + \tfrac{3}{16}\gamma^3 \sin(2nt + 2\omega - 3\Omega) \\
&\pm \tfrac{3}{16}\gamma^3 \sin(2nt - 3\Omega) + \tfrac{27}{16}e'^2\gamma \sin(2nt - 2n't + 2\omega' - \Omega) \\
&- \tfrac{27}{16}e'^2\gamma \sin(2n't - 2\omega' + \Omega) + \tfrac{27}{16}e'^2\gamma \sin(2nt + 2n't - 2\omega' - \Omega) \\
&\pm \tfrac{27}{16}e'^2\gamma \sin(2n't - 2\omega' - \Omega) + \tfrac{9}{8}ee'\gamma \sin(nt + n't - \omega - \omega' + \Omega) \\
&- \tfrac{27}{8}ee'\gamma \sin(nt - n't + \omega + \omega' - \Omega) \\
&- \tfrac{27}{8}ee'\gamma \sin(nt + n't + \omega - \omega' - \Omega) \\
&+ \tfrac{9}{8}ee'\gamma \sin(nt - n't - \omega + \omega' + \Omega) - \tfrac{3}{2}h_{,} \sin(nt + \alpha_{,}t - \beta_{,}) \\
&+ \tfrac{9}{8}ee'\gamma \sin(3nt + n't - \omega - \omega' - \Omega) + \tfrac{3}{2}h_{,} \sin(nt - \alpha_{,}t + \beta_{,}) \\
&+ \tfrac{9}{8}ee'\gamma \sin(3nt - n't - \omega + \omega' - \Omega) \\
&\mp \tfrac{9}{8}ee'\gamma \sin(nt + n't - \omega - \omega' - \Omega) \\
&\pm \tfrac{9}{8}ee'\gamma \sin(nt - n't - \omega + \omega' - \Omega) + \tfrac{3}{8}e\gamma^3 \sin(3nt + \omega - 3\Omega) \\
&\mp \tfrac{9}{16}e\gamma^3 \sin(nt + \omega - 3\Omega) \pm \tfrac{3}{16}e\gamma^3 \sin(3nt - \omega - 3\Omega) \\
&- \tfrac{9}{16}e\gamma^3 \sin(nt + 3\omega + 3\Omega) + \tfrac{25}{32}e^3\gamma \sin(5nt - 3\omega - \Omega) \\
&\mp \tfrac{3}{32}e^3\gamma \sin(3nt - 3\omega - \Omega) - \tfrac{3}{32}e\gamma\{2\gamma^2 - e^2\} \sin(3nt - 3\omega + \Omega) \\
&\pm \tfrac{7}{32}e^3\gamma \sin(nt - 3\omega + \Omega) + \tfrac{159}{64}e'^3\gamma \sin(2nt + 3n't - 3\omega' - \Omega) \\
&\pm \tfrac{159}{64}e'^3\gamma \sin(3n't - 3\omega' - \Omega) + \tfrac{159}{64}e'^3\gamma \sin(2nt - 3n't + 3\omega' - \Omega) \\
&- \tfrac{159}{64}e'^3\gamma \sin(3n't - 3\omega' + \Omega) + \tfrac{27}{16}ee'^2\gamma \sin(nt + 2n't - \omega - 2\omega' + \Omega) \\
&- \tfrac{81}{16}ee'^2\gamma \sin(nt - 2n't + \omega + 2\omega' - \Omega) \\
&- \tfrac{81}{16}ee'^2\gamma \sin(nt + 2n't + \omega - 2\omega' - \Omega) \\
&+ \tfrac{27}{16}ee'^2\gamma \sin(nt - 2n't - \omega + 2\omega' + \Omega) \\
&+ \tfrac{27}{16}ee'^2\gamma \sin(3nt + 2n't - \omega - 2\omega' - \Omega)
\end{aligned}\right\} \quad (279)
$$

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$$
\left.\begin{aligned}
&+\tfrac{27}{16}ee'^2\gamma\sin(3nt-2n't-\omega+2\omega'-\Omega)\\
&\mp\tfrac{27}{16}ee'^2\gamma\sin(nt+2n't-\omega-2\omega'-\Omega)\\
&\mp\tfrac{27}{16}ee'^2\gamma\sin(nt-2n't-\omega+2\omega'-\Omega)\\
&+\tfrac{9}{8}e^2e'\gamma\sin(4nt+n't-2\omega-\omega'-\Omega)\\
&\mp\tfrac{9}{32}e^2e'\gamma\sin(2nt+n't-2\omega-\omega'-\Omega)\\
&+\tfrac{9}{8}e^2e'\gamma\sin(4nt-n't-2\omega+\omega'-\Omega)\\
&\mp\tfrac{9}{32}e^2e'\gamma\ \sin(2nt-n't-2\omega+\omega'-\Omega)\\
&-\tfrac{9}{32}e'\gamma\{e^2+\gamma^2\}\sin(2nt-n't-2\omega+\omega'+\Omega)\\
&-\tfrac{9}{32}e'\gamma\{e^2+\gamma^2\}\sin(2nt+n't-2\omega-\omega'+\Omega)\\
&\mp\tfrac{45}{16}e^2e'\gamma\sin(n't-2\omega-\omega'+\Omega)+\tfrac{45}{16}e^2e'\gamma\sin(n't+2\omega-\omega'-\Omega)\\
&\pm\tfrac{9}{32}e'\gamma^3\sin(2nt+n't-\omega'-3\Omega)\pm\tfrac{9}{32}e'\gamma^3\sin(2nt-n't+\omega'-3\Omega)\\
&+\tfrac{9}{32}e'\gamma^3\sin(2nt+n't+2\omega-\omega'-3\Omega)\\
&+\tfrac{9}{32}e'\gamma^3\sin(2nt-n't+2\omega+\omega'-3\Omega)\\
&\pm\tfrac{3}{4}\gamma\{1-\tfrac{5}{2}e^2-\tfrac{5}{2}e'^2-\tfrac{3}{2}\gamma^2\}\sin(2nt-2n't-\Omega)\\
&+\tfrac{9}{64}\gamma^3\sin(2nt-2n't+\Omega)\\
&+\tfrac{3}{4}\gamma\{1+\tfrac{3}{2}e^2-\tfrac{5}{2}e'^2-\tfrac{13}{16}\gamma^2\}\sin(2n't-\Omega)\\
&-\tfrac{27}{32}\gamma^3\sin(4nt-2n't-\Omega)+e\gamma\{e^2-\tfrac{27}{64}\gamma^2\}\sin(5nt-2n't-\omega-\Omega)\\
&\pm\tfrac{3}{4}e\gamma\{1-\tfrac{19}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{9}{16}\gamma^2\}\sin(3nt-2n't-\omega-\Omega)\\
&-\tfrac{3}{4}e\gamma\{\tfrac{1}{8}e^2-\gamma^2\}\sin(3nt-2n't+\omega-\Omega)\\
&\mp\tfrac{9}{4}e\gamma\{1-\tfrac{13}{24}e^2-\tfrac{11}{8}e'^2-\tfrac{9}{16}\gamma^2\}\sin(nt-2n't+\omega-\Omega)\\
&-\tfrac{3}{4}e\gamma\{1-\tfrac{1}{8}e^2-2e'^2-\tfrac{13}{16}\gamma^2\}\sin(nt+2n't-\omega-\Omega)\\
&-\tfrac{27}{64}e\gamma\{6e'^2+\gamma^2\}\sin(nt-2n't+\omega+\Omega)\\
&\pm\tfrac{3}{4}e\gamma\{1+\tfrac{1}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{16}\gamma^2\}\sin(nt-2n't-\omega+\Omega)\\
&+\tfrac{9}{64}e\gamma^3\sin(3nt-2n't-\omega+\Omega)\\
&\mp\tfrac{3}{8}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{9}{16}\gamma^2\}\sin(2nt-n't-\omega'-\Omega)\\
&-\tfrac{9}{128}e'\gamma^3\sin(2nt-n't-\omega'+\Omega)\\
&-\tfrac{3}{8}e'\gamma\{1+\tfrac{3}{2}e^2-\tfrac{1}{8}e'^2-\tfrac{13}{16}\gamma^2\}\sin(n't+\omega'-\Omega)\\
&-\tfrac{63}{128}e'\gamma^3\sin(4nt-3n't+\omega'-\Omega)+\tfrac{63}{128}e'\gamma^3\sin(2nt-3n't+\omega'+\Omega)\\
&\pm\tfrac{21}{8}e'\gamma\{1-\tfrac{5}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{9}{16}\gamma^2\}\sin(2nt-3n't+\omega'-\Omega)\\
&+\tfrac{21}{8}e'\gamma\{1+\tfrac{3}{2}e^2-\tfrac{123}{56}e'^2-\tfrac{13}{16}\gamma^2\}\sin(3n't-\omega'-\Omega)\\
&\pm\tfrac{3}{4}e^2\gamma\sin(4nt-2n't-2\omega-\Omega)\\
&\pm\tfrac{3}{16}\gamma\{e^2-\gamma^2\}\sin(2nt-2n't-2\omega+\Omega)\\
&-\tfrac{3}{16}e^2\gamma\sin(2nt+2n't-2\omega-\Omega)-\tfrac{15}{8}e^2\gamma\sin(2n't-2\omega+\Omega)\\
&\pm\tfrac{51}{8}e'^2\gamma\sin(2nt-4n't+2\omega'-\Omega)+\tfrac{51}{8}e'^2\gamma\sin(4n't-2\omega'-\Omega)\\
&+\tfrac{3}{64}\gamma^3\sin(6nt-2n't-3\Omega)\mp\tfrac{3}{64}\gamma^3\sin(4nt-2n't-3\Omega)\\
&+\tfrac{9}{64}\gamma^3\sin(2nt+2n't-3\Omega)\pm\tfrac{3}{64}\gamma^3\sin(2n't-3\Omega)'
\end{aligned}\right\}\quad(279)
$$

(Continued on the next page.)

19

$$
\left.\begin{aligned}
&\pm \tfrac{3}{16}\gamma^3 \sin(2nt - 2n't + 2\omega - 3\Omega) \mp \tfrac{3}{8}ee'\gamma \sin(3nt - n't - \omega - \omega' - \Omega) \\
&\mp \tfrac{63}{8}ee'\gamma \sin(nt - 3n't + \omega + \omega' - \Omega) \\
&- \tfrac{21}{8}ee'\gamma \sin(nt + 3n't - \omega - \omega' - \Omega) \\
&+ \tfrac{3}{8}ee'\gamma \sin(nt + n't - \omega + \omega' - \Omega) \\
&\pm \tfrac{21}{8}ee'\gamma \sin(3nt - 3n't - \omega + \omega' - \Omega) \\
&\pm \tfrac{3}{8}ee'\gamma \sin(nt - n't + \omega - \omega' - \Omega) \mp \tfrac{3}{8}ee'\gamma \sin(nt - n't - \omega - \omega' + \Omega) \\
&\pm \tfrac{21}{8}ee'\gamma \sin(nt - 3n't - \omega + \omega' + \Omega) \\
&\pm \tfrac{25}{32}e^3\gamma \sin(5nt - 2n't - 3\omega - \Omega) - \tfrac{3}{32}e^3\gamma \sin(3nt + 2n't - 3\omega - \Omega) \\
&\pm \tfrac{3}{32}e\gamma\{e^2 - 2\gamma^2\} \sin(3nt - 2n't - 3\omega + \Omega) \\
&+ \tfrac{7}{32}e^3\gamma \sin(nt + 2n't - 3\omega + \Omega) \pm \tfrac{1}{64}e'^3\gamma \sin(2nt + n't - 3\omega' - \Omega) \\
&\mp \tfrac{1}{64}e'^3\gamma \sin(n't - 3\omega' + \Omega) \pm \tfrac{845}{64}e'^3\gamma \sin(2nt - 5n't + 3\omega' - \Omega) \\
&+ \tfrac{845}{64}e'^3\gamma \sin(5n't - 3\omega' - \Omega) - \tfrac{21}{64}e\gamma^3 \sin(5nt - 2n't + \omega - 3\Omega) \\
&\pm \tfrac{27}{64}e\gamma^3 \sin(3nt - 2n't + \omega - 3\Omega) - \tfrac{27}{64}e\gamma^3 \sin(nt + 2n't + \omega - 3\Omega) \\
&+ \tfrac{3}{64}e\gamma^3 \sin(nt - 2n't - \omega + 3\Omega) + \tfrac{15}{64}e\gamma^3 \sin(7nt - 2n't - \omega - 3\Omega) \\
&\mp \tfrac{9}{64}e\gamma^3 \sin(5nt - 2n't - \omega - 3\Omega) \mp \tfrac{9}{16}e\gamma^3 \sin(nt - 2n't + 3\omega - 3\Omega) \\
&+ \tfrac{9}{64}e\gamma^3 \sin(3nt + 2n't - \omega - 3\Omega) \mp \tfrac{3}{64}e\gamma^3 \sin(nt + 2n't - \omega - 3\Omega) \\
&- \tfrac{3}{128}e'\gamma^3 \sin(6nt - n't - \omega' - 3\Omega) \pm \tfrac{3}{128}e'\gamma^3 \sin(4nt - n't - \omega' - 3\Omega) \\
&+ \tfrac{63}{128}e'\gamma^3 \sin(2nt + 3n't - \omega' - 3\Omega) \pm \tfrac{21}{128}e'\gamma^3 \sin(3n't - \omega' - 3\Omega) \\
&+ \tfrac{21}{128}e'\gamma^3 \sin(6nt - 3n't + \omega' - 3\Omega) \mp \tfrac{21}{128}e'\gamma^3 \sin(4nt - 3n't + \omega' - 3\Omega) \\
&- \tfrac{9}{128}e'\gamma^3 \sin(2nt + n't + \omega' - 3\Omega) \mp \tfrac{3}{128}e'\gamma^3 \sin(n't + \omega' - 3\Omega) \\
&\pm \tfrac{21}{32}e'\gamma^3 \sin(2nt - 3n't + 2\omega + \omega' - 3\Omega) \\
&\mp \tfrac{3}{32}e'\gamma^3 \sin(2nt - n't + 2\omega - \omega' - 3\Omega) \\
&\pm \tfrac{3}{32}e'\gamma\{\gamma^2 - e^2\} \sin(2nt - n't - 2\omega - \omega' + \Omega) \\
&- \tfrac{105}{16}e^2e'\gamma \sin(3n't - 2\omega - \omega' + \Omega) + \tfrac{15}{16}e^2e'\gamma \sin(n't - 2\omega + \omega' + \Omega) \\
&\pm \tfrac{21}{32}e'\gamma\{e^2 - \gamma^2\} \sin(2nt - 3n't - 2\omega + \omega' + \Omega) \\
&\mp \tfrac{3}{8}e^2e'\gamma \sin(4nt - n't - 2\omega - \omega' - \Omega) \\
&- \tfrac{21}{32}e^2e'\gamma \sin(2nt + 3n't - 2\omega - \omega' - \Omega) \\
&\pm \tfrac{21}{8}e^2e'\gamma \sin(4nt - 3n't - 2\omega + \omega' - \Omega) \\
&+ \tfrac{3}{32}e^2e'\gamma \sin(2nt + n't - 2\omega + \omega' - \Omega) \\
&\mp \tfrac{153}{8}ee'^2\gamma \sin(nt - 4n't + \omega + 2\omega' - \Omega) \\
&- \tfrac{51}{8}ee'^2\gamma \sin(nt + 4n't - \omega - 2\omega' - \Omega) \\
&\pm \tfrac{51}{8}ee'^2\gamma \sin(3nt - 4n't - \omega + 2\omega' - \Omega) \\
&\pm \tfrac{51}{8}ee'^2\gamma \sin(nt - 4n't - \omega + 2\omega' + \Omega) \\
&- \tfrac{3}{4}\gamma\{1 + \tfrac{3}{2}e^2 + \tfrac{3}{2}e'^2 - \gamma^2\} \sin\Omega + \tfrac{15}{8}e^2\gamma \sin(2\omega - \Omega) \Big\}
\end{aligned}\right\} \quad (279)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
+\frac{\overline{m}^2}{a'}\Big\{ &+ \tfrac{15}{16}\gamma \sin(3nt - n't - \Omega) \pm \tfrac{9}{8}\gamma \sin(nt - n't - \Omega) \\
&+ \tfrac{9}{8}\gamma \sin(nt + n't - \Omega) - \tfrac{15}{16}\gamma \sin(nt - n't + \Omega) \\
&+ \tfrac{45}{32}e\gamma \sin(4nt - n't - \omega - \Omega) \mp \tfrac{9}{16}e\gamma \sin(2nt - n't - \omega - \Omega) \\
&+ \tfrac{15}{32}e\gamma \sin(2nt - n't - \omega + \Omega) - \tfrac{45}{16}e\gamma \sin(n't + \omega - \Omega) \\
&- \tfrac{15}{32}e\gamma \sin(2nt + n't - \omega - \Omega) \mp \tfrac{75}{32}e\gamma \sin(n't - \omega - \Omega) \\
&- \tfrac{135}{32}e\gamma \sin(2nt - n't + \omega - \Omega) + \tfrac{45}{16}e\gamma \sin(n't - \omega + \Omega) \\
&+ \tfrac{3}{32}e'\gamma \sin(nt + \omega' - \Omega) + \tfrac{3}{32}e'\gamma \sin(nt - \omega' + \Omega) \\
&+ \tfrac{45}{16}e'\gamma \sin(3nt - 2n't + \omega' - \Omega) \pm \tfrac{27}{8}e'\gamma \sin(nt - 2n't + \omega' - \Omega) \\
&+ \tfrac{27}{8}e'\gamma \sin(nt + 2n't - \omega' - \Omega) - \tfrac{45}{16}e'\gamma \sin(nt - 2n't + \omega' + \Omega) \\
&+ \tfrac{15}{16}e'\gamma \sin(3nt - \omega' - \Omega) \pm \tfrac{9}{8}e'\gamma \sin(nt - \omega' - \Omega) \\
&\pm \tfrac{15}{16}\gamma \sin(3nt - 3n't - \Omega) \mp \tfrac{15}{16}\gamma \sin(nt - 3n't + \Omega) \\
&\pm \tfrac{45}{32}e\gamma \sin(4nt - 3n't - \omega - \Omega) \pm \tfrac{15}{32}e\gamma \sin(2nt - 3n't - \omega + \Omega) \\
&\mp \tfrac{135}{32}e\gamma \sin(2nt - 3n't + \omega - \Omega) - \tfrac{75}{32}e\gamma \sin(3n't - \omega - \Omega) \\
&\pm \tfrac{75}{16}e'\gamma \sin(3nt - 4n't + \omega' - \Omega) \mp \tfrac{75}{16}e'\gamma \sin(nt - 4n't + \omega' + \Omega) \\
&\mp \tfrac{15}{16}e'\gamma \sin(3nt - 2n't - \omega' - \Omega) \pm \tfrac{15}{16}e'\gamma \sin(nt - 2n't - \omega' + \Omega) \Big\}
\end{aligned}
\right\} . \quad (279)
$$

Equation (279) will give the value of $\tan\theta_1 \cos v_1 \left(\frac{dR}{dv}\right) - \sin v_1 \left(\frac{dR}{d\theta}\right)$ by using the lower signs and changing *sin* to *cos* in the second member.

If we now multiply equation (279) by dt, and take the integral, we shall obtain

$$
\int \left\{ \tan\theta_1 \sin v_1 \left(\frac{dR}{dv}\right) + \cos v_1 \left(\frac{dR}{d\theta}\right) \right\} dt =
$$

$$
\left.
\begin{aligned}
\frac{\overline{m}^2}{an}\Big\{ &\mp \tfrac{3}{8}\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(2nt - \Omega) \\
&\pm \tfrac{3}{4}e\gamma\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(nt + \omega - \Omega) \\
&\pm \tfrac{3}{4}e\gamma\{1 - \tfrac{11}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{5}{4}\gamma^2\} \cos(nt - \omega + \Omega) \\
&\mp \tfrac{1}{4}e\gamma\{1 - \tfrac{19}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{4}\gamma^2\} \cos(3nt - \omega - \Omega) \\
&+ \tfrac{3}{4}e\gamma\{1 - \tfrac{1}{8}e^2 + \tfrac{3}{2}e'^2 - \gamma^2\} \cos(nt - \omega - \Omega) \\
&\mp e'\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{4}\gamma^2\}(0.5422208) \cos(2nt + n't - \omega' - \Omega) \\
&- e'\gamma\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2 - \gamma^2\}(15.03985) \cos(n't - \omega' - \Omega) \\
&\mp e'\gamma\{1 - \tfrac{5}{2}e^2 + \tfrac{9}{8}e'^2 - \tfrac{3}{4}\gamma^2\}(0.5843553) \cos(2nt - n't + \omega' - \Omega) \\
&\pm e'\gamma\{1 + \tfrac{3}{2}e^2 + \tfrac{9}{8}e'^2 - \gamma^2\}(15.03985) \cos(n't - \omega' + \Omega) \\
&\mp \tfrac{3}{16}e^2\gamma \cos(4nt - 2\omega - \Omega) + \tfrac{3}{32}e^2\gamma \cos(2nt - 2\omega - \Omega)
\end{aligned}
\right\} . \quad (280)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\pm \tfrac{3}{32}\gamma\{e^2+\gamma^2\}\cos(2nt-2\omega+\Omega) \mp \tfrac{3}{32}\gamma^3\cos(2nt+2\omega-3\Omega)\\
&-\tfrac{3}{32}\gamma^3\cos(2nt-3\Omega) \mp e'^2\gamma\,(0.9119663)\cos(2nt-2n't+2\omega'-\Omega)\\
&\pm e'^2\gamma\,(11.27988)\cos(2n't-2\omega'+\Omega)\\
&\mp e'^2\gamma\,(0.7850287)\cos(2nt+2n't-2\omega'-\Omega)\\
&-e'^2\gamma\,(11.27988)\cos(2n't-2\omega'-\Omega)\\
&\mp ee'\gamma\,(1.046705)\cos(nt+n't-\omega-\omega'+\Omega)\\
&\pm ee'\gamma\,(3.647866)\cos(nt-n't+\omega+\omega'-\Omega)\\
&\pm ee'\gamma\,(3.140115)\cos(nt+n't+\omega-\omega'-\Omega)\\
&\mp ee'\gamma\,(1.215955)\cos(nt-n't-\omega+\omega'+\Omega)\\
&\mp ee'\gamma\,(0.3638772)\cos(3nt+n't-\omega-\omega'-\Omega)\\
&\mp ee'\gamma\,(0.3845892)\cos(3nt-n't-\omega+\omega'-\Omega)\\
&+ee'\gamma\,(1.046705)\cos(nt+n't-\omega-\omega'-\Omega)\\
&+ee'\gamma\,(1.215955)\cos(nt-n't-\omega+\omega'-\Omega)\\
&\mp \tfrac{5}{32}e^3\gamma\cos(5nt-3\omega-\Omega)+\tfrac{1}{32}e^3\gamma\cos(3nt-3\omega-\Omega)\\
&\pm \tfrac{1}{32}e\gamma\{2\gamma^2-e^2\}\cos(3nt-3\omega+\Omega)-\tfrac{7}{32}e^3\gamma\cos(nt-3\omega+\Omega)\\
&\mp e'^3\gamma\,(1.116872)\cos(2nt+3n't-3\omega'-\Omega)\\
&-e'^3\gamma\,(11.27100)\cos(3n't-3\omega'-\Omega)\\
&\mp e'^3\gamma\,(1.399178)\cos(2nt-3n't+3\omega'-\Omega)\\
&\pm e'^3\gamma\,(11.27100)\cos(3nt-3\omega'+\Omega)\\
&\mp ee'^2\gamma\,(1.467898)\cos(nt+2n't-\omega-2\omega'+\Omega)\\
&\pm ee'^2\gamma\,(5.953100)\cos(nt-2n't+\omega+2\omega'-\Omega)\\
&\pm ee'^2\gamma\,(4.403696)\cos(nt+2n't+\omega-2\omega'-\Omega)\\
&\mp ee'^2\gamma\,(1.984366)\cos(nt-2n't-\omega+2\omega'+\Omega)\\
&\mp ee'^2\gamma\,(0.5357819)\cos(3nt+2n't-\omega-2\omega'-\Omega)\\
&\mp ee'^2\gamma\,(0.5920227)\cos(3nt-2n't-\omega+2\omega'-\Omega)\\
&+ee'^2\gamma\,(1.467898)\cos(nt+2n't-\omega-2\omega'-\Omega)\\
&+ee'^2\gamma\,(1.984366)\cos(nt-2n't-\omega+2\omega'-\Omega)\\
&\mp e^2e'\gamma\,(0.2760871)\cos(4nt+n't-2\omega-\omega'-\Omega)\\
&+e^2e'\gamma\,(0.1355552)\cos(2nt+n't-2\omega-\omega'-\Omega)\\
&\mp e^2e'\gamma\,(0.2866097)\cos(4nt-n't-2\omega+\omega'-\Omega)\\
&+e^2e'\gamma\,(0.14060888)\cos(2nt-n't-2\omega+\omega'-\Omega)\\
&\pm e'\gamma\{e^2+\gamma^2\}(0.1460888)\cos(2nt-n't-2\omega+\omega'+\Omega)\\
&\pm e'\gamma\{e^2+\gamma^2\}(0.1355552)\cos(2nt+n't-2\omega-\omega'+\Omega)\\
&+e^2e'\gamma\,(37.59962)\cos(n't-2\omega-\omega'+\Omega)\\
&\mp e^2e'\gamma\,(37.59962)\cos(n't+2\omega-\omega'-\Omega)-\tfrac{1}{16}e\gamma^3\cos(3nt-\omega-3\Omega)\\
&\pm \tfrac{9}{16}e\gamma^3\cos(nt+3\omega-3\Omega)\mp\tfrac{1}{8}e\gamma^3\cos(3nt+\omega-3\Omega)\\
&\pm \tfrac{3}{8}\frac{h_1}{n+a_1}\cos(nt+a_1t-\beta_1)\\
&\mp \tfrac{3}{8}\frac{h_1}{n-a_1}\cos(nt-a_1t+\beta_1)
\end{aligned}
\right\} \quad . \quad (280)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+ \tfrac{9}{16} e\gamma^3 \cos(nt + \omega - 3\Omega) - e'\gamma^3 (0.1460888) \cos(2nt - n't + \omega' - 3\Omega) \\
&- e'\gamma^3 (0.1355552) \cos(2nt + n't - \omega' - 3\Omega) \\
&\mp e'\gamma^3 (0.1355552) \cos(2nt + n't + 2\omega - \omega' - 3\Omega) \\
&\mp e'\gamma^3 (0.1460888) \cos(2nt - n't + 2\omega + \omega' - 3\Omega) \\
&- \gamma\{1 - \tfrac{5}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{3}{2}\gamma^2\}(0.4053184) \cos(2nt - 2n't - \Omega) \\
&\pm \gamma^3 (0.2191333) \cos(4nt - 2n't - \Omega) \\
&\mp \gamma\{1 + \tfrac{3}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{13}{16}\gamma^2\}(5.013282) \cos(2n't - \Omega) \\
&\mp \gamma^3 (0.07599720) \cos(2nt - 2n't + \Omega) \\
&\mp e\gamma\{e^2 - \tfrac{27}{64}\gamma^2\}(0.2061687) \cos(5nt - 2n't - \omega - \Omega) \\
&\pm e\gamma\{\tfrac{1}{8}e^2 - \gamma^2\}(0.2631212) \cos(3nt - 2n't + \omega - \Omega) \\
&- e\gamma\{1 - \tfrac{19}{8}e^2 - \tfrac{5}{2}e^2 - \tfrac{9}{16}\gamma^2\}(0.2631212) \cos(3nt - 2n't - \omega - \Omega) \\
&+ e\gamma\{1 - \tfrac{13}{24}e^2 - \tfrac{11}{8}e'^2 - \tfrac{9}{16}\gamma^2\}(2.645821) \cos(nt - 2n't + \omega - \Omega) \\
&\pm e\gamma\{1 - \tfrac{1}{8}e^2 - 2e'^2 - \tfrac{13}{16}\gamma^2\}(0.6523992) \cos(nt + 2n't - \omega - \Omega) \\
&\pm e\gamma\{6e'^2 + \gamma^2\}(0.4960915) \cos(nt - 2n't + \omega + \Omega) \\
&\mp e\gamma^3 (0.04933523) \cos(3nt - 2n't - \omega + \Omega) \\
&- e\gamma\{1 + \tfrac{1}{8}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{16}\gamma^2\}(0.8819406) \cos(nt - 2n't - \omega + \Omega) \\
&+ e'\gamma\{1 - \tfrac{5}{2}e^2 - \tfrac{1}{8}e'^2 - \tfrac{9}{16}\gamma^2\}(0.1947851) \cos(2nt - n't - \omega' - \Omega) \\
&\pm e'\gamma\{1 + \tfrac{3}{2}e^2 - \tfrac{1}{8}e'^2 - \tfrac{13}{16}\gamma^2\}(5.013282) \cos(n't + \omega' - \Omega) \\
&- e'\gamma\{1 - \tfrac{5}{2}e^2 - \tfrac{123}{56}e'^2 - \tfrac{9}{16}\gamma^2\}(1.478377) \cos(2nt - 3n't + \omega' - \Omega) \\
&\mp e'\gamma\{1 + \tfrac{3}{2}e^2 - \tfrac{123}{56}e'^2 - \tfrac{13}{16}\gamma^2\}(11.69765) \cos(3n't - \omega' - \Omega) \\
&\pm e'\gamma^3 (0.03652220) \cos(2nt - n't - \omega' + \Omega) \\
&\pm e'\gamma^3 (0.1303602) \cos(4nt - 3n't + \omega' - \Omega) \\
&\mp e'\gamma^3 (0.2771956) \cos(2nt - 3n't + \omega' + \Omega) \\
&- e^2\gamma (0.1947851) \cos(4nt - 2n't - 2\omega - \Omega) \\
&- \gamma\{e^2 - \gamma^2\}(0.1013296) \cos(2nt - 2n't - 2\omega + \Omega) \\
&\pm e^2\gamma (0.08722542) \cos(2nt + 2n't - 2\omega - \Omega) \\
&\pm e^2\gamma (12.53321) \cos(2n't - 2\omega + \Omega) \\
&- e'^2\gamma (3.748248) \cos(2nt - 4n't + 2\omega' - \Omega) \\
&\mp e'^2\gamma (21.30645) \cos(4n't - 2\omega' - \Omega) \\
&\mp \gamma^3 (0.008012275) \cos(6nt - 2n't - 3\Omega) \\
&+ \gamma^3 (0.01217407) \cos(4nt - 2n't - 3\Omega) \\
&\mp \gamma^3 (0.06541906) \cos(2nt + 2n't - 3\Omega) \\
&- \gamma^3 (0.3133301) \cos(2n't - 3\Omega) \\
&- \gamma^3 (0.1013296) \cos(2nt - 2n't + 2\omega - 3\Omega) \\
&+ ee'\gamma (0.1251964) \cos(3nt - n't - \omega - \omega' - \Omega)
\end{aligned}
\right\} . \quad (280)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ ee'\gamma\,(10.15348)\cos(nt - 3n't + \omega + \omega' - \Omega)\\
&\pm ee'\gamma\,(2.143900)\cos(nt + 3n't - \omega - \omega' - \Omega)\\
&- ee'\gamma\,(0.9457426)\cos(3nt - 3n't - \omega + \omega' - \Omega)\\
&- ee'\gamma\,(1.215955)\cos(nt - n't + \omega - \omega' - \Omega)\\
&\mp ee'\gamma\,(0.3489017)\cos(nt + n't - \omega + \omega' - \Omega)\\
&+ ee'\gamma\,(0.4053184)\cos(nt - n't - \omega - \omega' + \Omega)\\
&- ee'\gamma\,(3.384494)\cos(nt - 3n't - \omega + \omega' + \Omega)\\
&- e^3\gamma\,(0.1610655)\cos(5nt - 2n't - 3\omega - \Omega)\\
&- e\gamma\{e^2 - 2\gamma^2\}(0.03289015)\cos(3nt - 2n't - 3\omega + \Omega)\\
&\pm e^3\gamma\,(0.02976566)\cos(3nt + 2n't - 3\omega - \Omega)\\
&\mp e^3\gamma\,(0.1902833)\cos(nt + 2n't - 3\omega + \Omega)\\
&- e'^3\gamma\,(0.007513522)\cos(2nt + n't - 3\omega' - \Omega)\\
&+ e'^3\gamma\,(0.2088868)\cos(n't - 3\omega' + \Omega)\\
&- e'^3\gamma\,(8.120036)\cos(2nt - 5n't + 3\omega' - \Omega)\\
&\mp e'^3\gamma\,(35.30187)\cos(5n't - 3\omega' - \Omega)\\
&\pm e\gamma^3\,(0.06764908)\cos(5nt - 2n't + \omega - 3\Omega)\\
&- e\gamma^3\,(0.1480057)\cos(3nt - 2n't + \omega - 3\Omega)\\
&\pm e\gamma^3\,(0.3669746)\cos(nt + 2n't + \omega - 3\Omega)\\
&\mp e\gamma^3\,(0.05512128)\cos(nt - 2n't - \omega + 3\Omega)\\
&\mp e\gamma^3\,(0.03421335)\cos(7nt - 2n't - \omega - 3\Omega)\\
&+ e\gamma^3\,(0.02899246)\cos(5nt - 2n't - \omega - 3\Omega)\\
&+ e\gamma^3\,(0.6613030)\cos(nt - 2n't + 3\omega - 3\Omega)\\
&\mp e\gamma^3\,(0.04464848)\cos(3nt + 2n't - \omega - 3\Omega)\\
&+ e\gamma^3\,(0.04077495)\cos(nt + 2n't - \omega - 3\Omega)\\
&\pm e'\gamma^3\,(0.003955564)\cos(6nt - n't - \omega - 3\Omega)\\
&- e'\gamma^3\,(0.005971034)\cos(4nt - n't - \omega' - 3\Omega)\\
&\mp e'\gamma^3\,(0.2212671)\cos(2nt + 3n't - \omega' - 3\Omega)\\
&- e'\gamma^3\,(0.7311035)\cos(3n't - \omega' - 3\Omega)\\
&\mp e'\gamma^3\,(0.02840616)\cos(6nt - 3n't + \omega' - 3\Omega)\\
&+ e'\gamma^3\,(0.04345340)\cos(4nt - 3n't + \omega' - 3\Omega)\\
&\pm e'\gamma^3\,(0.0338888)\cos(2nt + n't + \omega' - 3\Omega)\\
&+ e'\gamma^3\,(0.3133301)\cos(n't + \omega' - 3\Omega)\\
&- e'\gamma^3\,(0.3695943)\cos(2nt - 3n't + 2\omega + \omega' - 3\Omega)\\
&+ e'\gamma^3\,(0.04869627)\cos(2nt - n't + 2\omega - \omega' - 3\Omega)\\
&- e'\gamma\{\gamma^2 - e^2\}(0.04869627)\cos(2nt - n't - 2\omega - \omega' + \Omega)\\
&\pm e^2e'\gamma\,(29.24414)\cos(3n't - 2\omega - \omega' + \Omega)
\end{aligned}\right\} . \quad (280)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp e^2e'\gamma\,(12.53321)\cos(n't-2\omega+\omega'+\Omega)\\
&-e'\gamma\{e^2-\gamma^2\}(0.3695943)\cos(2nt-3n't-2\omega+\omega'+\Omega)\\
&+e^2e'\gamma\,(0.09553654)\cos(4nt-n't-2\omega-\omega'-\Omega)\\
&\pm e^2e'\gamma\,(0.2950229)\cos(2nt+3n't-2\omega-\omega'-\Omega)\\
&-e^2e'\gamma\,(0.6952545)\cos(4nt-3n't-2\omega+\omega'-\Omega)\\
&\mp e^2e'\gamma\,(0.04518506)\cos(2nt+n't-2\omega+\omega'-\Omega)\\
&+ee'^2\gamma\,(27.29044)\cos(nt-4n't+\omega+2\omega'-\Omega)\\
&\pm ee'^2\gamma\,(4.906846)\cos(nt+4nt-\omega-2\omega'-\Omega)\\
&-ee'^2\gamma\,(2.360416)\cos(3nt-4n't-\omega+2\omega'-\Omega)\\
&-ee'^2\gamma\,(9.096815)\cos(nt-4n't-\omega+2\omega'+\Omega)\\
&\left.-\tfrac{3}{4}\int\gamma\{1+\tfrac{3}{2}e^2+\tfrac{3}{2}e'^2-\gamma^2\}\sin\Omega\,dt+\tfrac{15}{8}\int e^2\gamma\sin(2\omega-\Omega)\,dt\right\}\\[1ex]
+\frac{\overline{m}^2}{a'n}&\Big\{\mp\gamma\,(0.3204910)\cos(3nt-n't-\Omega)\\
&-\gamma\,(1.215954)\cos(nt-n't-\Omega)\mp\gamma\,(1.046705)\cos(nt+n't-\Omega)\\
&\pm\gamma\,(1.013296)\cos(nt-n't+\Omega)\\
&\mp e\gamma\,(0.3582621)\cos(4nt-n't-\omega-\Omega)\\
&+e\gamma\,(0.2921776)\cos(2nt-n't-\omega-\Omega)\\
&\mp e\gamma\,(0.2434813)\cos(2nt-n't-\omega+\Omega)\\
&\pm e\gamma\,(37.59961)\cos(n't+\omega-\Omega)\\
&\pm e\gamma\,(0.2259253)\cos(2nt+n't-\omega-\Omega)\\
&+e\gamma\,(31.33301)\cos(n't-\omega-\Omega)\\
&\pm e\gamma\,(2.191333)\cos(2nt-n't+\omega-\Omega)\\
&\mp e\gamma\,(37.59961)\cos(n't-\omega+\Omega)\mp e'\gamma\,(0.093750)\cos(nt+\omega'-\Omega)\\
&\pm e'\gamma\,(0.093750)\cos(nt-\omega'+\Omega)\\
&\mp e'\gamma\,(0.9867045)\cos(3nt-2n't+\omega'-\Omega)\\
&-e'\gamma\,(3.968733)\cos(nt-2n't+\omega'-\Omega)\\
&\mp e'\gamma\,(2.935797)\cos(nt+2n't-\omega'-\Omega)\\
&\pm e'\gamma\,(3.307276)\cos(nt-2n't+\omega'+\Omega)\\
&\mp e'\gamma\,(0.312500)\cos(3nt-\omega'-\Omega)-e'\gamma\,(1.125000)\cos(nt-\omega'-\Omega)\\
&-\gamma\,(0.3377653)\cos(3nt-3n't-\Omega)+\gamma\,(1.208748)\cos(nt-3n't+\Omega)\\
&-e\gamma\,(0.3724577)\cos(4nt-3n't-\omega-\Omega)\\
&-e\gamma\,(0.2639959)\cos(2nt-3n't-\omega+\Omega)\\
&+e\gamma\,(2.375963)\cos(2nt-3n't+\omega-\Omega)\\
&\pm e\gamma\,(10.44434)\cos(3n't-\omega-\Omega)\\
&-e'\gamma\,(1.735600)\cos(3nt-4n't+\omega'-\Omega)
\end{aligned}
\right\}. \quad (280)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+e'\gamma(6.688835)\cos(nt-4n't+\omega'+\Omega)\mp\tfrac{33}{32}\frac{h_{,,}}{n+\alpha_{,,}}\cos(nt+\alpha_{,,}t-\beta_{,,})\\
&+e'\gamma(0.3289015)\cos(3nt-2n't-\omega'-\Omega)\pm\tfrac{33}{32}\frac{h_{,,}}{n-\alpha_{,,}}\cos(nt-\alpha_{,,}t+\beta_{,,})\\
&-e'\gamma(1.102426)\cos(nt-2n't-\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (280)$$

This equation will give the value of $\int\left\{\tan\theta_1\cos v_1\left(\frac{dR}{dv}\right)-\sin v_1\left(\frac{dR}{d\theta}\right)\right\}dt$ by using the lower signs and changing *cos* to *sin* in the second member. We shall designate the equation so changed as equatión (281).

The values of $\frac{\sin v_1}{r_1^2}$ and $\frac{\cos v_1}{r_1^2}$ are given by means of equation (242); and if we multiply equation (280) by $\frac{\cos v_1}{r_1^2}$ and equation (281) by $\frac{\sin v_1}{r_1^2}$ and substitute the products in equation (263), we shall obtain the value of $\frac{d\delta_0\theta}{dt}$ as follows:

$$\left.\begin{aligned}
\frac{d\delta_0\theta}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{&+\gamma\{\tfrac{3}{8}+\tfrac{7}{8}e^2+\tfrac{9}{16}e'^2-\tfrac{21}{64}\gamma^2\}\cos(nt-\Omega)\\
&-e\gamma\{\tfrac{3}{4}-\tfrac{41}{32}e^2+\tfrac{3}{4}e'^2+\tfrac{87}{64}\gamma^2\}\cos(\omega-\Omega)\\
&-\tfrac{1}{2}e\gamma\{1-\tfrac{19}{4}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{2}\gamma^2\}\cos(2nt-\omega-\Omega)\\
&-\tfrac{3}{64}\gamma\{71e^2+\gamma^2\}\cos(nt-2\omega+\Omega)\\
&+e'\gamma\{15.58207+16.57685e^2+17.529829e'^2\\
&\qquad\qquad-16.58621\gamma^2\}\cos(nt+n't-\omega'-\Omega)\\
&-e'\gamma\{14.455495+13.930327e^2+16.252431e'^2\\
&\qquad\qquad-16.413789\gamma^2\}\cos(nt-n't+\omega'-\Omega)\\
&-e^2\gamma(1.359375)\cos(3nt-2\omega-\Omega)+\gamma^3(0.140625)\cos 3(nt-\Omega)\\
&+\gamma^3(0.046875)\cos(nt+2\omega-3\Omega)\qquad+3\frac{nh_{,}\alpha_{,}}{n^2-\alpha_{,}^2}\cos(\alpha_{,}t-\beta_{,})\\
&+e'^2\gamma(12.06491)\cos(nt+2n't-2\omega'-\Omega)\\
&-e'^2\gamma(10.36791)\cos(nt-2n't+2\omega'-\Omega)\\
&-ee'\gamma(0.839718)\cos(n't+\omega-\omega'-\Omega)\\
&-ee'\gamma(1.432455)\cos(n't-\omega-\omega'+\Omega)\\
&+ee'\gamma(29.39887)\cos(2nt+n't-\omega-\omega'-\Omega)\\
&-ee'\gamma(30.91107)\cos(2nt-n't-\omega+\omega'-\Omega)\\
&-\tfrac{5}{2}e^3\gamma\cos(4nt-3\omega-\Omega)-\tfrac{1}{4}e\gamma\{21e^2+\tfrac{1}{4}\gamma^2\}\cos(2nt-3\omega+\Omega)\\
&-e\gamma^3(0.984375)\cos(2nt+\omega-3\Omega)+e\gamma^3(0.46875)\cos(4nt-\omega-3\Omega)\\
&-\tfrac{3}{32}e\gamma^3\cos 3(\omega-\Omega)+e'^3\gamma(12.387872)\cos(nt+3n't-3\omega'-\Omega)
\end{aligned}\right\}. \quad (282)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&- e'^3\gamma\,(9.871822)\cos(nt - 3n't + 3\omega' - \Omega)\\
&- ee'^2\gamma\,(0.849273)\cos(2n't + \omega - 2\omega' - \Omega)\\
&- ee'^2\gamma\,(2.661269)\cos(2n't - \omega - 2\omega' + \Omega)\\
&- ee'^2\gamma\,(21.62765)\cos(2nt + 2n't - \omega - 2\omega' - \Omega)\\
&- ee'^2\gamma\,(23.95210)\cos(2nt - 2n't - \omega + 2\omega' - \Omega)\\
&+ e^2e'\gamma\,(48.874394)\cos(3nt + n't - 2\omega - \omega' - \Omega)\\
&- e^2e'\gamma\,(52.977839)\cos(3nt - n't - 2\omega + \omega' - \Omega)\\
&- e'\gamma\{44.93869e^2 + 1.947759\gamma^2\}\cos(nt + n't - 2\omega - \omega' + \Omega)\\
&+ e'\gamma\{34.88328e^2 + 1.806937\gamma^2\}\cos(nt - n't - 2\omega + \omega' + \Omega)\\
&- e'\gamma^3\,(1.676648)\cos(3nt + n't - \omega' - 3\Omega)\\
&+ e'\gamma^3\,(2.099114)\cos(3nt - n't + \omega' - 3\Omega)\\
&+ e'\gamma^3\,(1.947759)\cos(nt + n't + 2\omega - \omega' - 3\Omega)\\
&- e'\gamma^3\,(1.806937)\cos(nt - n't + 2\omega + \omega' - 3\Omega)\\
&+ \gamma\,\{0.4053184 - 6.607597e^2 - 1.0013296e'^2\\
&\qquad\qquad - 1.4537711\gamma^2\}\cos(3nt - 2n't - \Omega)\\
&+ \gamma\,\{5.013282 + 3.708484e^2 - 12.533205e'^2\\
&\qquad\qquad - 3.946629\gamma^2\}\cos(nt - 2n't + \Omega)\\
&+ e\gamma\,\{1.0737580 - 12.11452e^2 - 2.684395e'^2\\
&\qquad\qquad - 4.0678225\gamma^2\}\cos(4nt - 2n't - \omega - \Omega)\\
&- e\gamma\,\{2.645821 + 22.31036e^2 - 3.638004e'^2\\
&\qquad\qquad - 4.638222\gamma^2\}\cos(2nt - 2n't + \omega - \Omega)\\
&- e\gamma\,\{0.6523992 - 0.3435413e^2 + 1.6718506e'^2\\
&\qquad\qquad + 0.0434208\gamma^2\}\cos(2n't - \omega - \Omega)\\
&+ e\gamma\,\{10.908467 + 4.9873198e^2 - 27.271167e'^2\\
&\qquad\qquad - 7.8393859\gamma^2\}\cos(2nt - 2n't - \omega + \Omega)\\
&- e'\gamma\,\{0.1947851 - 3.016265e^2 - 0.0243481e'^2\\
&\qquad\qquad - 0.7362268\gamma^2\}\cos(3nt - n't - \omega' - \Omega)\\
&- e'\gamma\,\{5.013282 + 4.215479e^2 - 0.626660e'^2\\
&\qquad\qquad + 4.134161\gamma^2\}\cos(nt - n't - \omega' + \Omega)\\
&+ e'\gamma\,\{1.478377 - 24.74209e^2 - 3.247150e'^2\\
&\qquad\qquad - 2.424153\gamma^2\}\cos(3nt - 3n't + \omega' - \Omega)\\
&+ e'\gamma\,\{11.69765 + 7.40985e^2 - 25.69305e'^2\\
&\qquad\qquad - 9.042347\gamma^2\}\cos(nt - 3n't + \omega' + \Omega)\\
&+ e^2\gamma\,(2.0889771)\cos(5nt - 2n't - 2\omega - \Omega)\\
&+ \gamma\,\{18.785038e^2 - 0.1519944\gamma^2\}\cos(3nt - 2n't - 2\omega + \Omega)\\
&+ e^2\gamma\,(0.5394348)\cos(nt + 2n't - 2\omega - \Omega)
\end{aligned}
\right\} . \quad (282)
$$

(Continued on the next page.)

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$$
\left.
\begin{aligned}
&-\gamma\,\{12.48255e^2-0.6266602\gamma^2\}\cos(nt-2n't+2\omega-\Omega)\\
&+e'^2\gamma\,(3.748248)\cos(3nt-4n't+2\omega'-\Omega)\\
&+e'^2\gamma\,(21.30645)\cos(nt-4n't+2\omega'+\Omega)\\
&-\gamma^3\,(0.0548266)\cos(5nt-2n't-3\Omega)\\
&+\gamma^3\,(1.0054094)\cos(nt+2n't-3\Omega)\\
&+\gamma^3\,(0.1519944)\cos(3nt-2n't+2\omega-3\Omega)\\
&-\gamma^3\,(0.6266602)\cos(nt-2n't-2\omega+3\Omega)\\
&-ee'\gamma\,(0.5147666)\cos(4nt-n't-\omega-\omega'-\Omega)\\
&-ee'\gamma\,(10.15348)\cos(2nt-3n't+\omega+\omega'-\Omega)\\
&-ee'\gamma\,(2.143900)\cos(3n't-\omega-\omega'-\Omega)\\
&+ee'\gamma\,(3.902497)\cos(4nt-3n't-\omega+\omega'-\Omega)\\
&+ee'\gamma\,(1.215955)\cos(2nt-n't+\omega-\omega'-\Omega)\\
&+ee'\gamma\,(0.3489017)\cos(n't-\omega+\omega'-\Omega)\\
&-ee'\gamma\,(10.43188)\cos(2nt-n't-\omega-\omega'+\Omega)\\
&+ee'\gamma\,(26.77979)\cos(2nt-3n't-\omega+\omega'+\Omega)\\
&+e^3\gamma\,(3.600368)\cos(6nt-2n't-3\omega-\Omega)\\
&+e\gamma\,\{29.949603e^2-0.4026594\gamma^2\}\cos(4nt-2n't-3\omega+\Omega)\\
&+e^3\gamma\,(0.724232)\cos(2nt+2n't-3\omega-\Omega)\\
&-e\gamma\,\{0.0728912e^2+0.0815499\gamma^2\}\cos(2n't-3\omega+\Omega)\\
&+e'^3\gamma\,(0.007513522)\cos(3nt+n't-3\omega'-\Omega)\\
&-e'^3\gamma\,(0.2088868)\cos(nt+n't-3\omega'+\Omega)\\
&+e'^3\gamma\,(8.120036)\cos(3nt-5n't+3\omega'-\Omega)\\
&+e'^3\gamma\,(35.30187)\cos(nt-5n't+3\omega'+\Omega)\\
&+e\gamma^3\,(0.915982)\cos(4nt-2n't+\omega-3\Omega)\\
&-e\gamma^3\,(0.6158829)\cos(2n't+\omega-3\Omega)\\
&-e\gamma^3\,(0.2547766)\cos(6nt-2n't-\omega-3\Omega)\\
&+e\gamma^3\,(1.802303)\cos(2nt+2n't-\omega-3\Omega)\\
&-e\gamma^3\,(0.9920306)\cos(2nt-2n't+3\omega-3\Omega)\\
&-e\gamma^3\,(1.363563)\cos(2nt-2n't-3\omega+3\Omega)\\
&+e'\gamma^3\,(0.02636357)\cos(5nt-n't-\omega'-3\Omega)\\
&+e'\gamma^3\,(2.414577)\cos(nt+3n't-\omega'-3\Omega)\\
&-e'\gamma^3\,(0.1998443)\cos(5nt-3n't+\omega'-3\Omega)\\
&-e'\gamma^3\,(0.9738791)\cos(nt+n't+\omega'-3\Omega)\\
&+e'\gamma^3\,(0.5543914)\cos(3nt-3n't+2\omega+\omega'-3\Omega)\\
&-e'\gamma^3\,(0.0730448)\cos(3nt-n't+2\omega-\omega'-3\Omega)\\
&-e'\gamma\,\{17.77916e^2-0.0730444\gamma^2\}\cos(3nt-n't-2\omega-\omega'+\Omega)
\end{aligned}
\right\}\ . \qquad (282)
$$

(Continued on the next page.)

$$
\begin{aligned}
&- e'\gamma\,\{29.05934e^2 - 1.462206\gamma^2\}\cos(nt - 3n't + 2\omega + \omega' - \Omega)\\
&+ e'\gamma\,\{12.50886e^2 - 0.6266602\gamma^2\}\cos(nt - n't + 2\omega - \omega' - \Omega)\\
&- e'\gamma\,\{46.61815e^2 - 0.5543914\gamma^2\}\cos(3nt - 3n't - 2\omega + \omega' + \Omega)\\
&- e^2e'\gamma\,(0.9833290)\cos(5nt - n't - 2\omega - \omega' - \Omega)\\
&+ e^2e'\gamma\,(1.167183)\cos(nt + 3n't - 2\omega - \omega' - \Omega)\\
&+ e^2e'\gamma\,(7.576562)\cos(5nt - 3n't - 2\omega + \omega' - \Omega)\\
&- e^2e'\gamma\,(0.5814751)\cos(nt + n't - 2\omega + \omega' - \Omega)\\
&+ e'\gamma^3\,(0.6266602)\cos(nt - n't - 2\omega - \omega' + 3\Omega)\\
&- e'\gamma^3\,(1.462206)\cos(nt - 3n't - 2\omega + \omega' + 3\Omega)\\
&- ee'^2\gamma\,(27.29044)\cos(2nt - 4n't + \omega + 2\omega' - \Omega)\\
&- ee'^2\gamma\,(4.906846)\cos(4n't - \omega - 2\omega' - \Omega)\\
&+ ee'^2\gamma\,(9.857012)\cos(4nt - 4n't - \omega + 2\omega' - \Omega)\\
&+ ee'^2\gamma\,(51.70972)\cos(2nt - 4n't - \omega + 2\omega' + \Omega)\Big\}\\
&+ \frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{+\gamma\,(1.536445)\cos(2nt - n't - \Omega) + \gamma\,(0.033409)\cos(n't - \Omega)\\
&+ \gamma\,(0.3377653)\cos(4nt - 3n't - \Omega) - \gamma\,(1.208748)\cos(2nt - 3n't + \Omega)\\
&+ e\gamma\,(2.497992)\cos(3nt - n't - \omega - \Omega)\\
&- e\gamma\,(35.26272)\cos(nt - n't - \omega + \Omega)\\
&- e\gamma\,(33.58553)\cos(nt + n't - \omega - \Omega)\\
&+ e\gamma\,(36.04926)\cos(nt - n't + \omega - \Omega)\\
&+ e\gamma\,(1.0479883)\cos(5nt - 3n't - \omega - \Omega)\\
&- e\gamma\,(2.153500)\cos(3nt - 3n't - \omega + \Omega)\\
&- e\gamma\,(2.375963)\cos(3nt - 3n't + \omega - \Omega)\\
&- e\gamma\,(10.44434)\cos(nt - 3n't + \omega + \Omega)\\
&+ e'\gamma\,(4.9554375)\cos(2nt - 2n't + \omega' - \Omega)\\
&+ e'\gamma\,(0.1875)\cos(\omega' - \Omega) - e'\gamma\,(0.371479)\cos(2n't - \omega' - \Omega)\\
&+ e'\gamma\,(1.4375)\cos(2nt - \omega' - \Omega)\\
&+ e'\gamma\,(1.73560)\cos(4nt - 4n't + \omega' - \Omega)\\
&- e'\gamma\,(6.688835)\cos(2nt - 4n't + \omega' + \Omega)\\
&- e'\gamma\,(0.3289015)\cos(4nt - 2n't - \omega' - \Omega)\\
&+ e'\gamma\,(1.102426)\cos(2nt - 2n't - \omega' + \Omega)\Big\}
\end{aligned}
\qquad - \frac{33}{16}\frac{h_{,,}n\alpha_{,,}}{n^2 - \alpha_{,,}^2}\cos(\alpha_{,,}t - \beta_{,,}) \quad . \qquad (282)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+\tfrac{3}{4}\frac{\overline{m}^2}{a}\frac{\cos v_1}{r_1^2}\int\{1+\tfrac{3}{2}e^2+\tfrac{3}{2}e'^2-\gamma^2\}\gamma\sin\Omega\,dt \\
&-\tfrac{3}{4}\frac{\overline{m}^2}{a}\frac{\sin v_1}{r_1^2}\int\{1+\tfrac{3}{2}e^2-\tfrac{3}{2}e'^2-\gamma^2\}\gamma\cos\Omega\,dt \\
&-\tfrac{15}{8}\frac{\overline{m}^2}{a}\frac{\cos v_1}{r_1^2}\int e^2\gamma\sin(2\omega-\Omega)\,dt+\tfrac{15}{8}\frac{\overline{m}^2}{a}\frac{\sin v_1}{r_1^2}\int e^2\gamma\cos(2\omega-\Omega)\,dt
\end{aligned}\right\} . \quad (282)$$

This is the whole value of $\frac{d\delta\theta}{dt}$ arising from the sun's direct action.

23. We have thus completely developed the perturbations of the moon's co-ordinates r, v, and θ, in so far as they depend on the sun's direct action; and we must now develop equations (260), (262), and (264), which give the indirect action of the sun on the same co-ordinates. The determination of the indirect effect of the sun's action is most readily effected by a series of successive approximations. Since equation (260) contains no terms which are independent of e, it follows that there is no indirect perturbation of the radius vector which is independent of the eccentricity of the orbit. If we therefore suppose that $e=0$, the whole perturbation of r will be given by the single term in the development of equation (259), which is given in equation (274). If we suppose $e=0$ and $\gamma=0$, in equation (274), we shall obtain the first approximate value of $\frac{d\delta_1 r}{dt}$, which is the complete value of $\frac{d\delta r}{dt}$ to terms of the first order. We shall therefore have

$$\frac{d\delta r}{dt}=a\frac{\overline{m}^2}{\mu}n\,(2.382700)\sin 2(nt-n't). \tag{283}$$

This equation gives by integration

$$\delta r=a\frac{\overline{m}^2}{\mu}\{-[0.1098044]\cos 2(nt-n't)+c\}. \tag{284}$$

The constant to be added to this integral may be determined so as to satisfy any required condition. In general the substitution of δr in equation (262) will give some terms of perturbation in the longitude which are proportional to the time. Now all such terms may be supposed to be included in the moon's mean motion as determined by observation; and we may therefore determine the constant part of δr so as to make the terms of perturbation which are proportional to the time disappear from the expression of the moon's longitude. We shall therefore determine the constant part of δr so as to satisfy the condition that the elliptical motion in the undisturbed orbit shall be equal to the mean motion in the dis-

turbed orbit. We may then omit all the terms in the expression of δv which are proportional to the time, and use for n the value of the mean motion of the moon derived from observation.

If we designate by a the distance at which the undisturbed mean motion should be equal to the disturbed motion at distance $a + c$, we shall have the following equation to determine c:

$$\frac{\sqrt{\mu}}{a^{\frac{3}{2}}\left\{1+\frac{c}{a}\right\}^{\frac{3}{2}}}\left\{1+\frac{a^2}{\mu}\left(1+\frac{c}{a}\right)^2\left(\frac{dR}{dr}\right)\right\}^{\frac{1}{2}}=\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}. \tag{285}$$

This equation gives

$$1+\frac{c}{a}=\left\{1+\frac{a^2}{\mu}\left(1+\frac{c}{a}\right)^2\left(\frac{dR}{dr}\right)\right\}^{\frac{1}{3}}. \tag{286}$$

Hence we get

$$\frac{c}{a}=\tfrac{1}{3}\frac{a^2}{\mu}\left\{1+\frac{c}{a}\right\}^2\left(\frac{dR}{dr}\right)-\tfrac{1}{9}\frac{a^4}{\mu^2}\left\{1+\frac{c}{a}\right\}^4\left(\frac{dR}{dr}\right)^2. \tag{287}$$

If we substitute the constant part of the value of $\left(\frac{dR}{dr}\right)$ in this equation, we shall find

$$c=-\tfrac{1}{6}a\frac{\overline{m}^2}{\mu}. \tag{288}$$

This value of c is correct when we neglect the square of the disturbing force. We shall therefore have for the first approximation to the complete value of δr,

$$\delta r = a\frac{\overline{m}^2}{\mu}\{-\tfrac{1}{6}-[0.1098044]\cos 2(nt-n't)\}, \tag{289}$$

the number in brackets being a logarithm.

The value of $\frac{dv_1}{r_1 dt}$ is given by the equation

$$\left.\begin{aligned}\frac{dv_1}{r_1dt}=\frac{n}{a}\Big\{&1+e^2+e^4+3e\{1+\tfrac{5}{8}e^2\}\cos(nt-\omega)\\
&+e^2\{\tfrac{9}{2}+\tfrac{5}{4}e^2+\tfrac{1}{8}\frac{\gamma^4}{e^2}\}\cos 2(nt-\omega)+\tfrac{53}{8}e^3\cos 3(nt-\omega)\\
&+\tfrac{77}{8}e^4\cos 4(nt-\omega)-\tfrac{1}{2}\gamma^2\{1-3e^2-\tfrac{1}{2}\gamma^2\}\cos 2(nt-\Omega)\\
&+\tfrac{1}{4}e\gamma^2\cos(nt+\omega-2\Omega)-\tfrac{7}{4}e\gamma^2\cos(3nt-\omega-2\Omega)\\
&-\tfrac{17}{4}e^2\gamma^2\cos(4nt-2\omega-2\Omega)-\tfrac{1}{8}\gamma^4\cos(2nt+2\omega-4\Omega)\\
&+\tfrac{1}{8}\gamma^4\cos 4(nt-\Omega)\Big\}\end{aligned}\right\}. \tag{290}$$

The preceding value of δr gives by substitution in equation (262),

$$\frac{d\delta_1 v}{dt}=\frac{\overline{m}^2}{\mu}n\left\{+2.575339\cos 2(nt-n't)\right\}. \tag{291}$$

But we have from equation (276)

$$\frac{d\delta_0 v}{dt}=\frac{\overline{m}^2}{\mu}n\left\{+0.8106367\cos 2(nt-n't)\right\}. \tag{292}$$

Whence we get by substitution in equation (255)

$$\frac{d\delta v}{dt}=\frac{\overline{m}^2}{\mu}n\left\{+\{3.385976\,..\,[0.5296839]\}\cos 2(nt-n't)\right\}. \tag{293}$$

This equation gives, by integration,

$$\delta v=\frac{\overline{m}^2}{\mu}\left\{+1.829864\sin 2(nt-n't)\right\}. \tag{294}$$

This value of δv being substituted in equation (260), will give the value of $\frac{d\delta_2 r}{dt}$ correct to terms $\overline{m}^2 e$; and then by substitution in equation (253) we shall obtain the value of $\frac{d\delta r}{dt}$ correct to terms depending on the first power of e; and the substitution of this new value of δr in equation (262) will give the value of $\frac{d\delta_1 v}{dt}$ to terms of the same order.

In making these successive substitutions we shall require the following equations:

$$\left.\begin{aligned}&\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\mu e\cos\theta_0\cos\theta_1\cos(v_1-\omega)\\
&\quad=\frac{\mu}{\sqrt{a\mu}}\Big\{-e^2\{1+\tfrac{1}{8}e^2\}+e\{1-\tfrac{5}{8}e^2\}\cos(nt-\omega)\\
&\quad+e^2\{1-\tfrac{5}{8}e^2\}\cos 2(nt-\omega)+\tfrac{9}{8}e^3\cos 3(nt-\omega)\\
&\quad+\tfrac{4}{3}e^4\cos 4(nt-\omega)+\tfrac{1}{2}e\gamma^2\cos(nt+\omega-2\Omega)\\
&\quad+\tfrac{1}{2}e^2\gamma^2\cos 2(nt-\Omega)-\tfrac{1}{2}e^2\gamma^2\cos 2(\omega-\Omega)\Big\}\end{aligned}\right\}. \tag{295}$$

$$\left.\begin{aligned}
&\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e \cos\theta_0 \sin\theta_1 \sin(v_1-\omega) \\
&\quad = \frac{\mu}{\sqrt{a\mu}}\Big\{ \tfrac{1}{2}e\gamma\{1+\tfrac{5}{8}e^2+\tfrac{3}{8}\gamma^2\}\cos(\omega-\Omega) \\
&\qquad - \tfrac{1}{2}e\gamma\{1-\tfrac{3}{8}e^2+\tfrac{3}{8}\gamma^2\}\cos(2nt-\omega-\Omega) + e^2\gamma\cos(nt-\Omega) \\
&\qquad - e^2\gamma\cos(3nt-2\omega-\Omega)\Big\}
\end{aligned}\right\} . \quad (296)$$

$$\left.\begin{aligned}
&2\frac{dv_1}{dt}\tan\theta_1 \\
&\quad = n\Big\{ 2\gamma\{1-e^2+\tfrac{1}{8}\gamma^2\}\sin(nt-\Omega) + 4e\gamma\sin(2nt-\omega-\Omega) \\
&\qquad + \tfrac{27}{4}e^2\gamma\sin(3nt-2\omega-\Omega) - \tfrac{1}{4}\gamma\{e^2+\gamma^2\}\sin(nt-2\omega+\Omega) \\
&\qquad + \tfrac{1}{4}\gamma^3\sin(nt+2\omega-3\Omega) - \tfrac{3}{4}\gamma^3\sin 3(nt-\Omega)\Big\}
\end{aligned}\right\} . \quad (297)$$

$$\left.\begin{aligned}
\frac{d\theta_1}{r_1 dt} &= \frac{n}{a}\Big\{ \gamma\{1-\tfrac{3}{8}\gamma^2\}\cos(nt-\Omega) + e\gamma\{\tfrac{5}{2}-\tfrac{11}{8}e^2-\tfrac{15}{16}\gamma^2\}\cos(2nt-\omega-\Omega) \\
&\quad + \tfrac{1}{2}e\gamma\{1+e^2-\tfrac{1}{2}\gamma^2\}\cos(\omega-\Omega) - \tfrac{1}{8}\gamma^3\cos 3(nt-\Omega) \\
&\quad + \tfrac{1}{8}\gamma^3\cos(nt+2\omega-3\Omega) + \tfrac{39}{8}e^2\gamma\cos(3nt-2\omega-\Omega) \\
&\quad + \tfrac{1}{8}\gamma\{5e^2-\gamma^2\}\cos(nt-2\omega+\Omega) + \tfrac{1}{16}e\gamma^3\cos 3(\omega-\Omega) \\
&\quad + \tfrac{1}{8}e\gamma\{\tfrac{3}{2}\gamma^2+5e^2\}\cos(2nt-3\omega+\Omega) + \tfrac{1}{2}e\gamma^3\cos(2nt+\omega-3\Omega) \\
&\quad - \tfrac{9}{16}e\gamma^3\cos(4nt-\omega-3\Omega) + \tfrac{103}{12}e^2\gamma\cos(4nt-3\omega-\Omega)\Big\}
\end{aligned}\right\} . \quad (298)$$

$$\left.\begin{aligned}
&\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\,\frac{\tan\theta_1}{r_1^2} \\
&\quad = \frac{\sqrt{a\mu}}{a^2}\Big\{ \gamma\{1-e^2-\tfrac{5}{8}\gamma^2\}\sin(nt-\Omega) \\
&\qquad + 2e\gamma\{1-\tfrac{5}{4}e^2-\tfrac{5}{8}\gamma^2\}\sin(2nt-\omega-\Omega) - \tfrac{1}{8}\gamma^3\sin 3(nt-\Omega) \\
&\qquad - \tfrac{1}{8}\gamma\{e^2+\gamma^2\}\sin(nt-2\omega+\Omega) + \tfrac{1}{8}\gamma^3\sin(nt+2\omega-3\Omega) \\
&\qquad + \tfrac{27}{8}e^2\gamma\sin(3nt-2\omega-\Omega) - \tfrac{1}{2}e\gamma^3\sin(4nt-\omega-3\Omega) \\
&\qquad + \tfrac{16}{8}e^2\gamma\sin(4nt-3\omega-\Omega) - e\gamma\{\tfrac{1}{8}e^2+\tfrac{1}{4}\gamma^2\}\sin(2nt-3\omega+\Omega) \\
&\qquad + \tfrac{1}{2}e\gamma^3\sin(2nt+\omega-3\Omega)\Big\}
\end{aligned}\right\} . \quad (299)$$

It is evident that each successive approximation includes terms of an order one degree higher than the approximation which precedes it; and as we have retained terms of the order e^4, it is plain that five successive approximations will be necessary, which will severally include the terms e^0, e, e^2, e^3, and e^4; but as the final approximation includes all that precede it, we may omit these successive steps of the work and only give the final approximation.

24. This being supposed, we shall obtain by means of equation (260),

$$
\left.
\begin{aligned}
\frac{d\delta_2 r}{dt} = a\frac{\overline{m}^2}{\mu}n\Big\{ & -e^2\{\tfrac{3}{8} + \tfrac{3}{8}e^2 + \tfrac{9}{16}e'^2 - \tfrac{1}{2}\gamma^2\}\sin 2(nt-\omega) \\
& + \tfrac{19}{32}e^3 \sin(nt-\omega) - \tfrac{29}{32}e^3 \sin 3(nt-\omega) \\
& - ee'\{20.16597 + 22.47526e^2 + 22.68672e'^2 \\
& \qquad\qquad - 22.71845\gamma^2\}\sin(nt+n't-n-\omega') \\
& + ee'\{20.16597 + 15.82730e^2 + 22.68672e'^2 \\
& \qquad\qquad - 22.71845\gamma^2\}\sin(nt-n't-\omega+\omega') \\
& - ee'^2(15.339485)\sin(nt+2n't-\omega-2\omega') \\
& + ee'^2(15.339485)\sin(nt-2n't-\omega+2\omega') \\
& - e^2e'(30.54686)\sin(n't-\omega') - e^2e'(53.89648)\sin(2nt+n't-2\omega-\omega') \\
& + e^2e'(57.31426)\sin(2nt-n't-2\omega+\omega') \\
& + \tfrac{1}{32}e\gamma^2\sin(3nt-\omega-2\Omega) + \tfrac{1}{32}e\gamma^2\sin(nt+\omega-2\Omega) \\
& + e^2\gamma^2(0.2592593)\sin(4nt-2\omega-2\Omega) \\
& + e^2\gamma^2(0.1550926)\sin 2(nt-\Omega) + \tfrac{19}{48}e^2\gamma^2\sin 2(\omega-\Omega) \\
& - \tfrac{5}{8}e^4\sin 4(nt-\omega) - ee'^3(15.54410)\sin(nt+3n't-\omega-3\omega') \\
& + ee'^3(15.54410)\sin(nt-3n't-\omega+3\omega') \\
& - e^2e'^2(40.01922)\sin(2nt+2n't-2\omega-2\omega') \\
& + e^2e'^2(45.33851)\sin(2nt-2n't-2\omega+2\omega') \\
& - e^2e'^2(23.99980)\sin 2(n't-\omega') \\
& - e^3e'(98.87359)\sin(3nt+n't-3\omega-\omega') \\
& + e^3e'(105.89675)\sin(3nt-n't-3\omega+\omega') \\
& + ee'\gamma^2(1.869978)\sin(3nt+n't-\omega-\omega'-2\Omega) \\
& - ee'\gamma^2(0.543450)\sin(3nt-n't-\omega+\omega'-2\Omega) \\
& - ee'\gamma^2(8.21300)\sin(nt+n't+\omega-\omega'-2\Omega) \\
& + ee'\gamma^2(9.53953)\sin(nt-n't+\omega+\omega'-2\Omega) \\
& + e^2\{33.33765 - 65.0357e^2 - 83.3439e'^2 - 28.30869\gamma^2\}\sin 2(nt-n't) \\
& + e\{0.914932 + 72.244471e^2 - 2.287330e'^2 \\
& \qquad\qquad - 1.666988\gamma^2\}\sin(3nt-2n't-\omega) \\
& + e\{0.914932 - 1.85057e^2 - 2.287330e'^2 \\
& \qquad\qquad - 1.666988\gamma^2\}\sin(nt-2n't+\omega) \\
& + e^2\{2.299648 + 125.85534e^2 - 5.749102e'^2 \\
& \qquad\qquad - 4.938012\gamma^2\}\sin(4nt-2n't-2\omega) \\
& - e^2\{34.69773 - 9.80336e^2 - 86.7441e'^2 - 30.038864\gamma^2\}\sin 2(n't-\omega) \\
& - ee'\{0.3944260 + 63.41879e^2 - 0.049303e'^2 \\
& \qquad\qquad - 1.436811\gamma^2\}\sin(3nt-n't-\omega-\omega')
\end{aligned}
\right\} . \quad (300)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&-ee'\{0.3944260 - 1.18848e^2 - 0.049303e'^2 \\
&\qquad\qquad\qquad - 1.436811\gamma^2\} \sin(nt - n't + \omega - \omega') \\
&+ee'\{3.753160 + 195.0705e^2 - 8.24355e'^2 \\
&\qquad\qquad\qquad - 4.575672\gamma^2\} \sin(3nt - 3n't - \omega + \omega') \\
&+ee'\{3.753160 - 7.0916e^2 - 8.24355e'^2 \\
&\qquad\qquad\qquad - 4.575672\gamma^2\} \sin(nt - 3n't + \omega + \omega') \\
&+e^3(4.331852)\sin(5nt - 2n't - 3\omega) \\
&-e^3(59.82891)\sin(nt + 2n't - 3\omega) \\
&+ee'^2(10.82207)\sin(3nt - 4n't - \omega + 2\omega') \\
&+ee'^2(10.82207)\sin(nt - 4n't + \omega + 2\omega') \\
&-e^2e'(28.71029)\sin(2nt - n't - \omega') \\
&+e^2e'(91.8010)\sin(2nt - 3n't + \omega') \\
&-e^2e'(0.998083)\sin(4nt - n't - 2\omega - \omega') \\
&+e^2e'(9.36834)\sin(4nt - 3n't - 2\omega + \omega') \\
&-e^2e'(97.4453)\sin(3nt - 2\omega - \omega') + e^2e'(29.28991)\sin(n't - 2\omega + \omega') \\
&+e\gamma^2(0.449313)\sin(3nt - 2n't + \omega - 2\Omega) \\
&+e\gamma^2(16.72352)\sin(nt - 2n't - \omega + 2\Omega) \\
&-e\gamma^2(0.2368861)\sin(5nt - 2n't - \omega - 2\Omega) \\
&-e\gamma^2(16.49478)\sin(nt + 2n't - \omega - 2\Omega) \\
&-e\gamma^2(0.2287330)\sin(3nt - 2n't - 3\omega + 2\Omega) \\
&+e\gamma^2(0.228733)\sin(nt - 2n't + 3\omega - 2\Omega) \\
&+e^4(7.314627)\sin(6nt - 2n't - 4\omega) - e^4(94.40501)\sin(2nt + 2n't - 4\omega) \\
&+ee'^3(0.01258043)\sin(3nt + n't - \omega - 3\omega') \\
&+ee'^3(0.01258043)\sin(nt + n't + \omega - 3\omega') \\
&+ee'^3(27.02481)\sin(3nt - 5n't - \omega + 3\omega') \\
&+ee'^3(27.02481)\sin(nt - 5n't + \omega + 3\omega') \\
&-e^2\gamma^2(6.37229)\sin(4nt - 2n't - 2\Omega) - e^2\gamma^2(11.29038)\sin 2(n't - \Omega) \\
&+e^2\gamma^2(16.78709)\sin(2nt - 2n't + 2\omega - 2\Omega) \\
&+e^2\gamma^2(29.66829)\sin(2nt - 2n't - 2\omega + 2\Omega) \\
&-e^2\gamma^2(1.063341)\sin(6nt - 2n't - 2\omega - 2\Omega) \\
&-e^2\gamma^2(30.82726)\sin(2nt + 2n't - 2\omega - 2\Omega) \\
&-e^2\gamma^2(0.5749105)\sin(4nt - 2n't - 4\omega + 2\Omega) \\
&-e^2\gamma^2(8.67441)\sin(2n't - 4\omega + 2\Omega)
\end{aligned}\right\} \quad . \quad (300)
$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
+\, e^2e'^2\,(201.2387)\sin(2nt-4n't+2\omega') \\
+\, e^2e'^2\,(26.81485)\sin(4nt-4n't-2\omega+2\omega') \\
-\, e^2e'^2\,(217.7121)\sin(4n't-2\omega-2\omega') \\
-\, e^3e'\,(1.8877255)\sin(5nt-n't-3\omega-\omega') \\
+\, e^3e'\,(17.57591)\sin(5nt-3n't-3\omega+\omega') \\
-\, e^3e'\,(167.7318)\sin(nt+3n't-3\omega-\omega') \\
+\, e^3e'\,(50.44729)\sin(nt+2n't-3\omega+\omega') \\
-\, ee'\gamma^2\,(0.1933645)\sin(3nt-n't+\omega-\omega'-2\Omega) \\
-\, ee'\gamma^2\,(29.05776)\sin(nt-n't-\omega-\omega'+2\Omega) \\
+\, ee'\gamma^2\,(1.846617)\sin(3nt-3n't+\omega+\omega'-2\Omega) \\
+\, ee'\gamma^2\,(29.95396)\sin(nt-3n't-\omega+\omega'+2\Omega) \\
-\, ee'\gamma^2\,(0.968254)\sin(5nt-3n't-\omega+\omega'-2\Omega) \\
+\, ee'\gamma^2\,(0.1025131)\sin(5nt-n't-\omega-\omega'-2\Omega) \\
+\, ee'\gamma^2\,(28.95921)\sin(nt+n't-\omega+\omega'-2\Omega) \\
-\, ee'\gamma^2\,(29.01567)\sin(nt+3n't-\omega+\omega'-2\Omega) \\
+\, ee'\gamma^2\,(0.9382905)\sin(nt-3n't+3\omega+\omega'-2\Omega) \\
+\, ee'\gamma^2\,(0.9382905)\sin(3nt-3n't-3\omega+\omega'+2\Omega) \\
+\, ee'\gamma^2\,(0.0986646)\sin(3nt-n't-3\omega-\omega'+2\Omega) \\
-\, ee'\gamma^2\,(0.0986646)\sin(nt-n't+3\omega-\omega'-2\Omega)\Big\} \\
+\, a\dfrac{\overline{m}^2}{\mu}\dfrac{a}{a'}n\Big\{-e\,(14.308905)\sin(2nt-n't-\omega') \\
+\, e\,(14.308905)\sin(n't-\omega)+e\,(0.2950661)\sin(4nt-3n't-\omega) \\
+\, e\,(0.2950661)\sin(2nt-3n't+\omega)-e^2\,(25.28781)\sin(nt-n't) \\
-\, e^2\,(4.318772)\sin 3(nt-n't)-e^2\,(32.24530)\sin(3nt-n't-2\omega) \\
+\, e^2\,(50.27814)\sin(nt+n't-2\omega)+e^2\,(0.848399)\sin(5nt-3n't-2\omega) \\
-\, e^2\,(3.986907)\sin(nt-3n't+2\omega) \\
-\, ee'\,(24.79800)\sin(2nt-2n't-\omega+\omega') \\
+\, ee'\,(24.79800)\sin(2n't-\omega-\omega')+ee'\,(0.2812500)\sin(2nt-\omega-\omega') \\
+\, ee'\,(0.2812500)\sin(\omega-\omega')+ee'\,(1.602496)\sin(4nt-4n't-\omega+\omega') \\
+\, ee'\,(1.602496)\sin(2nt-4n't+\omega+\omega') \\
-\, ee'\,(0.2724958)\sin(4nt-2n't-\omega-\omega') \\
-\, ee'\,(0.2724958)\sin(2nt-2n't+\omega-\omega')\Big\}
\end{array}\right\} \quad . \quad (300)
$$

$$\left.\begin{aligned}
\frac{d\delta_3 r}{dt} = a\frac{\overline{m}^2}{\mu}n\Big\{ & -\tfrac{17}{48}e\gamma^2 \sin(nt-\omega) - \tfrac{17}{96}e\gamma^2 \sin(nt+\omega-2\Omega) \\
& + \tfrac{17}{96}e\gamma^2 \sin(3nt-\omega-2\Omega) + e\gamma^2(1.354629)\sin(3nt-2n't-\omega) \\
& - e\gamma^2(1.354649)\sin(nt-2n't+\omega) \\
& - e\gamma^2(0.2287330)\sin(3nt-2n't+\omega-2\Omega) \\
& - e\gamma^2(1.583382)\sin(nt-2n't-\omega+2\Omega) \\
& + e\gamma^2(0.2287330)\sin(5nt-2n't-\omega-2\Omega) \\
& - e\gamma^2(1.583382)\sin(nt+2n't-\omega-2\Omega) \\
& - \tfrac{23}{96}e^2\gamma^2\sin 2(nt-\Omega) - \tfrac{23}{96}e^2\gamma^2\sin 2(nt-\omega) \\
& + \tfrac{23}{96}e^2\gamma^2\sin(4nt-2\omega-2\Omega) - e^2\gamma^2(6.10863)\sin 2(nt-n't) \\
& + e^2\gamma^2(4.147520)\sin(4nt-2n't-2\omega) \\
& - e^2\gamma^2(1.96111)\sin 2(n't-\omega) \\
& + e^2\gamma^2(7.184577)\sin(4nt-2n't-2\Omega) \\
& + e^2\gamma^2(8.32440)\sin 2(n't-\Omega) \\
& - e^2\gamma^2(8.216965)\sin(2nt-2n't+2\omega-2\Omega) \\
& - e^2\gamma^2(1.555677)\sin(2nt-2n't-2\omega+2\Omega) \\
& + e^2\gamma^2(1.032388)\sin(6nt-2n't-2\omega-2\Omega) \\
& - e^2\gamma^2(9.88008)\sin(2nt+2n't-2\omega-2\Omega) \\
& + ee'\gamma^2(0.281647)\sin(nt+n't-\omega-\omega') \\
& - ee'\gamma^2(1.281647)\sin(nt-n't-\omega+\omega') \\
& - ee'\gamma^2(1.302099)\sin(3nt-n't-\omega-\omega') \\
& + ee'\gamma^2(1.302099)\sin(nt-n't+\omega-\omega') \\
& + ee'\gamma^2(3.294007)\sin(3nt-3n't-\omega+\omega') \\
& - ee'\gamma^2(3.294007)\sin(nt-3n't+\omega+\omega') \\
& + ee'\gamma^2(1.417084)\sin(nt+n't+\omega-\omega'-2\Omega) \\
& - ee'\gamma^2(1.1354375)\sin(nt-n't+\omega+\omega'-2\Omega) \\
& + ee'\gamma^2(0.09864475)\sin(3nt-n't+\omega-\omega'-2\Omega) \\
& + ee'\gamma^2(1.400744)\sin(nt-n't-\omega-\omega'+2\Omega) \\
& - ee'\gamma^2(0.938288)\sin(3nt-3n't+\omega+\omega'-2\Omega) \\
& - ee'\gamma^2(4.232295)\sin(nt-3n't-\omega+\omega'+2\Omega) \\
& - ee'\gamma^2(1.417084)\sin(3nt+n't-\omega-\omega'-2\Omega) \\
& + ee'\gamma^2(1.1354375)\sin(3nt-n't-\omega+\omega'-2\Omega) \\
& - ee'\gamma^2(0.09864475)\sin(5nt-n't-2\omega-\omega'-2\Omega) \\
& + ee'\gamma^2(1.400744)\sin(nt+n't-\omega+\omega'-2\Omega) \\
& + ee'\gamma^2(0.938288)\sin(5nt-3n't-\omega+\omega'-2\Omega) \\
& - ee'\gamma^2(4.232295)\sin(nt+3n't-\omega-\omega'-2\Omega)\Big\}
\end{aligned}\right\} . \quad (301)$$

If we now take the sum of equations (267), (274), (300), and (301), we shall obtain the complete value of $\frac{d\delta r}{dt}$, as follows:

$$
\left.
\begin{aligned}
\frac{d\delta r}{dt} = a\frac{\overline{m}^2}{\mu}n\Big\{ & -e\{0.875 - 1.359375e^2 + 1.3125e'^2 \\
& \qquad\qquad - 0.7708333\gamma^2\}\sin(nt-\omega) \\
& - e'\{0.1128333 + 0.311865e^2 + 0.1269375e'^2 - 0.16925\gamma^2\}\sin(n't-\omega') \\
& - e^2\{1 - 3.192708e^2 + \tfrac{3}{2}e'^2 - 0.869792\gamma^2 + 0.234375\frac{\gamma^4}{e^2}\}\sin 2(nt-\omega) \\
& - e'^2\{0.343312 + 0.96586e^2 + 0.2677982e'^2 - 0.516469\gamma^2\}\sin 2(n't-\omega') \\
& - ee'\{36.53352 + 4.31901e^2 + 41.10022e'^2 \\
& \qquad\qquad - 54.80023\gamma^2\}\sin(nt+n't-\omega-\omega') \\
& + ee'\{33.89207 - 0.01528e^2 + 38.12808e'^2 \\
& \qquad\qquad - 50.83793\gamma^2\}\sin(nt-n't-\omega+\omega') \\
& + \gamma^2\{1 - 1.389177e^2 + \tfrac{3}{2}e'^2 - \tfrac{7}{4}\gamma^2\}\sin 2(nt-\Omega) \\
& - e^3(1.57809)\sin 3(nt-\omega) - e'^3(0.782756)\sin 3(n't-\omega') \\
& - ee'^2(28.65920)\sin(nt+2n't-\omega-2\omega') \\
& + ee'^2(24.62185)\sin(nt-2n't-\omega+2\omega') \\
& - e^2e'(69.93758)\sin(2nt+n't-2\omega-\omega') \\
& + e^2e'(71.47053)\sin(2nt-n't-2\omega+\omega') \\
& + \tfrac{35}{24}e\gamma^2\sin(3nt-\omega-2\Omega) - \tfrac{5}{6}e\gamma^2\sin(nt+\omega-2\Omega) \\
& + e'\gamma^2(1.387119)\sin(2nt+n't-\omega'-2\Omega) \\
& + e'\gamma^2(1.631638)\sin(2nt-n't+\omega'-2\Omega) \\
& - \tfrac{1}{6}e^2\gamma^2\sin 2(\omega-\Omega) + e^2\gamma^2(1.998842)\sin(4nt-2\omega-2\Omega) \\
& + e^2\gamma^2(0.0234370)\sin(2nt-4\omega+2\Omega) + \tfrac{1}{4}\gamma^4\sin(2nt+2\omega-4\Omega) \\
& - ee'^3(29.73096)\sin(nt+3n't-\omega-3\omega') \\
& + ee'^3(23.59334)\sin(nt-3n't-\omega+3\omega') \\
& - e^3e'(116.88282)\sin(3nt+n't-3\omega-\omega') \\
& + e^3e'(121.87937)\sin(3nt-n't-3\omega+\omega') \\
& - e^2e'^2(52.91511)\sin(2nt+2n't-2\omega-2\omega') \\
& + e^2e'^2(55.36150)\sin(2nt-2n't-2\omega+2\omega') \\
& + e'^2\gamma^2(1.933957)\sin(2nt+2n't-2\omega'-2\Omega) \\
& + e'^2\gamma^2(2.680538)\sin(2nt-2n't+2\omega'-2\Omega) \\
& - e^4(2.53125)\sin 4(nt-\omega) - e'^4(1.581507)\sin 4(n't-\omega') \\
& + ee'\gamma^2(2.200914)\sin(3nt+n't-\omega-\omega'-2\Omega) \\
& + ee'\gamma^2(2.8315022)\sin(3nt-n't-\omega+\omega'-2\Omega)
\end{aligned}
\right\} \quad (302)
$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
-ee'\gamma^2\,(33.91977)\sin(nt+n't+\omega-\omega'-2\Omega) \\
+ee'\gamma^2\,(37.34411)\sin(nt-n't+\omega+\omega'-2\Omega) \\
-ee'\gamma^2\,(0.014105)\sin(nt+n't-3\omega-\omega'+2\Omega) \\
+ee'\gamma^2\,(0.014105)\sin(nt-n't-3\omega+\omega'+2\Omega) \\
+\{2.382700+55.75647e^2-5.95675e'^2-0.881940\gamma^2 \\
\qquad\quad -85.4495e^4+3.51041e'^4+0.912851\gamma^4-143.5570e^2e'^2 \\
\qquad\quad +2.204852e'^2\gamma^2-53.84484e^2\gamma^2\}\sin 2(nt-n't) \\
+e\{3.839622+96.75047e^2-9.599055e'^2 \\
\qquad\qquad\qquad -1.370622\gamma^2\}\sin(3nt-2n't-\omega) \\
+e\{24.69368-17.14267e^2-61.73422e'^2 \\
\qquad\qquad\qquad -22.768828\gamma^2\}\sin(nt-2n't+\omega) \\
-e'\{1.087759+52.55989e^2-0.135969e'^2 \\
\qquad\qquad\qquad -0.405318\gamma^2\}\sin(2nt-n't-\omega') \\
+e'\{9.20774+140.13880e^2-20.22414e'^2 \\
\qquad\qquad\qquad -3.384493\gamma^2\}\sin(2nt-3n't+\omega') \\
+e^2\{5.785218+154.2274e^2-15.463028e'^2 \\
\qquad\quad -1.979323\gamma^2-0.2094615\,\dfrac{\gamma^4}{e^2}\}\sin(4nt-2n't-2\omega) \\
-e^2\{5.31456-10.80775e^2-12.2862e'^2 \\
\qquad\quad -8.21729\gamma^2-0.1253682\,\dfrac{\gamma^4}{e^2}\}\sin 2(n't-\omega) \\
+e'^2\{24.93002+280.5456e^2-56.21175e'^2 \\
\qquad\qquad\qquad -9.09681\gamma^2\}\sin(2nt-4n't+2\omega') \\
-ee'\{1.7427374+90.04396e^2-0.217842e'^2 \\
\qquad\qquad\qquad +0.019255\gamma^2\}\sin(3nt-n't-\omega-\omega') \\
+ee'\{57.05573-44.4399e^2-125.3188e'^2 \\
\qquad\qquad\qquad -34.637142\gamma^2\}\sin(nt-3n't+\omega+\omega') \\
+ee'\{14.94025+245.7136e^2-32.81521e'^2 \\
\qquad\qquad\qquad -5.042187\gamma^2\}\sin(3nt-3n't-\omega+\omega') \\
-ee'\{24.89540-5.93963e^2-3.111922e'^2 \\
\qquad\qquad\qquad -7.901604\gamma^2\}\sin(nt-n't+\omega-\omega') \\
-\gamma^2\{0.0271238-2.885468e^2-0.0678085e'^2 \\
\qquad\qquad\qquad +0.00966416\gamma^2\}\sin(4nt-2n't-2\Omega) \\
-\gamma^2\{0.4983562-4.355539e^2-1.245890e'^2-0.5942523\gamma^2\}\sin 2(n't-\Omega) \\
+\gamma^2\{0.595675+7.02312e^2-1.489189e'^2 \\
\qquad\qquad\qquad -0.5083229\gamma^2\}\sin(2nt-2n't+2\omega-2\Omega)
\end{array}\right\} . \quad (302)
$$

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$$
\left.\begin{aligned}
&-\gamma^2\{0.595675-29.25068e^2-1.489189e'^2\\
&\qquad\qquad\qquad-0.807709\gamma^2\}\sin(2nt-2n't-2\omega+2\Omega)\\
&+e^3(8.782085)\sin(5nt-2n't-3\omega)-e^3(41.38248)\sin(nt+2n't-3\omega)\\
&+e'^3(0.03853108)\sin(2nt+n't-3\omega')\\
&+e'^3(58.24657)\sin(2nt-5n't+3\omega')\\
&+e\gamma^2(1.156170)\sin(3nt-2n't+\omega-2\Omega)\\
&+e\gamma^2(12.82417)\sin(nt-2n't-\omega+2\Omega)\\
&-e\gamma^2(0.9599083)\sin(3nt-2n't-3\omega+2\Omega)\\
&+e\gamma^2(6.173419)\sin(nt-2n't+3\omega-2\Omega)\\
&-e\gamma^2(24.79057)\sin(nt+2n't-\omega-2\Omega)\\
&-e\gamma^2(0.0859481)\sin(5nt-2n't-\omega-2\Omega)\\
&+e'\gamma^2(2.301935)\sin(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-e'\gamma^2(2.301935)\sin(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+e'\gamma^2(0.2719396)\sin(2nt-n't-2\omega-\omega'+2\Omega)\\
&-e'\gamma^2(0.2719396)\sin(2nt-n't+2\omega-\omega'-2\Omega)\\
&-e'\gamma^2(2.002395)\sin(3n't-\omega'-2\Omega)+e'\gamma^2(0.2167634)\sin(n't+\omega'-2\Omega)\\
&+e'\gamma^2(0.01301434)\sin(4nt-n't-\omega'-2\Omega)\\
&-e'\gamma^2(0.0990182)\sin(4nt-3n't+\omega'-2\Omega)\\
&+ee'^2(40.77597)\sin(3nt-4n't-\omega'+2\omega')\\
&+ee'^2(102.6964)\sin(nt-4n't+\omega+2\omega')\\
&-e^2e'(2.610552)\sin(4nt-n't-2\omega-\omega')\\
&+e^2e'(22.65144)\sin(4nt-3n't-2\omega+\omega')\\
&-e^2e'(22.4698)\sin(3n't-2\omega-\omega')+e^2e'(2.22209)\sin(n't-2\omega+\omega')\\
&+e^4(12.68360)\sin(6nt-2n't-4\omega)-e^4(76.36348)\sin(2nt+2n't-4\omega)\\
&+e'^4(0.07162802)\sin(2nt+2n't-4\omega')\\
&+e'^4(126.1904)\sin(2nt-6n't+4\omega')\\
&+\gamma^4(0.1764083)\sin(2nt-2n't+4\omega-4\Omega)\\
&-\gamma^4(0.00205562)\sin(2nt-2n't-4\omega+4\Omega)\\
&+\gamma^4(0.1139151)\sin(2nt+2n't-4\Omega)\\
&-\gamma^4(0.01356185)\sin(4nt-2n't+2\omega-4\Omega)\\
&+\gamma^4(0.00493759)\sin(6nt-2n't-4\Omega)\\
&-e^2\gamma^2(1.4245611)\sin(4nt-2n't-4\omega+2\Omega)\\
&-e^2\gamma^2(1.41489)\sin(2n't-4\omega+2\Omega)\\
&-e^2\gamma^2(47.35141)\sin(2nt+2n't-2\omega-2\Omega)\\
&-e^2\gamma^2(0.187174)\sin(6nt-2n't-2\omega-2\Omega)\\
&+ee'^3(0.06113652)\sin(3nt+n't-\omega-3\omega')
\end{aligned}\right\} \quad . \quad (302)
$$

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$$
\left.\begin{aligned}
&- ee'^2 (1.065486) \sin (nt + n't + \omega - 3\omega') \\
&+ ee'^3 (83.02541) \sin (3nt - 5n't - \omega + 3\omega') \\
&+ ee'^3 (167.7119) \sin (2nt - 5n't + \omega + 3\omega') \\
&- e^3e' (119.1115) \sin (nt + 3n't - 3\omega - \omega') \\
&+ e^3e' (33.55644) \sin (nt + n't - 3\omega + \omega') \\
&- e^3e' (3.843757) \sin (5nt - n't - 3\omega - \omega') \\
&+ e^3e' (39.32116) \sin (5nt - 3n't - 3\omega + \omega') \\
&+ e^2e'^2 (62.29238) \sin (4nt - 4n't - 2\omega + 2\omega') \\
&- e^2e'^2 (66.9172) \sin (4n't - 2\omega - 2\omega') \\
&+ e'^2\gamma^2 (6.232506) \sin (2nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&- e'^2\gamma^2 (6.232506) \sin (2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&- e'^2\gamma^2 (5.595899) \sin (4n't - 2\omega' - 2\Omega) \\
&- e'^2\gamma^2 (0.251066) \sin (4nt - 4n't + 2\omega' - 2\Omega) \\
&+ ee'\gamma^2 (4.445505) \sin (3nt - 3n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (25.37201) \sin (nt - n't - \omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 (0.5280468) \sin (3nt - n't + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (18.61989) \sin (nt - 3n't - \omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (3.735063) \sin (3nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&- ee'\gamma^2 (6.224016) \sin (nt - n't + 3\omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.4356306) \sin (3nt - n't - 3\omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 (15.675648) \sin (nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 (36.58055) \sin (nt + n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (45.33064) \sin (nt + 3n't - \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (0.934731) \sin (5nt - n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.311994) \sin (5nt - 3n't - \omega + \omega' - 2\Omega) \\
&- h'\alpha' \frac{n}{n^2 - \alpha'^2} \sin (\alpha' t - \beta') \Big\} \\
&+ a \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ -\{12.43582 + 35.72637e^2 + 24.87164e'^2 \\
&\qquad\qquad - 55.04355\gamma^2\} \sin (nt - n't) \\
&+ \{1.335670 - 21.41849e^2 - 8.01402e'^2 - 1.001752\gamma^2\} \sin 3(nt - n't) \\
&- e (27.96008) \sin (2nt - n't - \omega) + e (1.22548) \sin (n't - \omega) \\
&+ e (2.480983) \sin (4nt - 3n't + \omega) - e (11.16261) \sin (2nt - 3n't + \omega) \\
&- e' (18.49582) \sin (nt - 2n't + \omega') + e' (0.09375) \sin (nt - \omega') \\
&+ e' (7.001572) \sin (3nt - 4n't + \omega') - e' (1.276463) \sin (3nt - 2n't - \omega') \\
&- e^2 (47.69566) \sin (3nt - n't - 2\omega) + e^2 (53.11098) \sin (nt + n't - 2\omega)
\end{aligned}\right\} . \quad (302)
$$

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$$\left.\begin{aligned}
&+ e^2 (3.775487) \sin (5nt-3n't-2\omega) - e^2 (34.74803) \sin (nt - 3n't + 2\omega) \\
&+ e'^2 (17.35742) \sin (nt + n't - 2\omega') - e'^2 (26.97790) \sin (nt - 3n't + 2\omega') \\
&+ e'^2 (0.1527558) \sin (3nt - n't + 2\omega') + e'^2 (23.35469) \sin (3nt - 5n't + 2\omega') \\
&- ee' (0.937500) \sin (\omega - \omega') + ee' (0.09375) \sin (2nt - \omega - \omega') \\
&+ ee' (4.651690) \sin (2n't - \omega - \omega') \\
&- ee' (47.24880) \sin (2nt - 2n't - \omega + \omega') \\
&+ ee' (12.983378) \sin (4nt - 4n't - \omega + \omega') \\
&+ ee' (10.15500) \sin (2nt - 2n't + \omega - \omega') \\
&- ee' (2.374966) \sin (4nt - 2n't - \omega - \omega') \\
&- ee' (61.93044) \sin (2nt - 4n't + \omega + \omega') \\
&- \gamma^2 (1.554476) \sin (nt - n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (0.252459) \sin (nt - n't - 2\omega + 2\Omega) \\
&+ \gamma^2 (1.102626) \sin (3nt - n't - 2\Omega) + \gamma^2 (9.874348) \sin (nt + n't - 2\Omega) \\
&- \gamma^2 (5.095170) \sin (nt - 3n't + 2\Omega) \\
&+ \gamma^2 (0.498376) \sin (3nt - 3n't + 2\omega - 2\Omega) \\
&- \gamma^2 (0.498376) \sin (3nt - 3n't - 2\omega + 2\Omega) \\
&+ \tfrac{3}{16} h'' n \frac{10n + 12\alpha''}{n^2 - \alpha''^2} \sin (\alpha'' t - \beta'') \Big\}
\end{aligned}\right\} \quad (302)$$

This equation will give by integration, the numbers in brackets being logarithms,

$$\left.\begin{aligned}
\delta r = a \frac{\overline{m}^2}{\mu} \Big\{ &- \tfrac{1}{6} \{1 + \tfrac{1}{2} e^2 + \tfrac{3}{2} e'^2 - \tfrac{3}{2} \gamma^2 + \tfrac{15}{8} e'^4 - \tfrac{3}{4} e^2 \gamma^2 - \tfrac{9}{4} e'^2 \gamma^2 + \tfrac{3}{2} \gamma^4 \} \\
&+ e \{ [9.9420081] - [0.1333393] e^2 + [0.1180993] e'^2 \\
&\qquad - [9.8869605] \gamma^2 \} \cos (nt - \omega) \\
&+ e' \{ [0.1785282] + [0.6200576] e^2 + [0.2296808] e'^2 \\
&\qquad - [0.3546196] \gamma^2 \} \cos (n't - \omega') \\
&+ e^2 \{ \tfrac{1}{2} - [0.2031291] e^2 + \tfrac{3}{4} e'^2 - [9.6383854] \gamma^2 \\
&\qquad + [9.0688814] \frac{\gamma^4}{e^2} \} \cos 2 (nt - \omega) \\
&+ e'^2 \{ [0.3607499] + [0.8099751] e^2 + [0.2528686] e'^2 \\
&\qquad - [0.5381052] \gamma^2 \} \cos 2 (n't - \omega') \\
&+ ee' \{ [1.5313633] + [0.6040560] e^2 + [1.5825159] e'^2 \\
&\qquad - [1.7074542] \gamma^2 \} \cos (nt + n't - \omega - \omega') \\
&- ee' \{ [1.5638631] - [8.2178884] e^2 + [1.6150099] e'^2 \\
&\qquad - [1.7399529] \gamma^2 \} \cos (nt - n't - \omega + \omega')
\end{aligned}\right\} \quad (303)$$

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$$
\left.
\begin{aligned}
&-\gamma^2\{\tfrac{1}{2}-[9.8417273]e^2+\tfrac{3}{4}e'^2-\tfrac{7}{8}\gamma^2\}\cos 2(nt-\Omega)\\
&+e^3[9.7210105]\cos 3(nt-\omega)+e'^3[0.5425960]\cos 3(n't-\omega')\\
&+e\gamma^2[9.9208188]\cos(nt+\omega-2\Omega)\\
&-e\gamma^2[9.6867356]\cos(3nt-\omega-2\Omega)\\
&+ee'^2[1.3967162]\cos(nt+2n't-\omega-2\omega')\\
&-ee'^2[1.4616987]\cos(nt-2n't-\omega+2\omega')\\
&+e^2e'[1.5277342]\cos(2nt+n't-2\omega-\omega')\\
&-e^2e'[1.5696514]\cos(2nt-n't-2\omega+\omega')\\
&-e'\gamma^2[9.8251373]\cos(2nt+n't-\omega'-2\Omega)\\
&-e'\gamma[9.9281482]\cos(2nt-n't+\omega'-2\Omega)\\
&-e^2\gamma^2[9.6987184]\cos(4nt-2\omega-2\Omega)\\
&-e^2\gamma^2[8.0688720]\cos(2nt-4\omega+2\Omega)\\
&-\gamma^4[9.0969100]\cos(2nt+2\omega-4\Omega)\\
&+ee'^3[1.3852842]\cos(nt+3n't-\omega-3\omega')\\
&-ee'^3[1.4831538]\cos(nt-3n't-\omega+3\omega')\\
&+e^3e'[1.5799336]\cos(3nt+n't-3\omega-\omega')\\
&-e^3e'[1.6207748]\cos(3nt-n't-3\omega+\omega')\\
&+e^2e'^2[1.3912215]\cos(2nt+2n't-2\omega-2\omega')\\
&-e^2e'^2[1.4759428]\cos(2nt-2n't-2\omega+2\omega')\\
&-e'^2\gamma^2[9.9540886]\cos(2nt+2n't-2\omega'-2\Omega)\\
&-e'^2\gamma^2[0.1609569]\cos(2nt-2n't+2\omega'-2\Omega)\\
&+e^4[9.8012750]\cos 4(nt-\omega)+e'^4[0.7231020]\cos 4(n't-\omega')\\
&-ee'\gamma^2[9.8547860]\cos(3nt+n't-\omega-\omega'-2\Omega)\\
&-ee'\gamma^2[9.9858616]\cos(3nt-n't-\omega+\omega'-2\Omega)\\
&+ee'\gamma^2[1.4991305]\cos(nt+n't+\omega-\omega'-2\Omega)\\
&-ee'\gamma^2[1.6059871]\cos(nt-n't+\omega+\omega'-2\Omega)\\
&+ee'\gamma^2[8.1180449]\cos(nt+n't-3\omega-\omega'+2\Omega)\\
&-ee'\gamma^2[8.1831381]\cos(nt-n't-3\omega+\omega'+2\Omega)\\
&-\{[0.1098044]+[1.4790302]e^2-[0.5077443]e'^2-[9.6781740]\gamma^2\\
&\quad-[1.6644445]e^4+[0.2780928]e'^4+[9.6931349]\gamma^4-[1.8897590]e^2e'^2\\
&\qquad\qquad+[0.0761145]e'^2\gamma^2-[1.4638792]e^2\gamma^2\}\cos 2(nt-n't)\\
&-e\{[0.1293831]+[1.5307477]e^2-[0.5273231]e'^2\\
&\qquad\qquad-[9.6820122]\gamma^2\}\cos(3nt-2n't-\omega)\\
&-e\{[1.4629638]-[1.3044564]e^2-[1.8609040]e'^2\\
&\qquad\qquad-[1.4277188]\gamma^2\}\cos(nt-2n't+\omega)
\end{aligned}
\right\}\quad .\ (303)
$$

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$$
\left.
\begin{aligned}
&+ e'\{[9.7520571] + [1.4361789]e^2 - [8.8489643]e'^2 \\
&\qquad\qquad - [9.3233203]\gamma^2\} \cos(2nt - n't - \omega') \\
&- e'\{[0.7148090] + [1.8972143]e^2 - [1.0565261]e'^2 \\
&\qquad\qquad - [0.2801495]\gamma^2\} \cos(2nt - 3n't + \omega') \\
&- e^2\{[0.1768142] + [1.6026561]e^2 - [0.6037891]e'^2 \\
&\qquad\qquad - [9.7110112]\gamma^2 - [8.7355987]\frac{\gamma^4}{e^2}\} \cos(4nt - 2n't - 2\omega) \\
&+ e^2\{[1.5505282] - [1.8587962]e^2 - [1.9144785]e'^2 \\
&\qquad\qquad - [1.7397895]\gamma^2 - [9.9232482]\frac{\gamma^4}{e^2}\} \cos 2(n't - \omega) \\
&- e'^2\{[1.1660707] + [2.2173516]e^2 - [1.5191752]e'^2 \\
&\qquad\qquad - [0.7282372]\gamma^2\} \cos(2nt - 4n't + 2\omega') \\
&+ ee'\{[9.7750764] + [1.4882992]e^2 - [8.8719862]e'^2 \\
&\qquad\qquad + [7.8183881]\gamma^2\} \cos(3nt - n't - \omega - \omega') \\
&- ee'\{[1.8666637] - [1.7581375]e^2 - [2.2083806]e'^2 \\
&\qquad\qquad - [1.6499074]\gamma^2\} \cos(nt - 3n't + \omega + \omega') \\
&- ee'\{[0.7310015] + [1.9470729]e^2 - [1.0727189]e'^2 \\
&\qquad\qquad - [0.2592627]\gamma^2\} \cos(3nt - 3n't - \omega + \omega') \\
&+ ee'\{[1.4298841] - [0.8075244]e^2 - [0.5267937]e'^2 \\
&\qquad\qquad - [0.9314802]\gamma^2\} \cos(nt - n't + \omega - \omega') \\
&+ \gamma^2\{[7.8478450] - [9.8747105]e^2 - [8.2457785]e'^2 \\
&\qquad\qquad + [7.3996586]\gamma^2\} \cos(4nt - 2n't - 2\Omega) \\
&+ \gamma^2\{[0.5226007] - [1.4640879]e^2 - [0.9205406]e'^2 \\
&\qquad\qquad - [0.5990318]\gamma^2\} \cos 2(n't - \Omega) \\
&- \gamma^2\{[9.5077443] + [0.5792652]e^2 - [9.9056848]e'^2 \\
&\qquad\qquad - [9.4388747]\gamma^2\} \cos(2nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2\{[9.5077443] - [1.1988710]e^2 - [9.9056848]e'^2 \\
&\qquad\qquad - [9.6399900]\gamma^2\} \cos(2nt - 2n't - 2\omega + 2\Omega) \\
&- e^3[0.2578203] \cos(5nt - 2n't - 3\omega) \\
&+ e^3[1.5562687] \cos(nt + 2n't - 3\omega) \\
&- e'^3[8.2688348] \cos(2nt + n't - 3\omega') \\
&- e'^3[1.5541515] \cos(2nt - 5n't + 3\omega') \\
&- e\gamma^2[9.6081164] \cos(3nt - 2n't + \omega - 2\Omega) \\
&- e\gamma^2[1.1784073] \cos(nt - 2n't - \omega + 2\Omega) \\
&+ e\gamma^2[9.5273243] \cos(3nt - 2n't - 3\omega + 2\Omega) \\
&- e\gamma^2[0.8609037] \cos(nt - 2n't + 3\omega - 2\Omega)
\end{aligned}
\right\} . \qquad (303)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ e\gamma^2\,[1.3337387]\cos(nt + 2n't - \omega - 2\Omega)\\
&+ e\gamma^2\,[8.2484589]\cos(5nt - 2n't - \omega - 2\Omega)\\
&- e'\gamma^2\,[0.1127490]\cos(2nt - 3n't + 2\omega + \omega' - 2\Omega)\\
&+ e'\gamma^2\,[0.1127490]\cos(2nt - 3n't - 2\omega + \omega' + 2\Omega)\\
&- e'\gamma^2\,[9.1499969]\cos(2nt - n't - 2\omega - \omega' + 2\Omega)\\
&+ e'\gamma^2\,[9.1499969]\cos(2nt - n't + 2\omega - \omega' - 2\Omega)\\
&+ e'\gamma^2\,[0.9505193]\cos(3n't - \omega' - 2\Omega)\\
&- e'\gamma^2\,[0.4620768]\cos(n't + \omega' - 2\Omega)\\
&- e'\gamma^2\,[7.5205604]\cos(4nt - n't - \omega' - 2\Omega)\\
&+ e'\gamma^2\,[8.4187296]\cos(4nt - 3n't + \omega' - 2\Omega)\\
&- ee'^2\,[1.1789127]\cos(3nt - 4n't - \omega + 2\omega')\\
&- ee'^2\,[2.1659646]\cos(nt - 4n't + \omega + 2\omega')\\
&+ e^2e'\,[9.8228707]\cos(4nt - n't - 2\omega - \omega')\\
&- e^2e'\,[0.7781103]\cos(4nt - 3n't - 2\omega + \omega')\\
&+ e^2e'\,[2.0005688]\cos(3n't - 2\omega - \omega')\\
&- e^2e'\,[1.4728525]\cos(n't - 2\omega + \omega')\\
&- e^4\,[0.3360572]\cos(6nt - 2n't - 4\omega)\\
&+ e^4\,[1.5505275]\cos(2nt + 2n't - 4\omega)\\
&- e'^4\,[8.5227247]\cos(2nt + 2n't - 4\omega')\\
&- e'^4\,[1.9103607]\cos(2nt - 6n't + 4\omega')\\
&- \gamma^4\,[8.9792540]\cos(2nt - 2n't + 4\omega - 4\Omega)\\
&+ \gamma^4\,[7.0456778]\cos(2nt - 2n't - 4\omega + 4\Omega)\\
&- \gamma^4\,[8.7242231]\cos(2nt + 2n't - 4\Omega)\\
&+ \gamma^4\,[7.5468134]\cos(4nt - 2n't + 2\omega - 4\Omega)\\
&- \gamma^4\,[96.9263296]\cos(6nt - 2n't - 4\Omega)\\
&+ e^2\gamma^2\,[9.5681756]\cos(4nt - 2n't - 4\omega + 2\Omega)\\
&+ e^2\gamma^2\,[0.9757835]\cos(2n't - 4\omega + 2\Omega)\\
&+ e^2\gamma^2\,[1.3429747]\cos(2nt + 2n't - 2\omega - 2\Omega)\\
&+ e^2\gamma^2\,[8.5050601]\cos(6nt - 2n't - 2\omega - 2\Omega)\\
&- ee'^3\,[8.2984836]\cos(3nt + n't - \omega - 3\omega')\\
&+ ee'^3\,[9.9962196]\cos(nt + n't + \omega - 3\omega')\\
&- ee'^3\,[1.4999175]\cos(3nt - 5n't - \omega + 3\omega')\\
&- ee'^3\,[2.4279941]\cos(nt - 5n't + \omega + 3\omega')\\
&+ e^3e'\,[1.9880290]\cos(nt + 3n't - 3\omega - \omega')\\
&- e^3e'\,[1.4944477]\cos(nt + n't - 3\omega + \omega')\\
&+ e^3e'\,[9.8923322]\cos(5nt - n't - 3\omega - \omega')
\end{aligned}\right\}\quad . \qquad (303)$$

(Continued on the next page.)

$$
\begin{aligned}
&- e^3e' [0.9155989] \cos(5nt - 3n't - 3\omega + \omega') \\
&- e^2e'^2 [1.2261400] \cos(4nt - 4n't - 2\omega + 2\omega') \\
&+ e^2e'^2 [2.3495687] \cos(4n't - 2\omega - 2\omega') \\
&- e'^2\gamma^2 [0.5640108] \cos(2nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 [0.5640108] \cos(2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
&+ e'^2\gamma^2 [1.2719007] \cos(4n't - 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 [8.8314929] \cos(4nt - 4n't + 2\omega' - 2\Omega) \\
&- ee'\gamma^2 [0.2045648] \cos(3nt - 3n't + \omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 [1.4381199] \cos(nt - n't - \omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 [9.2565170] \cos(3nt - n't + \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 [1.3803415] \cos(nt - 3n't - \omega + \omega' + 2\Omega) \\
&+ ee'\gamma^2 [0.1289416] \cos(3nt - 3n't - 3\omega + \omega' + 2\Omega) \\
&+ ee'\gamma^2 [0.8278357] \cos(nt - n't + 3\omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 [9.1729630] \cos(3nt - n't - 3\omega - \omega' + 2\Omega) \\
&- ee'\gamma^2 [1.3055899] \cos(nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 [1.5319221] \cos(nt + n't - \omega + \omega' - 2\Omega) \\
&+ ee'\gamma^2 [1.5684671] \cos(nt + 3n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 [9.2782629] \cos(5nt - n't - \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 [8.8151188] \cos(5nt - 3n't - \omega + \omega' - 2\Omega) \\
&+ \frac{n^2}{n^2 - \alpha'^2} h' \cos(\alpha' t - \beta') - \tfrac{1}{6}\int e^2\gamma^2 \sin 2(\omega - \Omega)\, ndt \Big\} \\
+ a\frac{\overline{m}^2}{\mu}\frac{a}{a'} \Big\{ &+ \{[1.1284394] + [1.5867539]e^2 \\
&\qquad + [1.4294693]e'^2 - [1.7744715]\gamma^2\} \cos(nt - n't) \\
&- \{[9.6823429] - [0.8874326]e^2 - [0.4604941]e'^2 \\
&\qquad - [9.5574040]\gamma^2\} \cos 3(nt - n't) \\
&+ e [1.1620628] \cos(2nt - n't - \omega) - e [1.2143971] \cos(n't - \omega) \\
&- e [9.8176384] \cos(4nt - 3n't - \omega) \\
&+ e [0.7984216] \cos(2nt - 3n't + \omega) \\
&+ e' [1.3374516] \cos(nt - 2n't + \omega') - e' [8.9719713] \cos(nt - \omega') \\
&- e' [0.4137039] \cos(3nt - 4n't + \omega') \\
&+ e' [9.6511032] \cos(3nt - 2n't - \omega') \\
&+ e^2 [1.2123235] \cos(3nt - n't - 2\omega) \\
&- e^2 [1.6938561] \cos(nt + n't - 2\omega) \\
&- e^2 [9.8979455] \cos(5nt - 3n't - 2\omega) \\
&+ e^2 [1.6512946] \cos(nt - 3n't + 2\omega)
\end{aligned}
\quad (303)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e'^2\,[1.1481570] \cos(nt + n't - 2\omega') \\
&+ e'^2\,[1.5413726] \cos(nt - 3n't + 2\omega') \\
&- e'^2\,[8.7178423] \cos(3nt - n't - 2\omega') \\
&- e'^2\,[0.9490806] \cos(3nt - 5n't + 2\omega') \\
&- ee'\,[8.6709413] \cos(2nt - \omega - \omega') - ee'\,[1.4926717] \cos(2n't - \omega - \omega') \\
&+ ee'\,[1.4071258] \cos(2nt - 2n't - \omega + \omega') \\
&- ee'\,[0.5450928] \cos(4nt - 4n't - \omega + \omega') \\
&- ee'\,[0.7394149] \cos(2nt - 2n't + \omega - \omega') \\
&+ ee'\,[9.7901519] \cos(4nt - 2n't - \omega - \omega') \\
&+ ee'\,[1.5612523] \cos(2nt - 4n't + \omega + \omega') \\
&+ \gamma^2\,[0.2253487] \cos(nt - n't + 2\omega - 2\Omega) \\
&- \gamma^2\,[9.4359559] \cos(nt - n't - 2\omega + 2\Omega) \\
&- \gamma^2\,[9.5762728] \cos(3nt - n't - 2\Omega) \\
&- \gamma^2\,[0.9631802] \cos(nt + n't - 2\Omega) \\
&+ \gamma^2\,[0.8175231] \cos(nt - 3n't + 2\Omega) \\
&- \gamma^2\,[9.2542008] \cos(3nt - 3n't + 2\omega - 2\Omega) \\
&+ \gamma^2\,[9.2542008] \cos(3nt - 3n't - 2\omega + 2\Omega) \\
&- \tfrac{3}{16} h'' n^2 \frac{10n + 12\alpha''}{(n^2 - \alpha''^2)\alpha''} \cos(\alpha'' t - \beta'') \Big\}
\end{aligned}\right\} \quad (303)
$$

25. The value of δr given by equation (303) includes the whole perturbation of the radius vector arising from the first power of the sun's disturbing force; and if we substitute it in equation (262), we shall obtain the following value of $\frac{d\delta_1 v}{dt}$:

$$
\left.\begin{aligned}
\frac{d\delta_1 v}{dt} = \frac{\overline{m}^2}{\mu} n \Big\{ &- e\,\{\tfrac{3}{4} + \tfrac{107}{32}e^2 + \tfrac{9}{8}e'^2 - \tfrac{1}{24}\gamma^2\} \cos(nt - \omega) \\
&- e'\{3.016882 + 3.43160e^2 + 3.393992e'^2 - 4.525322\gamma^2\} \cos(n't - \omega') \\
&- e^2\{2.125 + 1.057257e^2 + 3.1875e'^2 - 0.932292\gamma^2 \\
&\qquad\qquad + 0.192709\frac{\gamma^4}{e^2}\} \cos 2(nt - \omega) \\
&- e'^2\{4.589654 + 5.43096e^2 + 3.580130e'^2 - 6.904548\gamma^2\} \cos 2(n't - \omega') \\
&- ee'\{72.50722 + 27.63419e^2 + 81.57065e'^2 \\
&\qquad\qquad - 108.76076\gamma^2\} \cos(nt + n't - \omega - \omega') \\
&+ ee'\{68.73910 + 16.30717e^2 + 77.33041e'^2 \\
&\qquad\qquad - 103.10828\gamma^2\} \cos(nt - n't - \omega + \omega') \\
&+ \gamma^2\{\tfrac{5}{6} - 0.298324e^2 + 1.25e'^2 - \tfrac{17}{12}\gamma^2\} \cos 2(nt - \Omega)
\end{aligned}\right\} \quad (304)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&- e^3 (4.281227) \cos 3(nt - \omega) - e'^3 (6.976314) \cos 3(n't - \omega') \\
&- ee'^2 (56.74380) \cos (nt + 2n't - \omega - 2\omega') \\
&+ ee'^2 (51.02220) \cos (nt - 2n't - \omega + 2\omega') \\
&- e^2e' (176.17703) \cos (2nt + n't - 2\omega - \omega') \\
&+ e^2e' (177.35607) \cos (2nt - n't - 2\omega + \omega') \\
&+ e\gamma^2 (2.3263889) \cos (3nt - \omega - 2\Omega) \\
&+ e\gamma^2 (0.35416667) \cos (nt + \omega - 2\Omega) \\
&+ e'\gamma^2 (2.0913311) \cos (2nt + n't - \omega' - 2\Omega) \\
&+ e'\gamma^2 (2.449254) \cos (2nt - n't + \omega' - 2\Omega) \\
&+ e^2\gamma^2 (5.0723376) \cos (4nt - 2\omega - 2\Omega) \\
&+ e^2\gamma^2 (0.0234370) \cos (2nt - 4\omega + 2\Omega) \\
&- e^2\gamma^2 (0.21875) \cos 2(\omega - \Omega) + \gamma^4 (0.2083333) \cos (2nt + 2\omega - 4\Omega) \\
&- \tfrac{5}{24}\gamma^4 \cos 4(nt - \Omega) - ee'^3 (59.02843) \cos (nt + 3n't - \omega - 3\omega') \\
&+ ee'^3 (50.37479) \cos (nt - 3n't - \omega + 3\omega') \\
&- e^3e' (340.10323) \cos (3nt + n't - 3\omega - \omega') \\
&+ e^3e' (349.74539) \cos (3nt - n't - 3\omega + \omega') \\
&- e^2e'^2 (134.34815) \cos (2nt + 2n't - 2\omega - 2\omega') \\
&+ e^2e'^2 (136.37073) \cos (2nt - 2n't - 2\omega + 2\omega') \\
&+ e'^2\gamma^2 (2.9467755) \cos (2nt + 2n't - 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 (4.044669) \cos (2nt - 2n't + 2\omega' - 2\Omega) \\
&- e^4 (7.682257) \cos 4(nt - \omega) - e'^4 (10.571388) \cos 4(n't - \omega') \\
&+ ee'\gamma^2 (23.072492) \cos (3nt + n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (11.197846) \cos (3nt - n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (79.80660) \cos (nt + nt + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (99.88761) \cos (nt - n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.02624672) \cos (nt + n't - 3\omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 (0.03049076) \cos (nt - n't - 3\omega + \omega' + 2\Omega) \\
&+ \{2.575339 + 153.99421e^2 - 6.438346e'^2 - 0.9532434\gamma^2 \\
&\qquad - 84.2927e^4 + 3.794222e'^4 + 2.655776\gamma^4 - 389.48813e^2e'^2 \\
&\qquad + 2.383112e'^2\gamma^2 - 140.91697e^2\gamma^2\} \cos 2(nt - n't) \\
&+ e\{6.557103 + 298.56818e^2 - 16.392759e'^2 \\
&\qquad\qquad - 2.3915711\gamma^2\} \cos (3nt - 2n't - \omega) \\
&+ e\{61.93863 + 10.05579e^2 - 154.84662e'^2 \\
&\qquad\qquad - 54.97855\gamma^2\} \cos (nt - 2n't + \omega) \\
&- e'\{1.1300226 + 138.24387e^2 - 0.1412519e'^2 \\
&\qquad\qquad - 0.4210660\gamma^2\} \cos (2nt - n't - \omega')
\end{aligned}
\right\} \quad . \quad (304)
$$

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$$\left.\begin{aligned}
&+ e'\{10.371440 + 405.06078e^2 - 22.78012e'^2 \\
&\qquad\qquad\qquad - 3.812234\gamma^2\} \cos(2nt - 3n't + \omega') \\
&+ e^2\{12.840652 + 524.4926e^2 - 32.62105e'^2 \\
&\qquad\qquad - 4.615471\gamma^2 - 0.1088\frac{\gamma^4}{e^2}\} \cos(4nt - 2n't - 2\omega) \\
&+ e^2\{21.85892 + 107.5565e^2 - 68.01873e'^2 \\
&\qquad\qquad + 27.13443\gamma^2 - 1.676016\frac{\gamma^4}{e^2}\} \cos 2(n't - \omega) \\
&+ e'^2\{29.31574 + 844.1369e^2 - 66.10056e'^2 \\
&\qquad\qquad\qquad - 10.697128\gamma^2\} \cos(2nt - 4n't + 2\omega') \\
&- ee'\{2.8865679 + 268.80031e^2 - 0.360820e'^2 \\
&\qquad\qquad\qquad - 0.618434\gamma^2\} \cos(3nt - n't - \omega - \omega') \\
&+ ee'\{162.68462 + 2.8591e^2 - 357.3250e'^2 \\
&\qquad\qquad\qquad - 95.03601\gamma^2\} \cos(nt - 3n't + \omega + \omega') \\
&+ ee'\{26.322592 + 783.35101e^2 - 57.81570e'^2 \\
&\qquad\qquad\qquad - 9.351579\gamma^2\} \cos(3nt - 3n't - \omega + \omega') \\
&- ee'\{55.51137 + 37.50035e^2 - 6.938914e'^2 \\
&\qquad\qquad\qquad - 17.712479\gamma^2\} \cos(nt - n't + \omega - \omega') \\
&- \gamma^2\{0.657924 + 59.59805e^2 - 1.644808e'^2 \\
&\qquad\qquad\qquad - 0.555208\gamma^2\} \cos(4nt - 2n't - 2\Omega) \\
&- \gamma^2\{7.306235 - 22.42989e^2 - 18.26559e'^2 - 8.504640\gamma^2\} \cos 2(n't - \Omega) \\
&+ \gamma^2\{0.6438346 + 56.21997e^2 - 1.609588e'^2 \\
&\qquad\qquad\qquad - 0.5494202\gamma^2\} \cos(2nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2\{0.6438346 - 59.79719e^2 - 1.609588e'^2 \\
&\qquad\qquad\qquad - 0.8710036\gamma^2\} \cos(2nt - 2n't - 2\omega + 2\Omega) \\
&+ e^3(22.721202) \cos(5nt - 2n't - 3\omega) \\
&- e^3(39.36698) \cos(nt + 2n't - 3\omega) \\
&+ e'^3(0.03714196) \cos(2nt + n't - 3\omega') \\
&+ e'^3(71.64428) \cos(2nt - 5n't + 3\omega') \\
&- e\gamma^2(12.44113) \cos(3nt - 2n't + \omega - 2\Omega) \\
&+ e\gamma^2(18.84947) \cos(nt - 2n't - \omega + 2\Omega) \\
&- e\gamma^2(69.89484) \cos(nt + 2n't - \omega - 2\Omega) \\
&- e\gamma^2(2.983516) \cos(5nt - 2n't - \omega - 2\Omega) \\
&- e\gamma^2(1.6392779) \cos(3nt - 2n't - 3\omega + 2\Omega) \\
&+ e\gamma^2(15.48465) \cos(nt - 2n't + 3\omega - 2\Omega) \\
&- e'\gamma^2(2.645312) \cos(4nt - 3n't + \omega' - 2\Omega)
\end{aligned}\right\} \quad (304)$$

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$$
\left.
\begin{aligned}
&+ e'\gamma^2 (0.2891368) \cos (4nt - n't - \omega' - 2\Omega) \\
&- e'\gamma^2 (20.439202) \cos (3n't - \omega' - 2\Omega) \\
&- e'\gamma^2 (6.078220) \cos (n't + \omega' - 2\Omega) \\
&+ e'\gamma^2 (2.592860) \cos (2nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&- e'\gamma^2 (2.592860) \cos (2nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&+ e'\gamma^2 (0.2825056) \cos (2nt - n't - 2\omega - \omega' + 2\Omega) \\
&- e'\gamma^2 (0.2825056) \cos (2nt - n't + 2\omega - \omega' - 2\Omega) \\
&+ ee'^2 (74.16913) \cos (3nt - 4n't - \omega + 2\omega') \\
&+ ee'^2 (337.0592) \cos (nt - 4n't + \omega + 2\omega') \\
&+ e^2e' (51.48276) \cos (4nt - 3n't - 2\omega + \omega') \\
&- e^2e' (5.660002) \cos (4nt - n't - 2\omega - \omega') \\
&+ e^2e' (43.7650) \cos (3n't - 2\omega - \omega') - e^2e' (23.85392) \cos (n't - 2\omega + \omega') \\
&+ e^4 (37.846015) \cos (6nt - 2n't - 4\omega) \\
&- e^4 (134.1316) \cos (2nt + 2n't - 4\omega) \\
&+ e'^4 (0.06664302) \cos (2nt + 2n't - 4\omega') \\
&+ e'^4 (162.70118) \cos (2nt - 6n't + 4\omega') \\
&+ \gamma^4 (1.9325457) \cos (2nt + 2n't - 4\Omega) \\
&+ \gamma^4 (0.1661686) \cos (6nt - 2n't - 4\Omega) \\
&+ \gamma^4 (0.19067072) \cos (2nt - 2n't + 4\omega - 4\Omega) \\
&- \gamma^4 (0.002221814) \cos (2nt - 2n't - 4\omega + 4\Omega) \\
&- \gamma^4 (0.3289616) \cos (4nt - 2n't + 2\omega - 4\Omega) \\
&- e^2\gamma^2 (3.198873) \cos (4nt - 2n't - 4\omega + 2\Omega) \\
&+ e^2\gamma^2 (4.311664) \cos (2n't - 4\omega + 2\Omega) \\
&- e^2\gamma^2 (161.26623) \cos (2nt + 2n't - 2\omega - 2\Omega) \\
&- e^2\gamma^2 (8.730023) \cos (6nt - 2n't - 2\omega - 2\Omega) \\
&+ ee'^3 (0.0954791) \cos (3nt + n't - \omega - 3\omega') \\
&- ee'^3 (1.9269535) \cos (nt + n't + \omega - 3\omega') \\
&+ ee'^3 (170.69996) \cos (3nt - 5n't - \omega + 3\omega') \\
&+ ee'^3 (643.2928) \cos (nt - 5n't + \omega + 3\omega') \\
&- e^3e' (129.56371) \cos (nt + 3n't - 3\omega - \omega') \\
&+ e^3e' (26.73187) \cos (nt + n't - 3\omega + \omega') \\
&- e^3e' (9.98022) \cos (5nt - n't - 3\omega - \omega') \\
&+ e^3e' (93.04337) \cos (5nt - 3n't - 3\omega + \omega') \\
&+ e^2e'^2 (144.91804) \cos (4nt - 4n't - 2\omega + 2\omega') \\
&+ e^2e'^2 (58.2890) \cos (4n't - 2\omega - 2\omega') \\
&+ e'^2\gamma^2 (7.328934) \cos (2nt - 4n't + 2\omega + 2\omega' - 2\Omega)
\end{aligned}
\right\} . \quad (304)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
& - e'^2\gamma^2 (7.328934) \cos (2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
& - e'^2\gamma^2 (44.73401) \cos (4n't - 2\omega' - 2\Omega) \\
& - e'^2\gamma^2 (7.464617) \cos (4nt - 4n't + 2\omega' - 2\Omega) \\
& - ee'\gamma^2 (28.471543) \cos (3nt - 3n't + \omega + \omega' - 2\Omega) \\
& - ee'\gamma^2 (45.57266) \cos (nt - n't - \omega - \omega' + 2\Omega) \\
& + ee'\gamma^2 (12.537988) \cos (3nt - n't + \omega - \omega' - 2\Omega) \\
& + ee'\gamma^2 (15.96067) \cos (nt - 3n't - \omega + \omega' + 2\Omega) \\
& + ee'\gamma^2 (91.20586) \cos (nt + n't - \omega + \omega' - 2\Omega) \\
& - ee'\gamma^2 (146.67160) \cos (nt + 3n't - \omega - \omega' - 2\Omega) \\
& + ee'\gamma^2 (1.6562773) \cos (5nt - n't - \omega - \omega' - 2\Omega) \\
& - ee'\gamma^2 (11.975707) \cos (5nt - 3n't - \omega + \omega' - 2\Omega) \\
& - ee'\gamma^2 (6.580650) \cos (3nt - 3n't - 3\omega + \omega' + 2\Omega) \\
& - ee'\gamma^2 (13.878202) \cos (nt - n't + 3\omega - \omega' - 2\Omega) \\
& + ee'\gamma^2 (0.7216052) \cos (3nt - n't - 3\omega - \omega' + 2\Omega) \\
& + ee'\gamma^2 (44.31149) \cos (nt - 3n't + 3\omega + \omega' - 2\Omega) \\
& \left. - 2 \frac{n^3}{n^2 - \kappa'^2} h' \cos (\kappa' t - \beta') \right\} \\
& + \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ - \{26.88248 + 98.53234 e^2 + 53.76496 e'^2 \\
& \qquad\qquad - 118.98754\gamma^2\} \cos (nt - n't) \\
& + \{0.9624384 - 31.35972 e^2 - 5.774630 e'^2 - 0.7218284\gamma^2\} \cos 3(nt - n't) \\
& - e (69.37016) \cos (2nt - n't - \omega) - e (7.55744) \cos (n't - \omega) \\
& + e (3.757878) \cos (4nt - 3n't - \omega) - e (11.129711) \cos (2nt - 3n't + \omega) \\
& - e' (43.49924) \cos (nt - 2n't + \omega') + e' (0.18750) \cos (nt - \omega') \\
& + e' (5.184824) \cos (3nt - 4n't + \omega') \\
& - e' (0.8956394) \cos (3nt - 2n't - \omega') \\
& - e^2 (136.66544) \cos (3nt - n't - 2\omega) + e^2 (87.49324) \cos (nt + n't - 2\omega) \\
& + e^2 (5.717976) \cos (5nt - 3n't - 2\omega) \\
& - e^2 (106.29796) \cos (nt - 3n't + 2\omega) \\
& + e'^2 (28.13112) \cos (nt + n't - 2\omega') \\
& - e'^2 (69.56690) \cos (nt - 3n't + 2\omega') \\
& + e'^2 (0.1044413) \cos (3nt - n't - 2\omega') \\
& + e'^2 (17.787326) \cos (3nt - 5n't + 2\omega') \\
& + ee' (0.37500) \cos (2nt - \omega - \omega') - ee' (3.06156) \cos (2n't - \omega - \omega') \\
& - ee' (116.31766) \cos (2nt - 2n't - \omega + \omega') \\
& + ee' (9.632561) \cos (2nt - 2n't + \omega - \omega')
\end{aligned}
\right\} . \quad (304)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ ee'(14.793774)\cos(4nt-4n't-\omega+\omega') \\
&- ee'(2.5770805)\cos(4nt-2n't-\omega-\omega') \\
&- ee'(65.04810)\cos(2nt-4n't+\omega+\omega') \\
&+ ee'(0.28125)\cos(\omega-\omega') - \gamma^2(3.360304)\cos(nt-n't+2\omega-2\Omega) \\
&+ \gamma^2(0.545740)\cos(nt-n't-2\omega+2\Omega) \\
&+ \gamma^2(7.47450)\cos(3nt-n't-2\Omega) + \gamma^2(25.09490)\cos(nt+n't-2\Omega) \\
&- \gamma^2(0.2406095)\cos(5nt-3n't-2\Omega) \\
&- \gamma^2(13.379329)\cos(nt-3n't+2\Omega) \\
&+ \gamma^2(0.3591126)\cos(3nt-3n't+2\omega-2\Omega) \\
&- \gamma^2(0.3591126)\cos(3nt-3n't-2\omega+2\Omega) \\
&+ \tfrac{3}{8}h''n^2\frac{10n+12\alpha''}{(n^2-\alpha''^2)\alpha''}\cos(\alpha''t-\beta'')\Big\}
\end{aligned}\right\} \quad (304)
$$

The value of $\delta\theta$ will give the following value of $\frac{d\delta_2 v}{dt}$:

$$
\left.\begin{aligned}
\frac{d\delta_2 v}{dt} = \frac{\overline{m}^2}{\mu}n\Big\{ &-\gamma^2\{0.708333 - 1.5416667e^2 + 1.0625e'^2 \\
&\qquad - 0.871639\gamma^2\}\cos 2(nt-\Omega) \\
&+ e\gamma^2(3.9583333)\cos(nt-\omega) - e'\gamma^2(1.126580)\cos(n't-\omega') \\
&- e\gamma^2(0.9583333)\cos(3nt-\omega-2\Omega) \\
&+ e'\gamma^2(5.668340)\cos(2nt+n't-\omega'-2\Omega) \\
&- e'\gamma^2(4.541760)\cos(2nt-n't+\omega'-2\Omega) \\
&+ \gamma^2\{(4.770834)e^2 + 0.1770833\gamma^2\}\cos 2(nt-\omega) \\
&- e^2\gamma^2(0.239583)\cos(4nt-2\omega-2\Omega) \\
&- e^2\gamma^2(3.614586)\cos 2(\omega-\Omega) - \gamma^4(0.475806)\cos 4(nt-\Omega) \\
&- \gamma^4(0.1770833)\cos(2nt+2\omega-4\Omega) + e'^2\gamma^2(7.992264)\cos 2(n't-\omega') \\
&- e'^2\gamma^2(4.844630)\cos(2nt+2n't-2\omega'-2\Omega) \\
&- e'^2\gamma^2(3.147634)\cos(2nt-2n't+2\omega'-2\Omega) \\
&+ ee'\gamma^2(2.07146)\cos(nt+n't-\omega-\omega') \\
&+ ee'\gamma^2(1.71307)\cos(nt-n't-\omega+\omega') \\
&+ ee'\gamma^2(51.06368)\cos(3nt+n't-\omega-\omega'-2\Omega) \\
&+ ee'\gamma^2(50.34174)\cos(3nt-n't-\omega+\omega'-2\Omega) \\
&+ ee'\gamma^2(28.20847)\cos(nt+n't+\omega-\omega'-2\Omega) \\
&- ee'\gamma^2(32.71494)\cos(nt-n't+\omega+\omega'-2\Omega) \\
&- \gamma^2\{5.418598 - 16.02403e^2 - 13.55069e'^2 + 14.57160\gamma^2\}\cos 2(nt-n't) \\
&- \gamma^2\{0.9149317 + 136.15422e^2 - 2.283133e'^2 \\
&\qquad - 3.69380\gamma^2\}\cos(4nt-2n't-2\Omega)
\end{aligned}\right\} \quad (305)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\gamma^2\{6.333530+17.85631e^2-15.83382e'^2-3.62048\gamma^2\}\cos 2(n't-\Omega)\\
&-e\gamma^2(16.590197)\cos(3nt-2n't-\omega)+e\gamma^2(7.90621)\cos(nt-2n't+\omega)\\
&-e\gamma^2(4.129510)\cos(5nt-2n't-\omega-2\Omega)\\
&+e\gamma^2(39.45857)\cos(nt+2n't-\omega-2\Omega)\\
&-e\gamma^2(32.86786)\cos(3nt-2n't+\omega-2\Omega)\\
&+e\gamma^2(6.222786)\cos(nt-2n't-\omega+2\Omega)\\
&-e'\gamma^2(5.208398)\cos(2nt-n't-\omega')-e'\gamma^2(13.17602)\cos(2nt-3n't+\omega')\\
&+e'\gamma^2(0.3945790)\cos(4nt-n't-\omega'-2\Omega)\\
&-e'\gamma^2(3.753162)\cos(4nt-3n't+\omega'-2\Omega)\\
&+e'\gamma^2(16.92918)\cos(3n't-\omega'-2\Omega)-e'\gamma^2(5.602977)\cos(n't+\omega'-2\Omega)\\
&-\gamma^2\{46.76502e^2-0.976193\gamma^2\}\cos(4nt-2n't-2\omega)\\
&+e^2\gamma^2(14.10645)\cos 2(n't-\omega)\\
&-e^2\gamma^2(12.019134)\cos(6nt-2n't-2\omega-2\Omega)\\
&+e^2\gamma^2(120.00338)\cos(2nt+2n't-2\omega-2\Omega)\\
&-e'^2\gamma^2(25.05469)\cos(2nt-4n't+2\omega')\\
&-e'^2\gamma^2(10.82207)\cos(4nt-4n't+2\omega'-2\Omega)\\
&+e'^2\gamma^2(35.87676)\cos(4n't-2\omega'-2\Omega)\\
&+\gamma^4(0.4614572)\cos(6nt-2n't-4\Omega)\\
&+\gamma^4(14.03683)\cos(2nt+2n't-4\Omega)\\
&+\gamma^2\{8.766409e^2-1.3546493\gamma^2\}\cos(2nt-2n't+2\omega-2\Omega)\\
&+\gamma^2\{18.18179e^2+0.8359221\gamma^2\}\cos(2nt-2n't-2\omega+2\Omega)\\
&-\gamma^4(0.4574660)\cos(4nt-2n't+2\omega-4\Omega)\\
&+ee'\gamma^2(15.769645)\cos(3nt-n't-\omega-\omega')\\
&-ee'\gamma^2(6.39584)\cos(nt-n't+\omega-\omega')\\
&+ee'\gamma^2(22.51650)\cos(nt-3n't+\omega+\omega')\\
&-ee'\gamma^2(40.90157)\cos(3nt-3n't-\omega+\omega')\\
&+ee'\gamma^2(1.784628)\cos(5nt-n't-\omega-\omega'-2\Omega)\\
&-ee'\gamma^2(16.874624)\cos(5nt-3n't-\omega+\omega'-2\Omega)\\
&-ee'\gamma^2(34.10034)\cos(nt+n't-\omega+\omega'-2\Omega)\\
&+ee'\gamma^2(108.78710)\cos(nt+3n't-\omega-\omega'-2\Omega)\\
&-ee'\gamma^2(89.93892)\cos(3nt-3n't+\omega+\omega'-2\Omega)\\
&+ee'\gamma^2(28.50107)\cos(3nt-n't+\omega-\omega'-2\Omega)\\
&-ee'\gamma^2(5.561238)\cos(nt-n't-\omega-\omega'+2\Omega)\\
&+ee'\gamma^2(16.41151)\cos(nt-3n't-\omega+\omega'+2\Omega)\ \Big\}
\end{aligned}
\right\} \quad (305)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{-\gamma^2(1.56992)\cos(nt-n't)+\gamma^2(0.8709828)\cos 3(nt-n't)\\
&\quad+\gamma^2(13.61610)\cos(3nt-n't-2\Omega)-\gamma^2(12.04618)\cos(nt+n't-2\Omega)\\
&\quad-\gamma^2(0.2950662)\cos(5nt-3n't-2\Omega)\\
&\quad-\gamma^2(0.5759166)\cos(nt-3n't+2\Omega)\Big\}
\end{aligned}\right\} . \quad (305)
$$

If we now take the sum of the partial values of $\frac{d\delta v}{dt}$, given in equations (276), (304), and (305), we shall obtain the following complete value of $\frac{d\delta v}{dt}$:

$$
\left.\begin{aligned}
\frac{d\delta v}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{&-e\{\tfrac{3}{4}+\tfrac{107}{32}e^2+\tfrac{9}{8}e'^2-4\gamma^2\}\cos(nt-\omega)\\
&-e'\{3.016882+3.43160e^2+2.393992e'^2-3.398742\gamma^2\}\cos(n't-\omega')\\
&-e^2\{2.125+1.057257e^2+3.1875e'^2-5.703126\gamma^2\\
&\qquad\qquad+0.015626\frac{\gamma^4}{e^2}\}\cos 2(nt-\omega)\\
&-e'^2\{4.589654+5.43096e^2+3.580130e'^2-14.896812\gamma^2\}\cos 2(n't-\omega')\\
&-ee'\{72.50722+27.63419e^2+81.57065e'^2\\
&\qquad\qquad-110.83222\gamma^2\}\cos(nt+n't-\omega-\omega')\\
&+ee'\{68.73910+16.30717e^2+77.33041e'^2\\
&\qquad\qquad-101.39521\gamma^2\}\cos(nt-n't-\omega+\omega')\\
&+\gamma^2\{0.125+1.839991e^2+0.1875e'^2-0.545028\gamma^2\}\cos 2(nt-\Omega)\\
&-e^3(4.281227)\cos 3(nt-\omega)-e'^3(6.976314)\cos 3(n't-\omega')\\
&-ee'^2(56.74380)\cos(nt+2n't-\omega-2\omega')\\
&+ee'^2(51.02220)\cos(nt-2n't-\omega+2\omega')\\
&-e^2e'(176.17703)\cos(2nt+n't-2\omega-\omega')\\
&+e^2e'(177.35607)\cos(2nt-n't-3\omega+\omega')\\
&+e\gamma^2(1.3680556)\cos(3nt-\omega-2\Omega)-e\gamma^2(2.64583333)\cos(nt+\omega-2\Omega)\\
&+e'\gamma^2(7.759671)\cos(2nt+n't-\omega'-2\Omega)\\
&-e'\gamma^2(2.092506)\cos(2nt-n't+\omega'-2\Omega)\\
&+e^2\gamma^2(4.8327546)\cos(4nt-2\omega-2\Omega)\\
&+e^2\gamma^2(0.0234370)\cos(2nt-4\omega+2\Omega)-\tfrac{23}{8}e^2\gamma^2\cos 2(\omega-\Omega)\\
&+\gamma^4(0.03125)\cos(2nt+2\omega-4\Omega)-\gamma^4(0.684139)\cos 4(nt-\Omega)\\
&-ee'^3(59.02843)\cos(nt+3n't-\omega-3\omega')\\
&+ee'^3(50.37479)\cos(nt-3n't-\omega+3\omega')\\
&-e^3e'(340.10323)\cos(3nt+n't-3\omega-\omega')
\end{aligned}\right\} . \quad (306)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ e^3e' (349.74539) \cos (3nt - n't - 3\omega + \omega') \\
&- e^2e'^2 (134.34815) \cos (2nt + 2n't - 2\omega - 2\omega') \\
&+ e^2e'^2 (136.37073) \cos (2nt - 2n't - 2\omega + 2\omega') \\
&- e'^2\gamma^2 (1.897855) \cos (2nt + 2n't - 2\omega' - 2\Omega) \\
&+ e'^2\gamma^2 (0.897035) \cos (2nt - 2n't + 2\omega' - 2\Omega) \\
&- e^4 (7.682257) \cos 4(nt - \omega) - e'^4 (10.571388) \cos 4(n't - \omega') \\
&+ ee'\gamma^2 (74.13617) \cos (3nt + n't - \omega - \omega' - 2\Omega) \\
&- ee'\gamma^2 (61.53959) \cos (3nt - n't - \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (51.59813) \cos (nt + n't + \omega - \omega' - 2\Omega) \\
&+ ee'\gamma^2 (67.17267) \cos (nt - n't + \omega + \omega' - 2\Omega) \\
&- ee'\gamma^2 (0.02624672) \cos (nt + n't - 3\omega - \omega' + 2\Omega) \\
&+ ee'\gamma^2 (0.03049076) \cos (nt - n't - 3\omega + \omega' + 2\Omega) \\
&+ \{3.385976 + 147.60754e^2 - 8.464938e'^2 - 6.169182\gamma^2 \\
&\qquad - 114.85443e^4 + 4.452864e'^4 - 11.10299\gamma^4 - 373.52145e^2e'^2 \\
&\qquad\qquad + 15.42715e'^2\gamma^2 - 126.48961e^2\gamma^2\} \cos 2(nt - n't) \\
&+ e\{7.893982 + 289.63231e^2 - 19.734957e'^2 \\
&\qquad\qquad - 18.647548\gamma^2\} \cos (3nt - 2n't - \omega) \\
&+ e\{57.45762 - 15.85494e^2 - 143.64410e'^2 \\
&\qquad\qquad - 48.19259\gamma^2\} \cos (nt - 2n't + \omega) \\
&- e'\{1.5186969 + 135.29100e^2 - 0.1898362e'^2 \\
&\qquad\qquad + 4.884501\gamma^2\} \cos (2nt - n't - \omega') \\
&+ e'\{13.328194 + 380.73188e^2 - 29.27442e'^2 \\
&\qquad\qquad - 16.24906\gamma^2\} \cos (2nt - 3n't + \omega') \\
&+ e^2\{14.768865 + 511.9869e^2 - 37.44158e'^2 \\
&\qquad\qquad - 51.86254\gamma^2 + 0.96872\frac{\gamma^4}{e^2}\} \cos (4nt - 2n't - 2\omega) \\
&- e^2\{7.48583 - 94.7428e^2 - 5.54316e'^2 \\
&\qquad\qquad - 33.9046\gamma^2 + 1.04936\frac{\gamma^4}{e^2}\} \cos 2(n't - \omega) \\
&+ e'^2\{36.81224 + 779.2839e^2 - 83.00442e'^2 \\
&\qquad\qquad - 33.87769\gamma^2\} \cos (2nt - 4n't + 2\omega') \\
&- ee'\{3.531635 + 264.6448e^2 - 0.441433e'^2 \\
&\qquad\qquad - 16.226812\gamma^2\} \cos (3nt - n't - \omega - \omega') \\
&+ ee'\{145.33441 - 58.70168e^2 - 319.7564e'^2 \\
&\qquad\qquad - 76.85706\gamma^2\} \cos (nt - 3n't + \omega + \omega')
\end{aligned}\right\} . \quad (306)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+ ee'\{31.170919 + 749.52802e^2 - 68.46460e'^2 \\
&\qquad\qquad\qquad - 49.04107\gamma^2\} \cos(3nt - 3n't - \omega + \omega') \\
&- ee'\{53.46813 + 12.02982e^2 - 6.683510e'^2 \\
&\qquad\qquad\qquad - 11.82776\gamma^2\} \cos(nt - n't + \omega - \omega') \\
&- \gamma^2\{1.824211 + 189.05906e^2 - 4.556330e'^2 \\
&\qquad\qquad\qquad - 4.257847\gamma^2\} \cos(4nt - 2n't - 2\Omega) \\
&- \gamma^2\{4.935325 - 39.42867e^2 - 12.33832e'^2 - 5.87481\gamma^2\} \cos 2(n't - \Omega) \\
&+ \gamma^2\{0.8464938 + 65.38412e^2 - 1.116236e'^2 \\
&\qquad\qquad\qquad - 1.954734\gamma^2\} \cos(2nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2\{0.8464938 - 75.58943e^2 - 2.116236e'^2 \\
&\qquad\qquad\qquad - 1.757590\gamma^2\} \cos(2nt - 2n't - 2\omega + 2\Omega) \\
&+ e^3(25.40710) \cos(5nt - 2n't - 3\omega) \\
&- e^3(69.35009) \cos(nt + 2n't - 3\omega) \\
&+ e'^3(0.05220364) \cos(2nt + n't - 3\omega') \\
&+ e'^3(87.88435) \cos(2nt - 5n't + 3\omega') \\
&- e\gamma^2(43.15700) \cos(3nt - 2n't + \omega - 2\Omega) \\
&+ e\gamma^2(21.84219) \cos(nt - 2n't - \omega + 2\Omega) \\
&- e\gamma^2(32.99200) \cos(nt + 2n't - \omega - 2\Omega) \\
&- e\gamma^2(8.017230) \cos(5nt - 2n't - \omega - 2\Omega) \\
&- e\gamma^2(1.9734977) \cos(3nt - 2n't - 3\omega + 2\Omega) \\
&+ e\gamma^2(14.36440) \cos(nt - 2n't + 3\omega - 2\Omega) \\
&- e'\gamma^2(7.311476) \cos(4nt - 3n't + \omega' - 2\Omega) \\
&+ e'\gamma^2(0.8047685) \cos(4nt - n't - \omega' - 2\Omega) \\
&- e'\gamma^2(13.022452) \cos(3n't - \omega' - 2\Omega) \\
&+ e'\gamma^2(4.332373) \cos(n't + \omega' - 2\Omega) \\
&+ e'\gamma^2(3.332049) \cos(2nt - 3n't + 2\omega + \omega' - 2\Omega) \\
&- e'\gamma^2(3.332049) \cos(2nt - 3n't - 2\omega + \omega' + 2\Omega) \\
&+ e'\gamma^2(0.3798981) \cos(2nt - n't - 2\omega - \omega' + 2\Omega) \\
&- e'\gamma^2(0.3798981) \cos(2nt - n't + 2\omega - \omega' - 2\Omega) \\
&+ ee'^2(86.38646) \cos(3nt - 4n't - \omega + 2\omega') \\
&+ ee'^2(289.9748) \cos(nt - 4n't + \omega + 2\omega') \\
&+ e^2e'(58.46078) \cos(4nt - 3n't - 2\omega + \omega') \\
&- e^2e'(6.593311) \cos(4nt - n't - 2\omega - \omega') \\
&- e^2e'(31.3343) \cos(3n't - 2\omega - \omega') \\
&+ e^2e'(3.15857) \cos(n't - 2\omega + \omega') + e^4(41.53707) \cos(6nt - 2n't - 4\omega) \\
&- e^4(171.8999) \cos(2nt + 2n't - 4\omega)
\end{aligned}
\right\} . \quad (306)
$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
- e'^4 (0.09571816) \cos (2nt + 2n't - 4\omega') \\
+ e'^4 (194.9143) \cos (2nt - 6n't + 4\omega') \\
+ \gamma^4 (16.94367) \cos (2nt + 2n't - 4\Omega) \\
+ \gamma^4 (0.6964739) \cos (6nt - 2n't - 4\Omega) \\
+ \gamma^4 (0.2413355) \cos (2nt - 2n't + 4\omega - 4\Omega) \\
- \gamma^4 (0.002221814) \cos (2nt - 2n't - 4\omega + 4\Omega) \\
- \gamma^4 (0.9121053) \cos (4nt - 2n't + 2\omega - 4\Omega) \\
- e^2\gamma^2 (3.681140) \cos (4nt - 2n't - 4\omega + 2\Omega) \\
- e^2\gamma^2 (3.024526) \cos (2n't - 4\omega + 2\Omega) \\
- e^2\gamma^2 (36.50003) \cos (2nt + 2n't - 2\omega - 2\Omega) \\
- e^2\gamma^2 (22.93544) \cos (6nt - 2n't - 2\omega - 2\Omega) \\
+ e'^2\gamma^2 (9.203058) \cos (2nt - 4n't + 2\omega + 2\omega' - 2\Omega) \\
- e'^2\gamma^2 (9.203058) \cos (2nt - 4n't - 2\omega + 2\omega' + 2\Omega) \\
- e'^2\gamma^2 (20.59146) \cos (4nt - 4n't + 2\omega' - 2\Omega) \\
- e'^2\gamma^2 (26.71121) \cos (4n't - 2\omega' - 2\Omega) \\
+ ee'^3 (0.1207040) \cos (3nt + n't - \omega - 3\omega') \\
- ee'^3 (2.0126727) \cos (nt + n't + \omega - 3\omega') \\
+ ee'^3 (196.99575) \cos (3nt - 5n't - \omega + 3\omega') \\
+ ee'^3 (532.9840) \cos (nt - 5n't + \omega + 3\omega') \\
- e^3e' (207.38035) \cos (nt + 3n't - 3\omega - \omega') \\
+ e^3e' (53.60305) \cos (nt + n't - 3\omega + \omega') \\
- e^3e' (11.28200) \cos (5nt - n't - 3\omega - \omega') \\
+ e^3e' (102.74821) \cos (5nt - 3n't - 3\omega + \omega') \\
+ e^2e'^2 (162.4547) \cos (4nt - 4n't - 2\omega + 2\omega') \\
- e^2e'^2 (93.4538) \cos (4n't - 2\omega - 2\omega') \\
- ee'\gamma^2 (110.37307) \cos (3nt - 3n't + \omega + \omega' - 2\Omega) \\
- ee'\gamma^2 (47.61361) \cos (nt - n't - \omega - \omega' + 2\Omega) \\
+ ee'\gamma^2 (40.03606) \cos (3nt - n't + \omega - \omega' - 2\Omega) \\
+ ee'\gamma^2 (26.02704) \cos (nt - 3n't - \omega + \omega' + 2\Omega) \\
+ ee'\gamma^2 (60.28733) \cos (nt + n't - \omega + \omega' - 2\Omega) \\
- ee'\gamma^2 (39.74047) \cos (nt + 3n't - \omega - \omega' - 2\Omega) \\
+ ee'\gamma^2 (4.486610) \cos (5nt - n't - \omega - \omega' - 2\Omega) \\
- ee'\gamma^2 (32.126856) \cos (5nt - 3n't - \omega + \omega' - 2\Omega) \\
- ee'\gamma^2 (7.792710) \cos (3nt - 3n't - 3\omega + \omega' + 2\Omega) \\
- ee'\gamma^2 (13.367617) \cos (nt - n't + 3\omega - \omega' - 2\Omega) \\
+ ee'\gamma^2 (0.8830959) \cos (3nt - n't - 3\omega - \omega' + 2\Omega)
\end{array}
\right\} \quad (306)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\quad + ee'\gamma^2 (39.61131) \cos (nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&\quad - 2 \frac{n^3}{n^2 - \alpha'^2} h' \cos (\alpha' t - \beta') \Big\} \\
&+ \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ - \{26.47716 + 85.08323 e^2 + 52.95432 e'^2 \\
&\qquad\qquad\qquad\qquad - 114.9857\gamma^2\} \cos (nt - n't) \\
&\quad + \{1.637969 - 39.082215 e^2 - 9.827814 e'^2 - 0.0197282\gamma^2\} \cos 3(nt - n't) \\
&\quad - e (69.06223) \cos (2nt - n't - \omega) + e (5.38109) \cos (n't - \omega) \\
&\quad + e (4.178324) \cos (4nt - 3n't - \omega) - e (15.206107) \cos (2nt - 3n't + \omega) \\
&\quad - e' (42.17633) \cos (nt - 2n't + \omega') + e' (0.5625) \cos (nt - \omega') \\
&\quad + e' (8.656024) \cos (3nt - 4n't + \omega') \\
&\quad - e' (1.553442) \cos (3nt - 2n't - \omega') \\
&\quad - e^2 (136.44848) \cos (3nt - n't - 2\omega) \\
&\quad + e^2 (100.05336) \cos (nt + n't - 2\omega) \\
&\quad + e^2 (8.043470) \cos (5nt - 3n't - 2\omega) \\
&\quad - e^2 (92.98083) \cos (nt - 3n't + 2\omega) + e'^2 (28.61086) \cos (nt + n't - 2\omega') \\
&\quad - e'^2 (66.36372) \cos (nt - 3n't + 2\omega') \\
&\quad + e'^2 (0.1845640) \cos (3nt - n't - 2\omega') \\
&\quad + e'^2 (29.12233) \cos (3nt - 5n't + 2\omega') + ee' (0.65625) \cos (2nt - \omega - \omega') \\
&\quad + ee' (17.06116) \cos (2n't - \omega - \omega') + h'' \{\tfrac{45}{16} \frac{n}{\alpha''} + \tfrac{9}{2}\} \cos (\alpha'' t - \beta'') \\
&\quad - ee' (115.29874) \cos (2nt - 2n't - \omega + \omega') \\
&\quad + ee' (13.53459) \cos (2nt - 2n't + \omega - \omega') \\
&\quad + ee' (22.064834) \cos (4nt - 4n't - \omega + \omega') \\
&\quad - ee' (3.965327) \cos (4nt - 2n't - \omega - \omega') \\
&\quad - ee' (86.38149) \cos (2nt - 4n't + \omega + \omega') + ee' (0.65625) \cos (\omega - \omega') \\
&\quad - \gamma^2 (3.309639) \cos (nt - n't + 2\omega - 2\Omega) \\
&\quad + \gamma^2 (0.5964048) \cos (nt - n't - 2\omega + 2\Omega) \\
&\quad + \gamma^2 (21.49121) \cos (3nt - n't - 2\Omega) \\
&\quad + \gamma^2 (11.87118) \cos (nt + n't - 2\Omega) \\
&\quad - \gamma^2 (0.5356757) \cos (5nt - 3n't + 2\Omega) \\
&\quad - \gamma^2 (12.142125) \cos (nt - 3n't + 2\Omega) \\
&\quad + \gamma^2 (0.6124366) \cos (3nt - 3n't + 2\omega - 2\Omega) \\
&\quad - \gamma^2 (0.6124366) \cos (3nt - 3n't - 2\omega + 2\Omega) \Big\}
\end{aligned}\right\} . \quad (306)
$$

This equation will give by integration

$$
\left.
\begin{aligned}
\delta v = \frac{\overline{m}^2}{\mu} \Big\{ & -e\{[9.8750613] + [0.5242338]e^2 \\
& \qquad + [0.0511525]e'^2 - \gamma^2\} \sin(nt - \omega) \\
& - e'\{[1.6056492] + [1.6615876]e^2 + [1.6568017]e'^2 \\
& \qquad - [1.6574090]\gamma^2\} \sin(n't - \omega') \\
& - e^2\{[0.0263289] + [9.7231506]e^2 + [0.2024202]e'^2 \\
& \qquad - [0.4550830]\gamma^2 + [7.8928178]\frac{\gamma^4}{e^2}\} \sin 2(nt - \omega) \\
& - e'^2\{[1.4868408] + [1.5599375]e^2 + [1.3789597]e'^2 \\
& \qquad - [1.9981542]\gamma^2\} \sin 2(n't - \omega') \\
& - ee'\{[1.8290530] + [1.4101186]e^2 + [1.8802058]e'^2 \\
& \qquad - [2.0133378]\gamma^2\} \sin(nt + n't - \omega - \omega') \\
& + ee'\{[1.8709688] + [1.2461436]e^2 + [1.9221153]e'^2 \\
& \qquad - [2.0397824]\gamma^2\} \sin(nt - n't - \omega + \omega') \\
& + \gamma^2\{[8.7858800] + [9.9637857]e^2 + [8.9719713]e'^2 \\
& \qquad - [9.4353888]\gamma^2\} \sin 2(nt - \Omega) \\
& - e^3[0.1544469] \sin 3(nt - \omega) - e'^3[1.4925956] \sin 3(n't - \omega') \\
& - ee'^2[1.6933706] \sin(nt + 2n't - \omega - 2\omega') \\
& + ee'^2[1.7781372] \sin(nt - 2n't - \omega + 2\omega') \\
& - e^2e'[1.9289728] \sin(2nt + n't - 2\omega - \omega') \\
& + e^2e'[1.9643703] \sin(2nt - n't - 2\omega + \omega') \\
& + e\gamma^2[9.6589823] \sin(3nt - \omega - 2\Omega) \\
& - e\gamma^2[0.4225625] \sin(nt + \omega - 2\Omega) \\
& + e'\gamma^2[0.5728669] \sin(2nt + n't - \omega' - 2\Omega) \\
& - e'\gamma^2[0.0361911] \sin(2nt - n't + \omega' - 2\Omega) \\
& + e^2\gamma^2[0.0821248] \sin(4nt - 2\omega - 2\Omega) \\
& + e^2\gamma^2[8.0688720] \sin(2nt - 4\omega + 2\Omega) \\
& - e^4[0.2834288] \sin 4(nt - \omega) - e'^4[1.5481629] \sin 4(n't - \omega') \\
& + \gamma^4[8.1938200] \sin(2nt + 2\omega - 4\Omega) - \gamma^4[9.2330844] \sin 4(nt - \Omega) \\
& - ee'^3[1.6831366] \sin(nt + 3n't - \omega - 3\omega') \\
& + ee'^3[1.8125777] \sin(nt - 3n't - \omega + 3\omega') \\
& - e^3e'[2.0437936] \sin(3nt + n't - 3\omega - \omega') \\
& + e^3e'[2.0775966] \sin(3nt - n't - 3\omega + \omega') \\
& - e^2e'^2[1.7958732] \sin(2nt + 2n't - 2\omega - 2\omega') \\
& + e^2e'^2[1.8674561] \sin(2nt - 2n't - 2\omega + 2\omega')
\end{aligned}
\right\} . \quad (307)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&- e'^2\gamma^2\,[9.9459047]\sin(2nt + 2n't - 2\omega' - 2\Omega)\\
&+ e'^2\gamma^2\,[9.6855444]\sin(2nt - 2n't + 2\omega' - 2\Omega)\\
&+ ee'\gamma^2\,[1.1822131]\sin(3nt + n't - \omega - \omega' - 2\Omega)\\
&- ee'\gamma^2\,[1.3229992]\sin(3nt - n't - \omega + \omega' - 2\Omega)\\
&- ee'\gamma^2\,[1.6813058]\sin(nt + n't + \omega - \omega' - 2\Omega)\\
&+ ee'\gamma^2\,[1.8609576]\sin(nt - n't + \omega + \omega' - 2\Omega)\\
&- ee'\gamma^2\,[8.3877468]\sin(nt + n't - 3\omega - \omega' + 2\Omega)\\
&+ ee'\gamma^2\,[8.5179333]\sin(nt - n't - 3\omega + \omega' + 2\Omega)\\
&+ \{[0.2624189] + [1.9018140]e^2 - [0.6603588]e'^2\\
&\qquad - [0.5229625]\gamma^2 - [1.7928827]e^4 + [0.3813745]e'^4\\
&\qquad - [0.7781749]\gamma^4 - [2.3050506]e^2e'^2 + [0.9210207]e'^2\gamma^2\\
&\qquad\qquad - [1.8347898]e^2\gamma^2\}\sin 2(nt - n't)\\
&+ e\{[0.4423907] + [2.0069415]e^2 - [0.8403307]e'^2\\
&\qquad\qquad - [0.8157165]\gamma^2\}\sin(3nt - 2n't - \omega)\\
&+ e\{[1.8297256] - [1.2705426]e^2 - [2.2276657]e'^2\\
&\qquad\qquad - [1.7533583]\gamma^2\}\sin(nt - 2n't + \omega)\\
&- e'\{[9.8969955] + [1.8467933]e^2 - [8.9939034]e'^2\\
&\qquad\qquad + [0.4043446]\gamma^2\}\sin(2nt - n't - \omega')\\
&+ e'\{[0.8754272] + [2.3312752]e^2 - [1.2171442]e'^2\\
&\qquad\qquad - [0.9614841]\gamma^2\}\sin(2nt - 3n't + \omega')\\
&+ e^2\{[0.5838415] + [2.1237534]e^2 - [0.9878486]e'^2\\
&\qquad - [1.1293483]\gamma^2 + [9.4006928]\frac{\gamma^4}{e^2}\}\sin(4nt - 2n't - 2\omega)\\
&- e^2\{[1.6993008] - [2.8016071]e^2 - [1.5528591]e'^2\\
&\qquad - [2.3353194]\gamma^2 + [0.8458856]\frac{\gamma^4}{e^2}\}\sin 2(n't - \omega)\\
&+ e'^2\{[1.3553403] + [2.6610437]e^2 - [1.6884493]e'^2\\
&\qquad\qquad - [1.2992619]\gamma^2\}\sin(2nt - 4n't + 2\omega')\\
&- ee'\{[0.0818204] + [1.9565080]e^2 - [9.1787192]e'^2\\
&\qquad\qquad - [0.7440777]\gamma^2\}\sin(3nt - n't - \omega - \omega')\\
&+ ee'\{[2.2727329] - [1.8790150]e^2 - [2.6151836]e'^2\\
&\qquad\qquad - [1.9960481]\gamma^2\}\sin(nt - 3n't + \omega + \omega')\\
&+ ee'\{[1.0503933] + [2.4314316]e^2 - [1.3921098]e'^2\\
&\qquad\qquad - [1.2472036]\gamma^2\}\sin(3nt - 3n't - \omega + \omega')\\
&- ee'\{[1.7618599] + [1.1140241]e^2 - [0.8587696]e'^2\\
&\qquad\qquad - [1.1066675]\gamma^2\}\sin(nt - n't + \omega - \omega')
\end{aligned}
\right\}\ . \quad (307)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&-\gamma^2\{[9.6755696]+[1.6910921]e^2-[0.0731097]e'^2\\
&\qquad\qquad -[0.0436846]\gamma^2\}\sin(4nt-2n't-2\Omega)\\
&-\gamma^2\{[1.5183767]-[2.4208731]e^2-[1.9163170]e'^2\\
&\qquad\qquad -[1.5940547]\gamma^2\}\sin 2(n't-\Omega)\\
&+\gamma^2\{[9.6603588]+[1.5482073]e^2-[0.0582991]e'^2\\
&\qquad\qquad -[0.0238227]\gamma^2\}\sin(2nt-2n't+2\omega-2\Omega)\\
&-\gamma^2\{[9.6603588]-[1.6111961]e^2-[0.0582991]e'^2\\
&\qquad\qquad -[9.9776525]\gamma^2\}\sin(2nt-2n't-2\omega+2\Omega)\\
&+e^3[0.7191777]\sin(5nt-2n't-3\omega)\\
&-e^3[1.7804993]\sin(nt+2n't-3\omega)\\
&+e'^3[8.4007244]\sin(2nt+n't-3\omega')\\
&+e'^3[1.7327927]\sin(2nt-5n't+3\omega')\\
&-e\gamma^2[1.1801458]\sin(3nt-2n't+\omega-2\Omega)\\
&+e\gamma^2[1.4096742]\sin(nt-2n't-\omega+2\Omega)\\
&-e\gamma^2[1.4578608]\sin(nt+2n't-\omega-2\Omega)\\
&-e\gamma^2[0.2182469]\sin(5nt-2n't-\omega-2\Omega)\\
&-e\gamma^2[9.8403312]\sin(3nt-2n't-3\omega+2\Omega)\\
&+e\gamma^2[1.2276654]\sin(nt-2n't+3\omega-2\Omega)\\
&-e'\gamma^2[0.2870196]\sin(4nt-3n't+\omega'-2\Omega)\\
&+e'\gamma^2[9.3118093]\sin(4nt-n't-\omega'-2\Omega)\\
&-e'\gamma^2[1.7636623]\sin(3n't-\omega'-2\Omega)\\
&+e'\gamma^2[1.7628167]\sin(n't+\omega'-2\Omega)\\
&+e'\gamma^2[0.2733674]\sin(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-e'\gamma^2[0.2733674]\sin(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+e'\gamma^2[9.2951915]\sin(2nt-n't-2\omega-\omega'+2\Omega)\\
&-e'\gamma^2[9.2951915]\sin(2nt-n't+2\omega-\omega'-2\Omega)\\
&+ee'^2[1.5049540]\sin(3nt-4n't-\omega+2\omega')\\
&+ee'^2[2.6167694]\sin(nt-4n't+\omega+2\omega')\\
&+e^2e'[1.1898791]\sin(4nt-3n't-2\omega+\omega')\\
&-e^2e'[0.2252418]\sin(4nt-n't-2\omega-\omega')\\
&-e^2e'[2.1449896]\sin(3n't-2\omega-\omega')\\
&+e^2e'[1.6255814]\sin(n't-2\omega+\omega')\\
&+e^4[0.8512504]\sin(6nt-2n't-4\omega)\\
&-e^4[1.9029174]\sin(2nt+2n't-4\omega)\\
&+e'^4[8.6457562]\sin(2nt+2n't-4\omega')\\
&+e'^4[2.0994009]\sin(2nt-6n't+4\omega')
\end{aligned}
\right\}\quad . \quad (307)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\gamma^4[0.8937693]\sin(2nt+2n't-4\Omega)+\gamma^4[9.0757195]\sin(6nt-2n't-4\Omega)\\
&+\gamma^4[0.1153561]\sin(2nt-2n't+4\omega-4\Omega)\\
&-\gamma^4[7.0794428]\sin(2nt-2n't-4\omega+4\Omega)\\
&-\gamma^4[9.3745395]\sin(4nt-2n't+2\omega-4\Omega)\\
&-e^2\gamma^2[9.7804768]\sin(4nt-2n't-4\omega+2\Omega)\\
&-e^2\gamma^2[1.3057183]\sin(2n't-4\omega+2\Omega)\\
&-e^2\gamma^2[1.2270551]\sin(2nt+2n't-2\omega-2\Omega)\\
&-e^2\gamma^2[0.5933216]\sin(6nt-2n't-2\omega-2\Omega)\\
&+e'^2\gamma^2[0.7332803]\sin(2nt-4n't+2\omega+2\omega'-2\Omega)\\
&-e'^2\gamma^2[0.7332803]\sin(2nt-4n't-2\omega+2\omega'+2\Omega)\\
&-e'^2\gamma^2[0.8453921]\sin(4nt-4n't+2\omega'-2\Omega)\\
&-e'^2\gamma^2[1.9507244]\sin(4n't-2\omega'-2\Omega)\\
&+ee'^3[8.5939046]\sin(3nt+n't-\omega-3\omega')\\
&-ee'^3[0.2724450]\sin(nt+n't+\omega-3\omega')\\
&+ee'^3[1.8731633]\sin(3nt-5n't-\omega+3\omega')\\
&+ee'^3[2.9301443]\sin(nt-5n't+\omega+3\omega')\\
&-e^3e'[2.2288429]\sin(nt+3n't-3\omega-\omega')\\
&+e^3e'[1.6978613]\sin(nt+n't-3\omega+\omega')\\
&-e^3e'[0.3599623]\sin(5nt-n't-3\omega-\omega')\\
&+e^3e'[1.3327466]\sin(5nt-3n't-3\omega+\omega')\\
&+e^2e'^2[1.6434373]\sin(4nt-4n't-2\omega+2\omega')\\
&-e^2e'^2[2.4946379]\sin(4n't-2\omega-2\omega')\\
&-ee'\gamma^2[1.5995068]\sin(3nt-3n't+\omega+\omega'-2\Omega)\\
&-ee'\gamma^2[1.7114961]\sin(nt-n't-\omega-\omega'+2\Omega)\\
&+ee'\gamma^2[1.1362960]\sin(3nt-n't+\omega-\omega'-2\Omega)\\
&+ee'\gamma^2[1.5257892]\sin(nt-3nt-\omega+\omega'+2\Omega)\\
&+ee'\gamma^2[1.7488979]\sin(nt+n't-\omega+\omega'-2\Omega)\\
&-ee'\gamma^2[1.5113083]\sin(nt+3n't-\omega-\omega'-2\Omega)\\
&+ee'\gamma^2[9.9596880]\sin(5nt-n't-\omega-\omega'-2\Omega)\\
&-ee'\gamma^2[0.8278408]\sin(5nt-3n't-\omega+\omega'-2\Omega)\\
&-ee'\gamma^2[0.4483323]\sin(3nt-3n't-3\omega+\omega'+2\Omega)\\
&-ee'\gamma^2[1.1598190]\sin(nt-n't+3\omega-\omega'-2\Omega)\\
&+ee'\gamma^2[9.4798545]\sin(3nt-n't-3\omega-\omega'+2\Omega)\\
&+ee'\gamma^2[1.7081836]\sin(nt-3n't+3\omega+\omega'-2\Omega)\\
&-2\frac{n^3h'}{(n^2-\alpha'^2)\alpha'}\sin(\alpha't-\beta')\Big\}
\end{aligned}\right\} \quad . \quad (307)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\frac{\overline{m}^2}{\mu}\frac{a}{a'}\Big\{-\{[1.4566364]+[1.9636090]e^2+[1.7576746]e'^2\\
&\qquad\qquad\qquad\qquad -[2.0944088]\gamma^2\}\sin(nt-n't)\\
&+\{[9.7709495]-[1.1486222]e^2-[0.5491006]e'^2\\
&\qquad\qquad\qquad\qquad -[7.8517312]\gamma^2\}\sin 3(nt-n't)\\
&-e[1.5547650]\sin(2nt-n't-\omega)+e[1.8569612]\sin(n't-\omega)\\
&+e[0.0440166]\sin(4nt-3n't-\omega)-e[0.9326740]\sin(2nt-3n't+\omega)\\
&-e'[1.6954468]\sin(nt-2n't+\omega')+e'[9.7501225]\sin(nt-\omega')\\
&+e'[0.5058268]\sin(3nt-4n't+\omega')\\
&-e'[9.7363897]\sin(3nt-2n't-\omega')\\
&-e^2[1.6688133]\sin(3nt-n't-2\omega)\\
&+e^2[1.9689036]\sin(nt+n't-2\omega)\\
&+e^2[0.2264160]\sin(5nt-3n't-2\omega)\\
&-e^2[2.0787578]\sin(nt-3n't+2\omega)\\
&+e'^2[1.4252027]\sin(nt+n't-2\omega')\\
&-e'^2[1.9322952]\sin(nt-3n't+2\omega')\\
&+e'^2[8.7999916]\sin(3nt-n't-2\omega')\\
&+e'^2[1.0449325]\sin(3nt-5n't+2\omega')\\
&+ee'[9.5160393]\sin(2nt-\omega-\omega')+ee'[2.0570694]\sin(2n't-\omega-\omega')\\
&-ee'[1.7945594]\sin(2nt-2n't-\omega+\omega')\\
&+ee'[0.8641801]\sin(2nt-2n't+\omega-\omega')\\
&+ee'[0.7754056]\sin(4nt-4n't-\omega+\omega')\\
&-ee'[0.0127735]\sin(4nt-2n't-\omega-\omega')\\
&-ee'[1.7057688]\sin(2nt-4n't+\omega+\omega')\\
&-\gamma^2[0.5535456]\sin(nt-n't+2\omega-2\Omega)\\
&+\gamma^2[9.8093061]\sin(nt-n't-2\omega+2\Omega)\\
&+\gamma^2[0.8661055]\sin(3nt-n't-2\Omega)\\
&+\gamma^2[1.0431657]\sin(nt+n't-2\Omega)\\
&-\gamma^2[9.0498744]\sin(5nt-3n't-2\Omega)\\
&-\gamma^2[1.1946591]\sin(nt-3n't+2\Omega)\\
&+\gamma^2[9.3437048]\sin(3nt-3n't+2\omega-2\Omega)\\
&-\gamma^2[9.3437048]\sin(3nt-3n't-2\omega+2\Omega)\\
&+h''\left\{\tfrac{45}{16}\frac{n^2}{\alpha''^2}+\tfrac{3}{2}\frac{n}{\alpha''}\right\}\sin(\alpha''t-\beta'')\Big\}
\end{aligned}
\right\} . \quad (307)
$$

26. The value of δr being substituted in the first of equations (264), will give the following value of $\frac{d\delta_1\theta}{dt}$:

$$
\left.\begin{aligned}
\frac{d\delta_1\theta}{dt} = \frac{\overline{m}^2}{\mu} n \Big\{ & + \gamma\{\tfrac{1}{3} - \tfrac{59}{24}e^2 + \tfrac{1}{2}e'^2 - \tfrac{1}{8}\gamma^2\} \cos(nt - \Omega) \\
& - \gamma\{\tfrac{35}{48}e^2 + \tfrac{1}{24}\gamma^2\} \cos(nt - 2\omega + \Omega) \\
& - e\gamma\{0.0416667 + 3.197917e^2 + 0.0625e'^2 \\
& \qquad - 0.2725694\gamma^2\} \cos(2nt - \omega - \Omega) \\
& - e\gamma\{0.708333 + 0.1875e^2 + 1.0625e'^2 - \tfrac{3}{2}\gamma^2\} \cos(\omega - \Omega) \\
& - e'\gamma\{1.508411 - 70.4158e^2 + 1.696996e'^2 \\
& \qquad - 3.49688\gamma^2\} \cos(nt + n't - \omega' - \Omega) \\
& - e'\gamma\{1.508441 + 70.83056' + 1.696996e'^2 \\
& \qquad - 3.675843\gamma^2\} \cos(nt - n't + \omega' - \Omega) \\
& - e^2\gamma(1.0625) \cos(3nt - 2\omega - \Omega) \\
& - e'^2\gamma(2.294827) \cos(nt + 2n't - 2\omega' - \Omega) \\
& - e'^2\gamma(2.294827) \cos(nt - 2n't + 2\omega' - \Omega) \\
& + \gamma^3(0.4586666) \cos 3(nt - \Omega) + \tfrac{1}{24}\gamma^3 \cos(nt + 2\omega - 3\Omega) \\
& - ee'\gamma(37.76205) \cos(2nt + n't - \omega - \omega' - \Omega) \\
& + ee'\gamma(32.86111) \cos(2nt - n't - \omega + \omega' - \Omega) \\
& - ee'\gamma(34.74517) \cos(n't - \omega - \omega' + \Omega) \\
& + ee'\gamma(35.87799) \cos(n't + \omega - \omega' - \Omega) \\
& - e^3\gamma(3.180544) \cos(4nt - 3\omega - \Omega) \\
& - e\gamma\{1.114575e^2 - 0.171875\gamma^2\} \cos(2nt - 3\omega + \Omega) \\
& + e\gamma^3(1.657986) \cos(4nt - \omega - 3\Omega) \\
& - e\gamma^3(0.4166667) \cos(2nt + \omega - 3\Omega) - \tfrac{17}{192}e\gamma^3 \cos 3(\omega - \Omega) \\
& - ee'^2\gamma(30.66673) \cos(2nt + 2n't - \omega - 2\omega' - \Omega) \\
& + ee'^2\gamma(23.21627) \cos(2nt - 2n't - \omega + 2\omega' - \Omega) \\
& + ee'^2\gamma(27.80593) \cos(2n't + \omega - 2\omega' - \Omega) \\
& - ee'^2\gamma(26.07707) \cos(2n't - \omega - 2\omega' + \Omega) \\
& - e'^3\gamma(3.488157) \cos(nt + 3n't - 3\omega' - \Omega) \\
& - e'^3\gamma(3.488157) \cos(nt - 3n't + 3\omega' - \Omega) \\
& - e^2e'\gamma(126.0391) \cos(3nt + n't - 2\omega - \omega' - \Omega) \\
& + e^2e'\gamma(121.3506) \cos(3nt - n't - 2\omega + \omega' - \Omega) \\
& - e'\gamma\{51.64635e^2 - 0.188555\gamma^2\} \cos(nt + n't - 2\omega - \omega' + \Omega) \\
& + e'\gamma\{54.49704e^2 + 0.188555\gamma^2\} \cos(nt - n't - 2\omega + \omega' + \Omega) \\
& - e'\gamma^3(0.188555) \cos(nt + n't + 2\omega - \omega' - 3\Omega)
\end{aligned}\right\} \quad (308)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&- e'\gamma^3 (0.188555) \cos (nt - n't + 2\omega + \omega' - 3\Omega) \\
&+ e'\gamma^3 (0.8571104) \cos (3nt + n't - \omega' - 3\Omega) \\
&+ e'\gamma^3 (1.0360718) \cos (3nt - n't + \omega' - 3\Omega) \\
&+ \gamma \{1.287669 + 103.40017e^2 - 3.219173e'^2 \\
&\qquad\qquad - 0.966541\gamma^2\} \cos (3nt - 2n't - \Omega) \\
&+ \gamma \{1.287669 + 48.01867e^2 - 3.219173e'^2 \\
&\qquad\qquad - 4.290697\gamma^2\} \cos (nt - 2n't + \Omega) \\
&+ e\gamma \{4.566218 + 249.8131e^2 - 11.41555e'^2 \\
&\qquad\qquad - 3.40598\gamma^2\} \cos (4nt - 2n't - \omega - \Omega) \\
&+ e\gamma \{1.990883 + 71.55762e^2 - 4.977206e'^2 \\
&\qquad\qquad - 1.49578\gamma^2\} \cos (2nt - 2n't - \omega + \Omega) \\
&+ e\gamma \{29.68164 - 92.41788e^2 - 74.20414e'^2 \\
&\qquad\qquad - 37.76268\gamma^2\} \cos (2nt - 2n't + \omega - \Omega) \\
&+ e\gamma \{32.25698 + 42.2060e^2 - 80.64248e'^2 \\
&\qquad\qquad - 63.29232\gamma^2\} \cos (2n't - \omega - \Omega) \\
&- e'\gamma \{2.5650113 + 94.86932e^2 - 0.07062595e'^2 \\
&\qquad\qquad - 0.4257278\gamma^2\} \cos (3nt - n't - \omega' - \Omega) \\
&- e'\gamma \{0.5650113 + 42.24452e^2 - 0.07062595e'^2 \\
&\qquad\qquad - 3.320269\gamma^3\} \cos (nt - n't - \omega' + \Omega) \\
&+ e'\gamma \{5.185720 + 265.52563e^2 - 11.39006e'^2 \\
&\qquad\qquad - 3.876988\gamma^2\} \cos (3nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma \{5.185720 + 129.16360e^2 - 11.39006e'^2 \\
&\qquad\qquad - 12.773933\gamma^2\} \cos (nt - 3n't + \omega' + \Omega) \\
&- \gamma^3 (0.1680028) \cos (5nt - 2n't - 3\Omega) \\
&- \gamma^3 (3.492158) \cos (nt + 2n't - 3\Omega) \\
&+ e^2\gamma (11.14751) \cos (5nt - 2n't - 2\omega - \Omega) \\
&+ e^2\gamma (43.3474) \cos (nt + 2n't - 2\omega - \Omega) \\
&+ \gamma \{2.980815e^2 - 0.4828757\gamma^2\} \cos (3nt - 2n't - 2\omega + \Omega) \\
&- \gamma \{20.20082e^2 - 0.1609587\gamma^2\} \cos (nt - 2n't + 2\omega - \Omega) \\
&+ e'^2\gamma (14.65787) \cos (3nt - 4n't + 2\omega' - \Omega) \\
&+ e'^2\gamma^2 (14.65787) \cos (nt - 4n't + 2\omega' + \Omega) \\
&+ \gamma^3 (0.4828757) \cos (3nt - 2n't + 2\omega - 3\Omega) \\
&- \gamma^3 (0.1609589) \cos (nt - 2n't - 2\omega + 3\Omega) \\
&- ee'\gamma (2.008295) \cos (4nt - n't - \omega - \omega' - \Omega) \\
&+ ee'\gamma (86.52803) \cos (3n't - \omega - \omega' - \Omega) \\
&- ee'\gamma (0.8782726) \cos (2nt - n't - \omega - \omega' + \Omega)
\end{aligned}
\right\} . \quad (308)
$$

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$$\left.\begin{aligned}
&+ ee'\gamma\,(76.15659)\cos(2nt - 3n't + \omega + \omega' - \Omega)\\
&+ ee'\gamma\,(18.34702)\cos(4nt - 3n't - \omega + \omega' - \Omega)\\
&+ ee'\gamma\,(7.975576)\cos(2nt - 3n't - \omega - \omega' + \Omega)\\
&- ee'\gamma\,(27.19068)\cos(2nt - n't + \omega - \omega' - \Omega)\\
&- ee'\gamma\,(28.32070)\cos(n't - \omega + \omega' - \Omega)\\
&+ e\gamma\{4.208538e^2 - 1.068500\gamma^2\}\cos(4nt - 2n't - 3\omega + \Omega)\\
&- e\gamma\{34.80602e^2 - 4.67595\gamma^2\}\cos(2n't - 3\omega + \Omega)\\
&+ e^3\gamma\,(23.18619)\cos(6nt - 2n't - 3\omega - \Omega)\\
&+ e^3\gamma\,(27.80334)\cos(2nt + 2n't - 3\omega - \Omega)\\
&- e\gamma^3\,(0.928026)\cos(6nt - 2n't - \omega - 3\Omega)\\
&- e\gamma^3\,(26.98704)\cos(2nt + 2n't - \omega - 3\Omega)\\
&- e\gamma^3\,(1.61063)\cos(4nt - 2n't + \omega - 3\Omega)\\
&+ e\gamma^3\,(16.72500)\cos(2n't + \omega - 3\Omega)\\
&+ e\gamma^3\,(11.050135)\cos(2nt - 2n't + 3\omega - 3\Omega)\\
&- e\gamma^3\,(0.392341)\cos(2nt - 2n't - 3\omega + 3\Omega)\\
&+ e'\gamma\{11.931868e^2 - 1.944645\gamma^2\}\cos(3nt - 3n't - 2\omega + \omega' + \Omega)\\
&+ e'\gamma\{15.89936e^2 - 0.0706264\gamma^2\}\cos(nt - n't + 2\omega - \omega' - \Omega)\\
&- e'\gamma\{1.3160905e^2 - 0.2118792\gamma^2\}\cos(3nt - n't - 2\omega - \omega' + \Omega)\\
&- e'\gamma\{60.1080e^2 - 0.648215\gamma^2\}\cos(nt - 3n't + 2\omega + \omega' - \Omega)\\
&+ e'^3\gamma\,(0.018571)\cos(3nt + n't - 3\omega' - \Omega)\\
&+ e'^3\gamma\,(0.018571)\cos(nt + n't - 3\omega' + \Omega)\\
&+ e'^3\gamma\,(35.82214)\cos(3nt - 5n't + 3\omega' - \Omega)\\
&+ e'^3\gamma\,(35.82214)\cos(nt - 5n't + 3\omega' + \Omega)\\
&+ e'\gamma^3\,(1.944645)\cos(3nt - 3n't + 2\omega + \omega' - 3\Omega)\\
&- e'\gamma^3\,(0.648215)\cos(nt - 3n't - 2\omega + \omega' + 3\Omega)\\
&+ e'\gamma^3\,(0.0706264)\cos(nt - n't - 2\omega - \omega' + 3\Omega)\\
&- e'\gamma^3\,(0.2118792)\cos(3nt - n't + 2\omega - \omega' - 3\Omega)\\
&- e'\gamma^3\,(9.571386)\cos(nt + 3n't - \omega' - 3\Omega)\\
&+ e'\gamma^3\,(2.968483)\cos(nt + n't + \omega' - 3\Omega)\\
&+ e'\gamma^3\,(0.0739423)\cos(5nt - n't - \omega' - 3\Omega)\\
&- e'\gamma^3\,(0.674441)\cos(5nt - 3n't + \omega' - 3\Omega)\\
&+ ee'^2\gamma\,(51.74244)\cos(4nt - 4n't - \omega + 2\omega' - \Omega)\\
&+ ee'^2\gamma\,(22.42669)\cos(2nt - 4n't - \omega + 2\omega' + \Omega)\\
&+ ee'^2\gamma\,(153.8717)\cos(2nt - 4n't + \omega + 2\omega' - \Omega)\\
&+ ee'^2\gamma\,(183.1875)\cos(4n't - \omega - 2\omega' - \Omega)\\
&- e^2e'\gamma\,(4.908923)\cos(5nt - n't - 2\omega - \omega' - \Omega)
\end{aligned}\right\} \quad (308)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ e^2e'\gamma\,(44.73660)\cos(5nt - 3n't - 2\omega + \omega' - \Omega)\\
&+ e^2e'\gamma\,(109.0587)\cos(nt + 3n't - 2\omega - \omega' - \Omega)\\
&\left. - e^2e'\gamma\,(40.31828)\cos(nt + n't - 2\omega + \omega' - \Omega)\right\}\\
&+\frac{\overline{m}'^2}{\mu}\frac{a}{a'}n\Big\{ -\gamma\,(13.44124)\cos(2nt - n't - \Omega) - \gamma\,(13.44124)\cos(n't - \Omega)\\
&+ \gamma\,(0.4812192)\cos(4nt - 3n't - \Omega)\\
&+ \gamma\,(0.4812192)\cos(2nt - 3n't + \Omega)\\
&- e\gamma\,(48.12632)\cos(3nt - n't - \omega - \Omega)\\
&- e\gamma\,(21.24384)\cos(nt - n't - \omega + \Omega)\\
&- e\gamma\,(17.21996)\cos(nt + n't - \omega - \Omega)\\
&+ e\gamma\,(9.66252)\cos(nt - n't + \omega - \Omega)\\
&+ e\gamma\,(1.860158)\cos(5nt - 3n't - \omega - \Omega)\\
&+ e\gamma\,(0.8977201)\cos(3nt - 3n't - \omega + \Omega)\\
&- e\gamma\,(6.046075)\cos(3nt - 3n't + \omega - \Omega)\\
&- e\gamma\,(5.083637)\cos(nt - 3n't + \omega + \Omega)\\
&- e'\gamma\,(21.74962)\cos(2nt - 2n't + \omega' - \Omega)\\
&- e'\gamma\,(21.74962)\cos(2n't - \omega' - \Omega)\\
&+ e'\gamma\,(0.09375)\cos(2nt - \omega' - \Omega) + e'\gamma\,(0.09375)\cos(\omega' - \Omega)\\
&+ e'\gamma\,(2.592412)\cos(4nt - 4n't + \omega' - \Omega)\\
&+ e'\gamma\,(2.592412)\cos(2nt - 4n't + \omega' + \Omega)\\
&- e'\gamma\,(0.4478197)\cos(4nt - 2n't - \omega' - \Omega)\\
&\left. - e'\gamma\,(0.4478197)\cos(2nt - 2n't - \omega' + \Omega)\right\}
\end{aligned}\right\} \quad (308)
$$

The value of δv being substituted in the second of equations (264), will give the following value of $\frac{d\delta_2\theta}{dt}$:

$$
\left.\begin{aligned}
\frac{d\delta_2\theta}{dt} = \frac{\overline{m}'^2}{\mu}n\Big\{ &- e\gamma\{0.375 + 0.03125e^2 + 0.5625e'^2\\
&\qquad - 0.506366\gamma^2\}\cos(2nt - \omega - \Omega)\\
&+ e\gamma\{0.375 + 2.3125e^2 + 0.5625e'^2 - 1.020833\gamma^2\}\cos(\omega - \Omega)\\
&- e'\gamma\{20.16597 + 77.06599e^2 + 22.68672e'^2\\
&\qquad - 33.45220\gamma^2\}\cos(nt + n't - \omega' - \Omega)\\
&+ e'\gamma\{20.16597 + 70.23043e^2 + 22.68672e'^2\\
&\qquad - 34.77873\gamma^2\}\cos(nt - n't + \omega' - \Omega)\\
&- e^2\gamma\,(1.28125)\cos(3nt - 2\omega - \Omega) + e^2\gamma\,(0.53125)\cos(nt - 2\omega + \Omega)
\end{aligned}\right\} \quad (309)
$$

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$$\left.\begin{aligned}
&+\gamma^3(0.03125)\cos 3(nt-\Omega)+\gamma\{0.75e^2-0.03125\gamma^2\}\cos(nt-\Omega)\\
&-e'^2\gamma(15.33948)\cos(nt+2n't-2\omega'-\Omega)\\
&+e'^2\gamma(15.33948)\cos(nt-2n't+2\omega'-\Omega)\\
&-ee'\gamma(74.06245)\cos(2nt+n't-\omega-\omega'-\Omega)\\
&+ee'\gamma(77.48024)\cos(2nt-n't-\omega+\omega'-\Omega)\\
&-ee'\gamma(37.14830)\cos(n't+\omega-\omega'-\Omega)\\
&+ee'\gamma(33.73051)\cos(n't-\omega-\omega'+\Omega)\\
&+e\gamma\{0.760413e^2+0.046875\gamma^2\}\cos(2nt-3\omega+\Omega)\\
&-e^3\gamma(3.041663)\cos(4nt-3\omega-\Omega)\\
&-e'^3\gamma(15.54410)\cos(nt+3n't-3\omega'-\Omega)\\
&+e'^3\gamma(15.54410)\cos(nt-3n't+3\omega'-\Omega)\\
&-ee'^2\gamma(55.35871)\cos(2nt+2n't-\omega-2\omega'-\Omega)\\
&+ee'^2\gamma(60.67800)\cos(2nt-2n't-\omega+2\omega-\Omega)\\
&-ee'^2\gamma(29.99903)\cos(2n't+\omega-2\omega'-\Omega)\\
&+ee'^2\gamma(24.67974)\cos(2n't-\omega-2\omega'+\Omega)\\
&-e^2e'\gamma(177.97753)\cos(3nt+n't-2\omega-\omega'-\Omega)\\
&+e^2e'\gamma(188.41847)\cos(3nt-n't-2\omega+\omega'-\Omega)\\
&+e'\gamma\{44.977111e^2+2.520746\gamma^2\}\cos(nt+n't-2\omega-\omega'+\Omega)\\
&-e'\gamma\{48.582249e^2+2.520746\gamma^2\}\cos(nt-n't-2\omega+\omega'+\Omega)\\
&+e'\gamma^3(4.390724)\cos(3nt+n't-\omega'-3\Omega)\\
&-e'\gamma^3(3.064196)\cos(3nt-n't+\omega'-3\Omega)\\
&+e\gamma^3(0.3373842)\cos(4nt-\omega-3\Omega)\\
&+\tfrac{1}{12}e\gamma^3\cos(2nt+\omega-3\Omega)+\tfrac{3}{64}e\gamma^3\cos 3(\omega-\Omega)\\
&-e'\gamma^3(2.520746)\cos(nt+n't+2\omega-\omega'-3\Omega)\\
&+e'\gamma^3(2.520746)\cos(nt-n't+2\omega+\omega'-3\Omega)\\
&+\gamma\{0.914932+106.53602e^2-2.28733e'^2\\
&\qquad\qquad\qquad-2.001935\gamma^2\}\cos(3nt-2n't-\Omega)\\
&-\gamma\{0.914932+41.73984e^2-2.28733e'^2\\
&\qquad\qquad\qquad-18.73361\gamma^2\}\cos(nt-2n't+\Omega)\\
&+e\gamma\{3.214581+240.9220e^2-8.03643e'^2\\
&\qquad\qquad\qquad-7.67276\gamma^2\}\cos(4nt-2n't-\omega-\Omega)\\
&-e\gamma\{35.61266+38.87515e^2-89.03115e'^2\\
&\qquad\qquad\qquad-68.29602\gamma^2\}\cos(2n't-\omega-\Omega)\\
&-e\gamma\{1.384716+57.4807e^2-3.461772e'^2\\
&\qquad\qquad\qquad-45.23052\gamma^2\}\cos(2nt-2n't-\omega+\Omega)
\end{aligned}\right\}\quad(309)$$

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$$
\left.
\begin{aligned}
&+ e\gamma\{33.78280 + 6.92507e^2 - 84.4568e'^2 \\
&\qquad\qquad - 41.44088\gamma^2\}\cos(2nt - 2n't + \omega - \Omega) \\
&- e'\gamma\{0.3944260 + 92.53381e^2 - 0.049303e'^2 \\
&\qquad\qquad - 1.580815\gamma^2\}\cos(3nt - n't - \omega' - \Omega) \\
&+ e'\gamma\{0.3944260 + 35.95015e^2 - 0.049303e'^2 \\
&\qquad\qquad - 30.642547\gamma^2\}\cos(nt - n't - \omega' + \Omega) \\
&+ e'\gamma\{3.753160 + 290.8393e^2 - 8.24355e'^2 \\
&\qquad\qquad - 5.55079\gamma^2\}\cos(3nt - 3n't + \omega' - \Omega) \\
&- e'\gamma\{3.753160 + 114.6854e^2 - 8.24355e'^2 \\
&\qquad\qquad - 35.66341\gamma^2\}\cos(nt - 3n't + \omega' + \Omega) \\
&+ e^2\gamma\,(7.775169)\cos(5nt - 2n't - 2\omega - \Omega) \\
&- e^2\gamma\,(95.67031)\cos(nt + 2n't - 2\omega - \Omega) \\
&- \gamma^3\,(0.3512527)\cos(5nt - 2n't - 3\Omega) \\
&- \gamma^3\,(16.38042)\cos(nt + 2n't - 3\Omega) \\
&- \gamma\{2.032205e^2 + 0.343100\gamma^2\}\cos(3nt - 2n't - 2\omega + \Omega) \\
&+ \gamma\{25.13118e^2 - 0.1143667\gamma^2\}\cos(nt - 2n't + 2\omega - \Omega) \\
&+ \gamma^3\,(0.343100)\cos(3nt - 2n't + 2\omega - 3\Omega) \\
&+ \gamma^3\,(0.1143667)\cos(nt - 2n't - 2\omega + 3\Omega) \\
&+ e'^2\gamma\,(10.82207)\cos(3nt - 4n't + 2\omega' - \Omega) \\
&- e'^2\gamma\,(10.82207)\cos(nt - 4n't + 2\omega' + \Omega) \\
&- ee'\gamma\,(1.392509)\cos(4nt - n't - \omega - \omega' - \Omega) \\
&+ ee'\gamma\,(13.12149)\cos(4nt - 3n't - \omega + \omega' - \Omega) \\
&- ee'\gamma\,(101.1984)\cos(3n't - \omega - \omega' - \Omega) \\
&+ ee'\gamma\,(29.68433)\cos(n't - \omega + \omega' - \Omega) \\
&+ ee'\gamma\,(93.6921)\cos(2nt - 3n't + \omega + \omega' - \Omega) \\
&+ ee'\gamma\,(0.603657)\cos(2nt - n't - \omega - \omega' + \Omega) \\
&- ee'\gamma\,(5.61517)\cos(2nt - 3n't - \omega + \omega' + \Omega) \\
&- ee'\gamma\,(28.89548)\cos(2nt - n't + \omega - \omega' - \Omega) \\
&- e\gamma\{2.914153e^2 + 1.2054662\gamma^2\}\cos(4nt - 2n't - 3\omega + \Omega) \\
&+ e\gamma\{34.65659e^2 - 4.45157\gamma^2\}\cos(2n't - 3\omega + \Omega) \\
&+ e^3\gamma\,(15.977308)\cos(6nt - 2n't - 3\omega - \Omega) \\
&- e^3\gamma\,(199.21146)\cos(2nt + 2n't - 3\omega - \Omega) \\
&- e\gamma^3\,(10.53298)\cos(4nt - 2n't + \omega - 3\Omega) \\
&- e\gamma^3\,(16.49198)\cos(2n't + \omega - 3\Omega) \\
&- e\gamma^3\,(1.930783)\cos(6nt - 2n't - \omega - 3\Omega) \\
&- e\gamma^3\,(42.64173)\cos(2nt + 2n't - \omega - 3\Omega)
\end{aligned}
\right\} . \quad (309)
$$

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$$
\left.
\begin{aligned}
&+ e\gamma^3 (12.66853) \cos(2nt - 2n't + 3\omega - 3\Omega) \\
&+ e\gamma^3 (0.1730879) \cos(2nt - 2n't - 3\omega + 3\Omega) \\
&+ e'^3\gamma (0.01258043) \cos(3nt + n't - 3\omega' - \Omega) \\
&- e'^3\gamma (0.01258043) \cos(nt + n't - 3\omega' + \Omega) \\
&+ e'^3\gamma (27.02481) \cos(3nt - 5n't + 3\omega' - \Omega) \\
&- e'^3\gamma (27.02481) \cos(nt - 5n't + 3\omega' + \Omega) \\
&- e'\gamma^3 (1.566093) \cos(5nt - 3n't + \omega' - 3\Omega) \\
&+ e'\gamma^3 (0\;151816) \cos(5nt - n't - \omega' - 3\Omega) \\
&- e'\gamma^3 (28.54653) \cos(nt + 3n't - \omega' - 3\Omega) \\
&+ e'\gamma^3 (28.90991) \cos(nt + n't + \omega' - 3\Omega) \\
&+ e'\gamma^3 (1.407435) \cos(3nt - 3n't + 2\omega + \omega' - 3\Omega) \\
&+ e'\gamma^3 (0.469145) \cos(nt - 3n't - 2\omega + \omega' + 3\Omega) \\
&- e'\gamma^3 (0.1479678) \cos(3nt - n't + 2\omega - \omega' - 3\Omega) \\
&- e'\gamma^3 (0.0493614) \cos(nt - n't - 2\omega - \omega' + 3\Omega) \\
&+ e'\gamma\{0.889172e^2 + 0.1479678\gamma^2\} \cos(3nt - n't - 2\omega - \omega' + \Omega) \\
&+ e'\gamma\{70.2865e^2 - 0.469145\gamma^2\} \cos(nt - 3n't + 2\omega + \omega' - \Omega) \\
&- e'\gamma\{16.82001e^2 - 0.0493634\gamma^2\} \cos(nt - n't + 2\omega - \omega' - \Omega) \\
&- e'\gamma\{8.21016e^2 + 1.407435\gamma^2\} \cos(3nt - 3n't - 2\omega + \omega' + \Omega) \\
&+ ee'^2\gamma (37.63692) \cos(4nt - 4n't - \omega + 2\omega' - \Omega) \\
&- ee'^2\gamma (15.99278) \cos(2nt - 4n't - \omega + 2\omega' + \Omega) \\
&+ ee'^2\gamma (206.8900) \cos(2nt - 4n't + \omega + 2\omega' - \Omega) \\
&- ee'^2\gamma (228.5341) \cos(4n't - \omega - 2\omega' - \Omega) \\
&- e^2e'\gamma (3.385371) \cos(5nt - n't - 2\omega - \omega' - \Omega) \\
&+ e^2e'\gamma (21.63828) \cos(5nt - 3n't - 2\omega + \omega' - \Omega) \\
&- e^2e'\gamma (269.8685) \cos(nt + 3n't - 2\omega - \omega' - \Omega) \\
&+ e^2e'\gamma (75.89287) \cos(nt + n't - 2\omega + \omega' - \Omega) \quad \Big\} \\
&+ \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ -\gamma (14.30890) \cos(2nt - n't - \Omega) + \gamma (14.30890) \cos(n't - \Omega) \\
&+ \gamma (0.2950661) \cos(4nt - 3n't - \Omega) \\
&- \gamma (0.2950661) \cos(2nt - 3n't + \Omega) \\
&- e\gamma (46.55420) \cos(3nt - n't - \omega - \Omega) \\
&+ e\gamma (17.93639) \cos(nt - n't - \omega + \Omega) \\
&+ e\gamma (64.58704) \cos(nt + n't - \omega - \Omega) \\
&- e\gamma (35.96923) \cos(nt - n't + \omega - \Omega) \\
&+ e\gamma (1.143465) \cos(5nt - 3n't - \omega - \Omega)
\end{aligned}
\right\} . \tag{309}
$$

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$$\left.\begin{aligned}
&-e\gamma\,(0.553333)\cos(3nt-3n't-\omega+\Omega)\\
&-e\gamma\,(4.2819735)\cos(3nt-3n't+\omega-\Omega)\\
&+e\gamma\,(3.6918412)\cos(nt-3n't+\omega+\Omega)\\
&-e'\gamma\,(24.79800)\cos(2nt-2n't+\omega'-\Omega)\\
&+e'\gamma\,(24.79800)\cos(2n't-\omega'-\Omega)\\
&+e'\gamma\,(0.2812500)\cos(2nt-\omega'-\Omega)-e'\gamma\,(0.2812500)\cos(\omega'-\Omega)\\
&+e'\gamma\,(1.6024955)\cos(4nt-4n't+\omega'-\Omega)\\
&-e'\gamma\,(1.6024955)\cos(2nt-4n't+\omega+\Omega)\\
&-e'\gamma\,(0.2724958)\cos(4nt-2n't-\omega'-\Omega)\\
&+e'\gamma\,(0.2724958)\cos(2nt-2n't-\omega'+\Omega)\Big\}
\end{aligned}\right\}\ . \quad (309)$$

If we now take the sum of equations (282), (308), and (309), we shall obtain the following complete value of $\frac{d\delta\theta}{dt}$:

$$\left.\begin{aligned}
\frac{d\delta\theta}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{&+\gamma\{\tfrac{17}{24}-\tfrac{5}{6}e^2+\tfrac{17}{16}e'^2-\tfrac{81}{64}\gamma^2\}\cos(nt-\Omega)\\
&-e\gamma\{\tfrac{11}{12}+\tfrac{41}{48}e^2+\tfrac{11}{8}e'^2-1.528936\gamma^2\}\cos(2nt-\omega-\Omega)\\
&-e\gamma\,\{\tfrac{13}{12}-3.46875e^2+\tfrac{5}{4}e'^2+0.098958\gamma^2\}\cos(\omega-\Omega)\\
&-e'\gamma\,\{6.09234-9.92666e^2+6.85389e'^2\\
&\qquad\qquad-20.36287\gamma^2\}\cos(nt+n't-\omega'-\Omega)\\
&+e'\gamma\{4.20203-14.5304e^2+4.73729e'^2\\
&\qquad\qquad-14.68911\gamma^2\}\cos(nt-n't+\omega'-\Omega)\\
&-\gamma\{3.526042e^2+0.0885417\gamma^2\}\cos(nt-2\omega+\Omega)\\
&-e^2\gamma\,(3.703125)\cos(3nt-2\omega-\Omega)\\
&+\gamma^3(0.630542)\cos 3(nt-\Omega)+\gamma^3(0.0885417)\cos(nt+2\omega-3\Omega)\\
&+e'^2\gamma\,(5.56940)\cos(nt+2n't-2\omega'-\Omega)\\
&+e'^2\gamma\,(2.67674)\cos(nt-2n't+2\omega'-\Omega)\\
&-ee'\gamma\,(2.11003)\cos(n't+\omega-\omega'-\Omega)\\
&-ee'\gamma\,(2.44712)\cos(n't-\omega-\omega'+\Omega)\quad+3\frac{nh_{,}\alpha_{,}}{n^2-\alpha_{,}^2}\cos(\alpha_{,}t-\beta_{,})\\
&-ee'\gamma\,(82.42563)\cos(2nt+n't-\omega-\omega'-\Omega)\\
&+ee'\gamma\,(79.43028)\cos(2nt-n't-\omega+\omega'-\Omega)\\
&-e\gamma\{5.604162e^2-0.28125\gamma^2\}\cos(2nt-3\omega+\Omega)\\
&-e^3\gamma\,(8.722210)\cos(4nt-3\omega-\Omega)\\
&-e\gamma^3\,(1.317708)\cos(2nt+\omega-3\Omega)\\
&+e\gamma^3(2.46412)\cos(4nt-\omega-3\Omega)-e\gamma^3(0.135417)\cos 3(\omega-\Omega)\\
&-e'^3\gamma\,(6.64438)\cos(nt+3n't-3\omega'-\Omega)
\end{aligned}\right\}\ . \quad (310)$$

(Continued on the next page)

$$
\left.\begin{aligned}
&+ e'^3\gamma\,(2.18412)\cos(nt - 3n't + 3\omega' - \Omega) \\
&- ee'^2\gamma\,(3.04237)\cos(2n't + \omega - 2\omega' - \Omega) \\
&- ee'^2\gamma\,(4.05860)\cos(2n't - \omega - 2\omega' + \Omega) \\
&- ee'^2\gamma\,(64.39779)\cos(2nt + 2n't - \omega - 2\omega' - \Omega) \\
&+ ee'^2\gamma\,(59.94217)\cos(2nt - 2n't - \omega + 2\omega' - \Omega) \\
&- e^2e'\gamma\,(255.1422)\cos(3nt + n't - 2\omega - \omega' - \Omega) \\
&+ e^2e'\gamma\,(256.7913)\cos(3nt - n't - 2\omega + \omega' - \Omega) \\
&- e'\gamma\{51.60793e^2 - 9.761542\gamma^2\}\cos(nt + n't - 2\omega - \omega' + \Omega) \\
&+ e'\gamma\{40.79783e^2 - 0.525254\gamma^2\}\cos(nt - n't - 2\omega + \omega' + \Omega) \\
&+ e'\gamma^3\,(3.571186)\cos(3nt + n't - \omega' - 3\Omega) \\
&+ e'\gamma^3\,(0.070936)\cos(3nt - n't + \omega' - 3\Omega) \\
&- e'\gamma^3\,(0.761542)\cos(nt + n't + 2\omega - \omega' - 3\Omega) \\
&+ e'\gamma^3\,(0.525254)\cos(nt - n't + 2\omega + \omega' - 3\Omega) \\
&+ \gamma\{2.607919 + 203.32859e^2 - 6.507836e'^2 \\
&\qquad\qquad - 4.422247\gamma^2\}\cos(3nt - 2n't - \Omega) \\
&+ \gamma\{5.386019 + 9.98730e^2 - 13.46504e'^2 \\
&\qquad\qquad + 10.49628\gamma^2\}\cos(nt - 2n't + \Omega) \\
&+ e\gamma\{8.854557 + 478.6206e^2 - 22.13637e'^2 \\
&\qquad\qquad - 15.14656\gamma^2\}\cos(4nt - 2n't - \omega - \Omega) \\
&+ e\gamma\{60.81862 - 107.80317e^2 - 155.0229e'^2 \\
&\qquad\qquad - 74.56534\gamma^2\}\cos(2nt - 2n't + \omega - \Omega) \\
&- e\gamma\{4.00808 - 3.6744e^2 - 6.7172e'^2 - 4.96028\gamma^2\}\cos(2n't - \omega - \Omega) \\
&+ e\gamma\{11.514634 + 19.0642e^2 - 28.786601e'^2 \\
&\qquad\qquad - 38.886091\gamma^2\}\cos(2nt - 2n't - \omega + \Omega) \\
&- e'\gamma\{1.1542224 + 184.38687e^2 - 0.144277e'^2 \\
&\qquad\qquad - 2.7427696\gamma^2\}\cos(3nt - n't - \omega' - \Omega) \\
&- e'\gamma\{5.183867 + 10.50985e^2 - 0.647983e'^2 \\
&\qquad\qquad + 31.45643\gamma^2\}\cos(nt - n't - \omega' + \Omega) \\
&+ e'\gamma\{10.417257 + 531.6228e^2 - 22.88076e'^2 \\
&\qquad\qquad - 11.85193\gamma^2\}\cos(3nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma\{13.13021 + 21.8880e^2 - 28.83956e'^2 \\
&\qquad\qquad + 13.84713\gamma^2\}\cos(nt - 3n't + \omega' + \Omega) \\
&+ e^2\gamma\,(21.01166)\cos(5nt - 2n't - 2\omega - \Omega) \\
&- e^2\gamma\,(51.7835)\cos(nt + 2n't - 2\omega - \Omega) \\
&+ \gamma\{19.733648e^2 - 0.9779701\gamma^2\}\cos(3nt - 2n't - 2\omega + \Omega) \\
&- \gamma\{7.55219e^2 - 0.6732522\gamma^2\}\cos(nt - 2n't + 2\omega - \Omega)
\end{aligned}\right\} . \quad (310)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+ e'^2\gamma\,(29.22819)\cos(3nt-4n't+2\omega'-\Omega)\\
&+ e'^2\gamma\,(25.14225)\cos(nt-4n't+2\omega'+\Omega)\\
&- \gamma^3(0.5740821)\cos(5nt-2n't-3\Omega)-\gamma^3(18.86717)\cos(nt+2n't-3\Omega)\\
&+ \gamma^3(0.9779701)\cos(3nt-2n't+2\omega-3\Omega)\\
&- \gamma^3(0.6732524)\cos(nt-2n't-2\omega+3\Omega)\\
&- ee'\gamma\,(3.915571)\cos(4nt-n't-\omega-\omega'-\Omega)\\
&+ ee'\gamma\,(159.6952)\cos(2nt-3n't+\omega+\omega'-\Omega)\\
&- ee'\gamma\,(16.8143)\cos(3n't-\omega-\omega'-\Omega)\\
&+ ee'\gamma\,(35.37091)\cos(4nt-3n't-\omega+\omega'-\Omega)\\
&- ee'\gamma\,(54.87021)\cos(2nt-n't+\omega-\omega'-\Omega)\\
&+ ee'\gamma\,(1.71253)\cos(n't-\omega+\omega'-\Omega)\\
&- ee'\gamma\,(10.70649)\cos(2nt-n't-\omega-\omega'+\Omega)\\
&+ ee'\gamma\,(29.14020)\cos(2nt-3n't-\omega+\omega'+\Omega)\\
&+ e^3\gamma\,(42.76387)\cos(6nt-2n't-3\omega-\Omega)\\
&- e^3\gamma\,(170.68389)\cos(2nt+2n't-3\omega-\Omega)\\
&+ e\gamma\{31.243988e^2-2.6766252\gamma^2\}\cos(4nt-2n't-3\omega'+\Omega)\\
&- e\gamma\{0.22232e^2+0.14283\gamma^2\}\cos(2n't-3\omega+\Omega)\\
&+ e'^3\gamma\,(0.0386649)\cos(3nt+n't-3\omega'-\Omega)\\
&- e'^3\gamma\,(0.2029001)\cos(nt+n't-3\omega'+\Omega)\\
&+ e'^3\gamma\,(70.96699)\cos(3nt-5n't+3\omega'-\Omega)\\
&+ e'^3\gamma\,(44.09920)\cos(nt-5n't+3\omega'+\Omega)\\
&- e\gamma^3(11.22763)\cos(4nt-2n't+\omega-3\Omega)\\
&- e\gamma^3(0.38286)\cos(2n't+\omega-3\Omega)\\
&- e\gamma^3(3.113586)\cos(6nt-2n't-\omega-3\Omega)\\
&- e\gamma^3(67.82647)\cos(2nt+2n't-\omega-3\Omega)\\
&+ e\gamma^3(22.72663)\cos(2nt-2n't+3\omega-3\Omega)\\
&- e\gamma^3(1.519816)\cos(2nt-2n't-3\omega+3\Omega)\\
&+ e'\gamma^3(0.252122)\cos(5nt-n't-\omega'-3\Omega)\\
&- e'\gamma^3(35.70334)\cos(nt+3n't-\omega'-3\Omega)\\
&- e'\gamma^3(2.440378)\cos(5nt-3n't+\omega'-3\Omega)\\
&+ e'\gamma^3(30.90451)\cos(nt+n't+\omega'-3\Omega)\\
&+ e'\gamma^3(3.906471)\cos(3nt-3n't+2\omega+\omega'-3\Omega)
\end{aligned}
\right\} \quad (310)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&- e'\gamma^3 (0.4328918) \cos (3nt - n't + 2\omega - \omega' - 3\Omega) \\
&+ e'\gamma^3 (0.6479252) \cos (nt - n't - 2\omega - \omega' + 3\Omega) \\
&- e'\gamma^3 (1.641276) \cos (nt - 3n't - 2\omega + \omega' + 3\Omega) \\
&- e^2e'\gamma (9.277623) \cos (5nt - n't - 2\omega - \omega' - \Omega) \\
&+ e^2e'\gamma (73.95144) \cos (5nt - 3n't - 2\omega + \omega' - \Omega) \\
&- e^2e'\gamma (159.6426) \cos (nt + 3n't - 2\omega - \omega' - \Omega) \\
&+ e^2e'\gamma (34.99311) \cos (nt + n't - 2\omega + \omega' - \Omega) \\
&+ ee'^2\gamma (333.4713) \cos (2nt - 4n't + \omega + 2\omega' - \Omega) \\
&- ee'^2\gamma (50.2534) \cos (4n't - \omega - 2\omega' - \Omega) \\
&+ ee'^2\gamma (99.23637) \cos (4nt - 4n't - \omega + 2\omega' - \Omega) \\
&+ ee'^2\gamma (58.14363) \cos (2nt - 4n't - \omega + 2\omega' + \Omega) \\
&- e'\gamma \{18.20608e^2 - 0.4328914\gamma^2\} \cos (3nt - n't - 2\omega - \omega' + \Omega) \\
&- e'\gamma \{18.8808e^2 - 1.641276\gamma^2\} \cos (nt - 3n't + 2\omega + \omega' - \Omega) \\
&+ e'\gamma \{11.58821e^2 - 0.5066704\gamma^2\} \cos (nt - n't + 2\omega - \omega' - \Omega) \\
&+ e'\gamma \{50.33986e^2 - 3.906471\gamma^2\} \cos (3nt - 3n't - 2\omega + \omega' + \Omega) \Big\} \\
+ \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ &- \gamma (26.21370) \cos (2nt - n't - \Omega) + \gamma (0.90107) \cos (n't - \Omega) \\
&+ \gamma (1.1140506) \cos (4nt - 3n't - \Omega) - \gamma (1.022595) \cos (2nt - 3n't + \Omega) \\
&- e\gamma (92.18253) \cos (3nt - n't - \omega - \Omega) \\
&\left.
\begin{aligned}
&- e\gamma (38.57017) \cos (nt - n't - \omega + \Omega) \\
&+ e\gamma (13.78155) \cos (nt + n't - \omega - \Omega)
\end{aligned}
\right\} - \tfrac{33}{16} \frac{h_{,,} n \alpha_{,,}}{n^2 - \alpha_{,,}^2} \cos (\alpha_{,,} t - \beta_{,,}) \\
&+ e\gamma (9.74255) \cos (nt - n't + \omega - \Omega) \\
&+ e\gamma (4.051591) \cos (5nt - 3n't - \omega - \Omega) \\
&- e\gamma (1.809113) \cos (3nt - 3n't - \omega + \Omega) \\
&- e\gamma (12.704011) \cos (3nt - 3n't + \omega - \Omega) \\
&- e\gamma (11.83614) \cos (nt - 3n't + \omega + \Omega) \\
&- e'\gamma (41.59218) \cos (2nt - 2n't + \omega' - \Omega) \\
&+ e'\gamma (2.67690) \cos (2n't - \omega' - \Omega) + e'\gamma (1.81250) \cos (2n't - \omega' - \Omega) \\
&+ e'\gamma (5.930507) \cos (4nt - 4n't + \omega' - \Omega) \\
&- e'\gamma (5.698918) \cos (2nt - 4n't + \omega' + \Omega) \\
&- e'\gamma (1.048217) \cos (4nt - 2n't - \omega' - \Omega) \\
&+ e'\gamma (0.927102) \cos (2nt - 2n't - \omega' + \Omega) \Big\}
\end{aligned}
\right\} . \quad (310)
$$

This equation gives by integration,

$$
\left.
\begin{aligned}
\delta\theta = \frac{\overline{m}^2}{\mu}\Big\{ & +\gamma\{[9.8502376]-[9.9208187]e^2+[0.0263289]e'^2 \\
& \qquad\qquad -[9.6851817]\gamma^2\}\sin(nt-\Omega) \\
& -e\gamma\{[9.6611814]+[9.6305126]e^2+[9.8372727]e'^2 \\
& \qquad\qquad -[9.8833593]\gamma^2\}\sin(2nt-\omega-\Omega) \\
& -e'\gamma\{[0.7534559]+[0.9654749]e^2+[0.08046090]e'^2 \\
& \qquad\qquad -[1.2775109]\gamma^2\}\sin(nt+n't-\omega'-\Omega) \\
& +e'\gamma\{[0.6572241]-[1.1960426]e^2+[0.7092950]e'^2 \\
& \qquad\qquad -[1.2007605]\gamma^2\}\sin(nt-n't+\omega'-\Omega) \\
& -\gamma\{[0.5472874]e^2+[8.9471477]\gamma^2\}\sin(nt-2\omega+\Omega) \\
& -e^2\gamma\,[0.0914471]\sin(3nt-2\omega-\Omega)+\gamma^3\,[9.3225927]\sin 3(nt-\Omega) \\
& +\gamma^3\,[8.9471477]\sin(nt+2\omega-3\Omega) \\
& +e'^2\gamma\,[0.6852606]\sin(nt+2n't-2\omega'-\Omega) \\
& +e'^2\gamma\,[0.4979842]\sin(nt-2n't+2\omega'-\Omega) \\
& -ee'\gamma\,[1.4503796]\sin(n't+\omega-\omega'-\Omega) \\
& -ee'\gamma\,[1.5147461]\sin(n't-\omega-\omega'+\Omega) \\
& -ee'\gamma\,[1.5990859]\sin(2nt+n't-\omega-\omega'-\Omega) \\
& +ee'\gamma\,[1.6155105]\sin(2nt-n't-\omega+\omega'-\Omega) \\
& -e^3\gamma\,[0.3385665]\sin(4nt-3\omega-\Omega) \\
& -e\gamma\{[0.4474807]e^2-[9.1480625]\gamma^2\}\sin(2nt-3\omega+\Omega) \\
& -e\gamma^3\,[9.8187891]\sin(2nt+\omega-3\Omega)+[3h_{\prime}\sin\alpha_{\prime}t-\beta_{\prime}) \\
& +e\gamma^3\,[9.7896018]\sin(4nt-\omega-3\Omega) \qquad =3e\gamma\sin(\omega-\Omega)]\ \text{nearly.} \\
& -e'^3\gamma\,[0.7345298]\sin(nt+3n't-3\omega'-\Omega) \\
& +e'^3\gamma\,[0.4496409]\sin(nt-3n't+3\omega'-\Omega) \\
& -ee'^2\gamma\,[1.3082728]\sin(2n't+\omega-2\omega'-\Omega) \\
& -ee'^2\gamma\,[1.4334372]\sin(2n't-\omega-2\omega'+\Omega) \\
& -ee'^2\gamma\,[1.4765128]\sin(2nt+2n't-\omega-2\omega'-\Omega) \\
& +ee'^2\gamma\,[1.5104674]\sin(2nt-2n't-\omega+2\omega'-\Omega) \\
& -e^2e'\gamma\,[1.9189653]\sin(3nt+n't-2\omega-\omega'-\Omega) \\
& +e^2e'\gamma\,[1.9434249]\sin(3nt-n't-2\omega+\omega'-\Omega) \\
& -e'\gamma\{[1.6813882]e^2-[9.8503657\gamma]^2\}\sin(nt+n't-2\omega-\omega'+\Omega) \\
& +e'\gamma\{[1.6444021]e^2-[9.7541344]\gamma^2\}\sin(nt-n't-2\omega+\omega'+\Omega) \\
& +e'\gamma^3\,[0.0649954]\sin(3nt+n't-\omega'-3\Omega) \\
& +e'\gamma^3\,[8.3847113]\sin(3nt-n't+\omega'-3\Omega) \\
& -e'\gamma^3\,[9.8503657]\sin(nt+n't+2\omega-\omega'-3\Omega)
\end{aligned}
\right\} . \quad (311)
$$

(Continued on the next page.)

26

$$
\left.
\begin{aligned}
&+ e'\gamma^3\,[9.7541344] \sin(nt - n't + 2\omega + \omega' - 3\Omega) \\
&+ \gamma\{[9.9613887] + [1.8532931]e^2 - [0.3585312]e'^2 \\
&\qquad\qquad - [0.1907376]\gamma^2\} \sin(3nt - 2n't - \Omega) \\
&+ \gamma\{[0.8016458] + [1.0698261]e^2 - [1.1995857]e'^2 \\
&\qquad\qquad + [1.0914134]\gamma^2\} \sin(nt - 2n't + \Omega) \\
&+ e\gamma\{[0.3616613] + [2.0944858]e^2 - [0.7596009]e'^2 \\
&\qquad\qquad - [0.5948085]\gamma^2\} \sin(4nt - 2n't - \omega - \Omega) \\
&+ e\gamma\{[1.5167715] - [1.7653668]e^2 - [1.9231308]e'^2 \\
&\qquad\qquad - [1.6052720]\gamma^2\} \sin(2nt - 2n't + \omega - \Omega) \\
&- e\gamma\{[1.4279972] - [1.3902473]e^2 - [1.6522492]e'^2 \\
&\qquad\qquad - [1.5205670]\gamma^2\} \sin(2n't - \omega - \Omega) \\
&+ e\gamma\{[0.7939850] + [1.0129536]e^2 - [1.1919253]e'^2 \\
&\qquad\qquad - [1.3225384]\gamma^2\} \sin(2nt - 2n't - \omega + \Omega) \\
&- e'\gamma\{[9.5961340] + [1.7995747]e^2 - [8.6930417]e'^2 \\
&\qquad\qquad - [9.9720339]\gamma^2\} \sin(3nt - n't - \omega' - \Omega) \\
&- e'\gamma\{[0.7484189] + [1.0553615]e^2 - [9.8453286]e'^2 \\
&\qquad\qquad + [1.5314744]\gamma^2\} \sin(nt - n't - \omega' + \Omega) \\
&+ e'\gamma\{[0.5743973] + [2.2817073]e^2 - [0.9161142]e'^2 \\
&\qquad\qquad - [0.6304327]\gamma^2\} \sin(3nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma\{[1.2286360] + [1.4505705]e^2 - [1.5703530]e'^2 \\
&\qquad\qquad + [1.2517241]\gamma^2\} \sin(nt - 3n't + \omega' + \Omega) \\
&+ e^2\gamma\,[0.6366830] \sin(5nt - 2n't - 2\omega - \Omega) \\
&- e^2\gamma\,[1.6536436] \sin(nt + 2n't - 2\omega - \Omega) \\
&+ e'^2\gamma\,[1.0343103] \sin(3nt - 4n't + 2\omega' - \Omega) \\
&+ e'^2\gamma\,[1.5548132] \sin(nt - 4n't + 2\omega' + \Omega) \\
&- \gamma^3\,[9.0731966] \sin(5nt - 2n't - 3\Omega) \\
&- \gamma^3\,[1.2151589] \sin(nt + 2n't - 3\Omega) \\
&+ \gamma^3\,[9.5354201] \sin(3nt - 2n't + 2\omega - 3\Omega) \\
&- \gamma^3\,[9.8985559] \sin(nt - 2n't - 2\omega + 3\Omega) \\
&- ee'\gamma\,[9.9989333] \sin(4nt - n't - \omega - \omega' - \Omega) \\
&+ ee'\gamma\,[1.9539477] \sin(2nt - 3n't + \omega + \omega' - \Omega) \\
&- ee'\gamma\,[1.8746483] \sin(3n't - \omega - \omega' - \Omega) \\
&+ ee'\gamma\,[0.9716607] \sin(4nt - 3n't - \omega + \omega' - \Omega) \\
&- ee'\gamma\,[1.4548611] \sin(2nt - n't + \omega - \omega' - \Omega) \\
&+ ee'\gamma\,[1.3597291] \sin(n't - \omega + \omega' - \Omega) \\
&- ee'\gamma\,[0.7451714] \sin(2nt - n't - \omega - \omega' + \Omega)
\end{aligned}
\right\} . \quad (311)
$$

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$$\left.\begin{aligned}
&+ee'\gamma\,[1.2151484]\sin(2nt-3n't-\omega+\omega'+\Omega)\\
&+\gamma\{[1.2403021]e^2-[9.9354201]\gamma^2\}\sin(3nt-2n't-2\omega+\Omega)\\
&-\gamma\{[0.9484509]e^2-[9.8985558]\gamma^2\}\sin(nt-2n't+2\omega-\Omega)\\
&+e\gamma\{[0.9092609]e^2-[9.8420821]\gamma^2\}\sin(4nt-2n't-3\omega+\Omega)\\
&-e\gamma\{[0.1720394]e^2+[9.9798803]\gamma^2\}\sin(2n't-3\omega+\Omega)\\
&+e^3\gamma\,[0.8638916]\sin(6nt-2n't-3\omega-\Omega)\\
&-e^3\gamma\,[1.8998342]\sin(2nt+2n't-3\omega-\Omega)\\
&+e'^3\gamma\,[8.0994998]\sin(3nt+n't-3\omega'-\Omega)\\
&-e'^3\gamma\,[9.2759540]\sin(nt+n't-3\omega'+\Omega)\\
&+e'^3\gamma\,[1.4317627]\sin(3nt-5n't+3\omega'-\Omega)\\
&+e'^3\gamma\,[1.9031277]\sin(nt-5n't+3\omega'+\Omega)\\
&-e\gamma^3\,[0.4647826]\sin(4nt-2n't+\omega-3\Omega)\\
&-e\gamma^3\,[0.4081009]\sin(2n't+\omega-3\Omega)\\
&-e\gamma^3\,[9.7260753]\sin(6nt-2n't-\omega-3\Omega)\\
&-e\gamma^3\,[1.4990409]\sin(2nt+2n't-\omega-3\Omega)\\
&+e\gamma^3\,[1.0892701]\sin(2nt-2n't+3\omega-3\Omega)\\
&-e\gamma^3\,[9.9145260]\sin(2nt-2n't-3\omega+3\Omega)\\
&-e'\gamma^3\,[8.7091869]\sin(5nt-n't-\omega'-3\Omega)\\
&-e'\gamma^3\,[1.4647841]\sin(nt+3n't-\omega'-3\Omega)\\
&-e'\gamma^3\,[9.7084296]\sin(5nt-3n't+\omega'-3\Omega)\\
&+e'\gamma^3\,[1.4586936]\sin(nt+n't+\omega'-3\Omega)\\
&+e'\gamma^3\,[0.1484283]\sin(3nt-3n't+2\omega+\omega'-3\Omega)\\
&-e'\gamma^3\,[9.1702240]\sin(3nt-n't+2\omega-\omega'-3\Omega)\\
&+e'\gamma^3\,[9.8452898]\sin(nt-n't-2\omega-\omega'+3\Omega)\\
&-e'\gamma^3\,[0.3255461]\sin(nt-3n't-2\omega+\omega'+3\Omega)\\
&-e^2e'\gamma\,[0.2750129]\sin(5nt-n't-2\omega-\omega'-\Omega)\\
&+e^2e'\gamma\,[1.1899191]\sin(5nt-3n't-2\omega+\omega'-\Omega)\\
&-e^2e'\gamma\,[2.1152240]\sin(nt+3n't-2\omega-\omega'-\Omega)\\
&+e^2e'\gamma\,[2.5126544]\sin(nt+n't-2\omega+\omega'-\Omega)\\
&+ee'^2\gamma\,[2.2924066]\sin(2nt-4n't+\omega+2\omega'-\Omega)\\
&-ee'^2\gamma\,[2.2251963]\sin(4n't-\omega-2\omega'-\Omega)\\
&+ee'^2\gamma\,[1.4283759]\sin(4nt-4n't-\omega+2\omega'-\Omega)
\end{aligned}\right\}. \quad (311)$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
+ee'^2\gamma\,[1.5338502]\sin(2nt-4n't-\omega+2\omega'+\Omega) \\
-e'\gamma\{[0.7940610]e^2-[9.1702236]\gamma^2\}\sin(3nt-n't-2\omega-\omega'+\Omega) \\
-e'\gamma\{[1.3863848]e^2-[0.3255460]\gamma^2\}\sin(nt-3n't+2\omega+\omega'-\Omega) \\
+e'\gamma\{[1.0977814]e^2-[9.7384905]\gamma^2\}\sin(nt-n't+2\omega-\omega'-\Omega) \\
+e'\gamma\{[1.2585557]e^2-[0.1484283]\gamma^2\}\sin(3nt-3n't-2\omega+\omega'+\Omega)\Big\} \\
-\frac{13}{12}\int e\gamma\cos(\omega-\Omega)\,ndt \\
\\
+\frac{\overline{m}^2}{\mu}\frac{a}{a'}\Big\{-\gamma\,[1.1340527]\sin(2nt-n't-\Omega) \\
+\gamma\,[1.0808494]\sin(n't-\Omega)+[9.4699194]\sin(4nt-3n't-\Omega) \\
-\gamma\,[9.7603596]\sin(2nt-3n't+\Omega)-[\frac{33}{16}h_{,,}\sin(\alpha_{,,}t-\beta_{,,}) \\
-e\gamma\,[1.4984933]\sin(3nt-n't-\omega-\Omega) \qquad =\frac{33}{16}e'\gamma\sin(\omega'-\Omega)] \\
-e\gamma\,[1.6200165]\sin(nt-n't-\omega+\Omega) \\
+e\gamma\,[1.1079699]\sin(nt+n't-\omega-\Omega) \\
+e\gamma\,[1.0224376]\sin(nt-n't+\omega-\Omega) \\
+e\gamma\,[9.9285981]\sin(5nt-3n't-\omega-\Omega) \\
-e\gamma\,[9.8141094]\sin(3nt-3n't-\omega+\Omega) \\
-e\gamma\,[0.6605846]\sin(3nt-3n't+\omega-\Omega) \\
-e\gamma\,[1.1835746]\sin(nt-3n't+\omega+\Omega) \\
-e'\gamma\,[1.3517467]\sin(2nt-2n't+\omega'-\Omega) \\
+e'\gamma\,[1.2526930]\sin(2n't-\omega'-\Omega)+e'\gamma\,[9.9572480]\sin(2nt-\omega'-\Omega) \\
+e'\gamma\,[0.2048115]\sin(4nt-4n't+\omega'-\Omega) \\
-e'\gamma\,[0.5251405]\sin(2nt-4n't+\omega'+\Omega) \\
-e'\gamma\,[9.4349458]\sin(4nt-2n't-\omega'-\Omega) \\
+e'\gamma\,[9.6998625]\sin(2nt-2n't-\omega'+\Omega)\Big\}
\end{array}\right\} \quad . \quad (311)
$$

We have thus completely developed the perturbations of the moon's co-ordinates, in so far as they depend on the first power of the sun's disturbing force; and if we substitute the numerical values of the elements of the orbits of the sun and moon, in equations (303), (307), and (311), we shall obtain the corrections to the elliptical values of these co-ordinates. By means of this first approximation we shall now develop the perturbations arising from the square of the disturbing force.

CHAPTER IV.

PERTURBATIONS ARISING FROM THE SQUARE OF THE DISTURBING FORCE.

27. We shall now consider the effect of the square of the disturbing force on the moon's co-ordinates r, v, and θ. For this purpose we must first determine the variations of the forces $\left(\frac{dR}{dr}\right)$, $\left(\frac{dR}{dv}\right)$, and $\left(\frac{dR}{d\theta}\right)$. If we put for a moment, for brevity, $u = \left(\frac{dR}{dr}\right)$, and use the value of $\left(\frac{dR}{dr}\right)$ given by equation (153), the partial differentials of u, with respect to r, v, and θ, being substituted in equation (157), will give

$$\left.\begin{aligned}
\delta\left(\frac{dR}{dr}\right) = {} & \frac{m'}{r'^3}\left\{1 - \tfrac{3}{2}\cos^2\theta - \tfrac{3}{2}\cos^2\theta\cos 2(v - v')\right\}\delta r \\
& + 3\,\frac{m'r}{r'^3}\cos^2\theta\sin 2(v - v')\,\delta v \\
& + 3\,\frac{m'r}{r'^3}\sin\theta\cos\theta\,\{1 + \cos 2(v - v')\}\,\delta\theta \\
& + 9\,\frac{m'r}{r'^4}\cos\theta\,\{1 - \tfrac{5}{4}\cos^2\theta\}\cos(v - v')\,\delta r \\
& - \tfrac{9}{2}\,\frac{m'r^2}{r'^4}\cos\theta\,\{1 - \tfrac{5}{4}\cos^2\theta\}\sin(v - v')\,\delta v \\
& - \tfrac{15}{4}\,\frac{m'r}{r'^4}\cos^3\theta\cos 3(v - v')\,\delta r \\
& + \tfrac{45}{8}\,\frac{m'r^2}{r'^4}\cos^3\theta\sin 3(v - v')\,\delta v
\end{aligned}\right\} . \quad (313)$$

In like manner we shall obtain the values of $\delta\left(\frac{dR}{dv}\right)$ and $\delta\left(\frac{dR}{d\theta}\right)$, as follows:

$$\left.\begin{aligned}\delta\left(\frac{dR}{dv}\right) = {} & 3\frac{m'r}{r'^3}\cos\theta\sin 2(v-v')\,\delta r + 3\frac{m'r^2}{r'^3}\cos\theta\cos 2(v-v')\,\delta v \\ & -\tfrac{3}{2}\frac{m'r^2}{r'^3}\sin\theta\sin 2(v-v')\,\delta\theta \\ & -\tfrac{9}{8}\frac{m'r^2}{r'^4}\cos\theta\,\{5\sin^2\theta-1\}\sin(v-v')\,\delta r \\ & -\tfrac{3}{8}\frac{m'r^3}{r'^4}\cos\theta\,\{5\sin^2\theta-1\}\cos(v-v')\,\delta v \\ & +\tfrac{45}{8}\frac{m'r^2}{r'^4}\cos^3\theta\sin 3(v-v')\,\delta r \\ & +\tfrac{45}{8}\frac{m'r^3}{r'^4}\cos^3\theta\cos 3(v-v')\,\delta v\end{aligned}\right\} \quad . \quad (314)$$

$$\left.\begin{aligned}\delta\left(\frac{dR}{d\theta}\right) = {} & 3\frac{m'r}{r'^3}\sin\theta\cos\theta\,\{1+\cos 2(v-v')\}\,\delta r \\ & -3\frac{m'r^2}{r'^3}\sin\theta\cos\theta\sin 2(v-v')\,\delta v \\ & +\tfrac{3}{2}\frac{m'r^2}{r'^3}\,\{1-2\sin^2\theta\}\,\{1+\cos 2(v-v')\}\,\delta\theta\end{aligned}\right\} \quad . \quad (315)$$

We also get for the variations of $c_1\cos\beta$, $c_2\cos\beta$, $c_3\cos\beta$, and $c_4\cos\beta$ given by equations (248–251):

$$\delta(c_1\cos\beta) = \frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\,\{-\sin\theta\cos v\,\delta\theta \mp \cos\theta\sin v\,\delta v\} \quad . \quad (316)$$

$$\left.\begin{aligned}\delta(c_2\cos\beta) = {} & 2r\{d\theta\sin\theta\sin v \mp dv\cos\theta\cos v\}\,\delta r \\ & + r^2\{dv\cos\theta\sin v \pm d\theta\sin\theta\cos v\}\,\delta v \\ & + r^2\{d\theta\cos\theta\sin v \pm dv\sin\theta\cos v\}\,\delta\theta \\ & \mp r^2\cos\theta\cos v\,d\delta v + r^2\sin\theta\sin v\,d\delta\theta\end{aligned}\right\} \quad . \quad (317)$$

$$\left.\begin{aligned}\delta(c_3\cos\beta) = {} & 2dv\cos\theta\cos v\,\delta r + \{dr\frac{\cos v}{\cos\theta} \mp 2rdv\cos\theta\sin v\}\,\delta v \\ & - \{2rdv\sin\theta\cos v \mp \frac{\sin\theta\sin v}{\cos^2\theta}dr\}\,\delta\theta \pm \frac{\sin v}{\cos\theta}d\delta r + 2r\cos\theta\cos v\,d\delta v\end{aligned}\right\} \quad . \quad (318)$$

$$\left.\begin{aligned}\delta(c_4\cos\beta) = {} & 2d\theta\cos\theta\cos v\,\delta r \mp \{2rd\theta\cos\theta + dr\sin\theta\}\sin v\,\delta v \\ & +\{dr\cos\theta - 2rd\theta\sin\theta\}\cos v\,\delta\theta + 2r\cos\theta\cos v\,d\delta\theta + \sin\theta\cos v\,d\delta r\end{aligned}\right\} \quad . \quad (319)$$

If we substitute the elliptical values of r, v, and θ, and of their differentials in equations (313–319), putting also $\overline{m}^2 = m'\dfrac{a^3}{a'^3}$, they will become

$$
\left.
\begin{aligned}
\delta\left(\frac{dR}{dr}\right) = \frac{\overline{m}^2}{a^3}\Big\{ & -\tfrac{1}{2}\{1 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2\} - \tfrac{3}{2}e' \cos(n't - \omega') \\
& - \tfrac{9}{4}e'^2 \cos 2(n't - \omega') - \tfrac{3}{4}\gamma^2 \cos 2(nt - \Omega) \\
& - \tfrac{3}{2}\{1 - 4e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2\} \cos 2(nt - n't) - 3e \cos(3nt - 2n't - \omega) \\
& + 3e \cos(nt - 2n't + \omega) + \tfrac{3}{4}e' \cos(2nt - n't - \omega') \\
& - \tfrac{21}{4}e' \cos(2nt - 3n't + \omega') - \tfrac{39}{8}e^2 \cos(4nt - 2n't - 2\omega) \\
& - \tfrac{9}{8}e^2 \cos 2(n't - \omega) - \tfrac{51}{8}e'^2 \cos(2nt - 4n't + 2\omega') - \tfrac{3}{4}\gamma^2 \cos 2(n't - \Omega) \\
& - \tfrac{3}{8}\gamma^2 \cos(2nt - 2n't + 2\omega - 2\Omega) + \tfrac{3}{8}\gamma^2 \cos(2nt - 2n't + 2\omega - 2\Omega) \\
& + \tfrac{3}{2}ee' \cos(3nt - n't - \omega - \omega') + \tfrac{21}{2}ee' \cos(nt - 3n't + \omega + \omega') \\
& - \tfrac{21}{2}ee' \cos(3nt - 3n't - \omega + \omega') - \tfrac{3}{2}ee' \cos(nt - n't + \omega - \omega') \Big\}\,\delta r \\
+ \frac{\overline{m}^2}{a^2}\Big\{ & 3\{1 - \tfrac{7}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{2}\gamma^2\} \sin 2(nt - n't) \\
& + \tfrac{9}{2}e \sin(3nt - 2n't - \omega) - \tfrac{15}{2}e \sin(nt - 2n't + \omega) \\
& - \tfrac{3}{2}e' \sin(2nt - n't - \omega') + \tfrac{21}{2}e' \sin(2nt - 3n't + \omega') \\
& + 6e^2 \sin(4nt - 2n't - 2\omega) - \tfrac{9}{2}e^2 \sin 2(n't - \omega) \\
& + \tfrac{51}{2}e'^2 \sin(2nt - 4n't + 2\omega') - \tfrac{3}{2}\gamma^2 \sin 2(n't - \Omega) \\
& + \tfrac{3}{4}\gamma^2 \sin(2nt - 2n't + 2\omega - 2\Omega) - \tfrac{3}{4}\gamma^2 \sin(2nt - 2n't - 2\omega + 2\Omega) \\
& - \tfrac{9}{4}ee' \sin(3nt - n't - \omega - \omega') - \tfrac{105}{4}ee' \sin(nt - 3n't + \omega + \omega') \\
& + \tfrac{63}{4}ee' \cos(3nt - 3n't - \omega + \omega') + \tfrac{15}{4}ee' \sin(nt - n't + \omega - \omega') \Big\}\,\delta v \\
+ \frac{\overline{m}^2}{a^2}\Big\{ & 3\gamma \sin(nt - \Omega) + \tfrac{3}{2}\gamma \sin(3nt - 2n't - \Omega) \\
& - \tfrac{3}{2}\gamma \sin(nt - 2n't + \Omega) \Big\}\,d\theta \\
+ \frac{\overline{m}^2}{a^2 a'}\Big\{ & - \tfrac{9}{4}\cos(nt - n't) - \tfrac{15}{4}\cos 3(nt - n't) \Big\}\,\delta r \\
+ \frac{\overline{m}^2}{aa'}\Big\{ & \tfrac{9}{8}\sin(nt - n't) + \tfrac{45}{8}\sin 3(nt - n't) \Big\}\,\delta v
\end{aligned}
\right\} \quad (320)
$$

$$
\left.\begin{aligned}
\delta\left(\frac{dR}{dv}\right) = \frac{\overline{m}^2}{a^2} \Big\{ & 3\{1 - \tfrac{7}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2\} \sin 2(nt - n't) \\
& + \tfrac{9}{2}e \sin(3nt - 2n't - \omega) - \tfrac{15}{2}e \sin(nt - 2n't + \omega) \\
& - \tfrac{3}{2}e' \sin(2nt - n't - \omega') + \tfrac{21}{2}e' \sin(2nt - 3n't + \omega') \\
& + 6e^2 \sin(4nt - 2n't - 2\omega) - \tfrac{9}{2}e^2 \sin 2(n't - \omega) \\
& + \tfrac{51}{2}e'^2 \sin(2nt - 4n't + 2\omega') - \tfrac{3}{8}\gamma^2 \sin(4nt - 2n't - 2\Omega) \\
& - \tfrac{9}{8}\gamma^2 \sin 2(n't - \Omega) + \tfrac{3}{4}\gamma^2 \sin(2nt - 2n't + 2\omega - 2\Omega) \\
& - \tfrac{3}{4}\gamma^2 \sin(2nt - 2n't - 2\omega + 2\Omega) - \tfrac{9}{4}ee' \sin(3nt - n't - \omega - \omega') \\
& - \tfrac{105}{4}ee' \sin(nt - 3n't + \omega + \omega') + \tfrac{63}{4}ee' \sin(3nt - 3n't - \omega + \omega') \\
& + \tfrac{15}{4}ee' \sin(nt - n't + \omega - \omega') \Big\} \delta r \\
+ \frac{\overline{m}^2}{a} \Big\{ & 3\{1 - \tfrac{5}{2}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{4}\gamma^2\} \cos 2(nt - n't) + 3e \cos(3nt - 2n't - \omega) \\
& - 9e \cos(nt - 2n't + \omega) - \tfrac{3}{2}e' \cos(2nt - n't - \omega') \\
& + \tfrac{21}{2}e' \cos(2nt - 3n't + \omega') + 3e^2 \cos(4nt - 2n't - 2\omega) \\
& + \tfrac{15}{2}e^2 \cos 2(n't - \omega) + \tfrac{51}{2}e'^2 \cos(2nt - 4n't + 2\omega') \\
& - \tfrac{3}{8}\gamma^2 \cos(4nt - 2n't - 2\Omega) + \tfrac{9}{8}\gamma^2 \cos 2(n't - \Omega) \\
& + \tfrac{3}{4}\gamma^2 \cos(2nt - 2n't + 2\omega - 2\Omega) - \tfrac{3}{4}\gamma^2 \cos(2nt - 2n't - 2\omega + 2\Omega) \\
& - \tfrac{3}{2}ee' \cos(3nt - n't - \omega - \omega') - \tfrac{63}{2}ee' \cos(nt - 3n't + \omega + \omega') \\
& + \tfrac{21}{2}ee' \cos(3nt - 3n't - \omega + \omega') + \tfrac{3}{2}ee' \cos(nt - n't + \omega - \omega') \Big\} \delta v \\
+ \frac{\overline{m}^2}{a} \Big\{ & - \tfrac{3}{4}\gamma \cos(nt - 2n't + \Omega) + \tfrac{3}{4}\gamma \cos(3nt - 2n't - \Omega) \Big\} \delta\theta \\
+ \frac{\overline{m}^2}{aa'} \Big\{ & \tfrac{3}{8} \sin(nt - n't) + \tfrac{45}{8} \sin 3(nt - n't) \Big\} \delta r \\
+ \frac{\overline{m}^2}{a'} \Big\{ & \tfrac{3}{8} \cos(nt - n't) + \tfrac{45}{8} \cos 3(nt - n't) \Big\} \delta v
\end{aligned}\right\} . \quad (321)
$$

$$
\left.\begin{aligned}
\delta\left(\frac{dR}{d\theta}\right) = \frac{\overline{m}^2}{a^2} \Big\{ & 3\gamma \sin(nt - \Omega) + \tfrac{3}{2}e\gamma \sin(2nt - \omega - \Omega) \\
& - \tfrac{9}{2}e\gamma \sin(\omega - \Omega) + \tfrac{9}{2}e'\gamma \sin(nt + n't - \omega' - \Omega) \\
& + \tfrac{9}{2}e'\gamma \sin(nt - n't + \omega' - \Omega) + \tfrac{3}{2}\gamma \sin(3nt - 2n't - \Omega) \\
& - \tfrac{3}{2}\gamma \sin(nt - 2n't + \Omega) + \tfrac{15}{4}e\gamma \sin(4nt - 2n't - \omega - \Omega) \\
& - \tfrac{3}{4}e\gamma \sin(2nt - 2n't - \omega + \Omega) - \tfrac{21}{4}e\gamma \sin(2nt - 2n't + \omega - \Omega) \\
& - \tfrac{9}{4}e\gamma \sin(2n't - \omega - \Omega) - \tfrac{3}{4}e'\gamma \sin(3nt - n't - \omega' - \Omega) \\
& + \tfrac{3}{4}e'\gamma \sin(nt - n't - \omega' + \Omega) + \tfrac{21}{4}e'\gamma \sin(3nt - 3n't + \omega' - \Omega) \\
& - \tfrac{21}{4}e'\gamma \sin(nt - 3n't + \omega' + \Omega) \Big\} \delta r
\end{aligned}\right\} . \quad (322)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\frac{\overline{m}^2}{a}\Big\{\tfrac{3}{2}\gamma\cos(3nt-2n't-\Omega)-\tfrac{3}{2}\gamma\cos(nt-2n't+\Omega)\\
&\quad+3e\gamma\cos(4nt-2n't-\omega-\Omega)-6e\gamma\cos(2nt-2n't+\omega-\Omega)\\
&\quad+3e\gamma\cos(2n't-\omega-\Omega)-\tfrac{3}{4}e'\gamma\cos(3nt-n't-\omega'-\Omega)\\
&\quad+\tfrac{3}{4}e'\gamma\cos(nt-n't-\omega'+\Omega)+\tfrac{21}{4}e'\gamma\cos(3nt-3n't+\omega'-\Omega)\\
&\quad-\tfrac{21}{4}e'\gamma\cos(nt-3n't+\omega'+\Omega)\Big\}\,\delta v\\
&+\frac{\overline{m}^2}{a}\Big\{\tfrac{3}{2}-3e\cos(nt-\omega)+\tfrac{3}{2}e'\cos(n't-\omega')+\tfrac{3}{2}\cos 2(nt-n't)\\
&\quad+\tfrac{3}{2}e\cos(3nt-2n't-\omega)-\tfrac{3}{2}e\cos(nt-2n't+\omega)\\
&\quad-\tfrac{3}{4}e'\cos(2nt-n't-\omega')+\tfrac{21}{4}e'\cos(2nt-3n't+\omega')\Big\}\,\delta\theta
\end{aligned}\right\}. \quad (322)
$$

$$
\left.\begin{aligned}
\delta(c_1\cos\beta)&=\mp\Big\{\{1-\tfrac{1}{2}e^2+\tfrac{1}{4}\gamma^2\}\sin nt+e\sin(2nt-\omega)-e\sin\omega\\
&\quad+\tfrac{3}{8}e^2\sin(3nt-2\omega)\pm\tfrac{1}{8}e^2\sin(nt-2\omega)\mp\tfrac{1}{4}\gamma^2\sin(nt-2\Omega)\\
&\quad+\tfrac{1}{8}\gamma^2\sin(nt+2\omega-2\Omega)-\tfrac{1}{8}\gamma^2\sin(nt-2\omega+2\Omega)\Big\}\,\delta v\\
&\quad\mp\{\tfrac{1}{2}\gamma\sin(2nt-\Omega)-\tfrac{1}{2}\gamma\sin\Omega\}\,\delta\theta
\end{aligned}\right\}. \quad (323)
$$

$$
\left.\begin{aligned}
\delta(c_2\cos\beta)&=andt\,\{\mp2\{1-\tfrac{3}{2}e^2-\tfrac{1}{4}\gamma^2\}\cos nt\mp3e\cos(2nt-\omega)\\
&\quad\pm e\cos\omega\mp\tfrac{17}{4}e^2\cos(3nt-2\omega)+\tfrac{1}{4}e^2\cos(nt-2\omega)+\tfrac{1}{2}\gamma^2\cos(nt-2\Omega)\\
&\quad\mp\tfrac{1}{4}\gamma^2\cos(nt+2n't-2\Omega)\pm\tfrac{1}{4}\gamma^2\cos(nt-2\omega+2\Omega)\}\,\delta r\\
&+a^2ndt\Big\{\{1-\tfrac{3}{2}e^2-\tfrac{1}{4}\gamma^2\}\sin nt+e\sin(2nt-\omega)-e\sin\omega\\
&\quad+\tfrac{3}{8}e^2\sin(3nt-2\omega)\pm\tfrac{1}{8}e^2\sin(nt-2\omega)\pm\tfrac{1}{4}\gamma^2\sin(nt-2\Omega)\\
&\quad+\tfrac{1}{8}\gamma^2\sin(nt+2\omega-2\Omega)-\tfrac{1}{8}\gamma^2\sin(nt-2\omega+2\Omega)\Big\}\,\delta v\\
&+a^2ndt\,\{\gamma\sin(2nt-\Omega)+2e\gamma\sin(3nt-\omega-\Omega)\\
&\qquad\qquad-2e\gamma\sin(nt+\omega-\Omega)\}\,\delta\theta\\
&+a^2\{\mp\{1+\tfrac{1}{2}e^2-\tfrac{1}{4}\gamma^2\}\cos nt\pm2e\cos\omega\pm\tfrac{1}{8}e^2\cos(3nt-2\omega)\\
&\quad-\tfrac{5}{8}e^2\cos(nt-2\omega)-\tfrac{1}{4}\gamma^2\cos(nt-2\Omega)\mp\tfrac{1}{8}\gamma^2\cos(nt+2\omega-2\Omega)\\
&\quad\pm\tfrac{1}{8}\gamma^2\cos(nt-2\omega+2\Omega)\}\,d\delta v\\
&+a^2\{\pm\tfrac{1}{2}\gamma\cos\Omega\mp\tfrac{1}{2}\gamma\cos(2nt-\Omega)\mp\tfrac{1}{2}e\gamma\cos(3nt-\omega-\Omega)\\
&\quad\pm\tfrac{3}{2}e\gamma\cos(nt+\omega-\Omega)\mp\tfrac{1}{2}e\gamma\cos(nt-\omega+\Omega)\\
&\qquad\qquad-\tfrac{1}{2}e\gamma\cos(nt-\omega-\Omega)\}\,d\delta\theta
\end{aligned}\right\}. \quad (324)
$$

$$
\left.
\begin{aligned}
\delta\,(c_3 \cos\beta) = ndt\,\{&2\{1 - e^2 - \tfrac{1}{4}\gamma^2\}\cos nt + 4e\cos(2nt - \omega) \\
&+ \tfrac{27}{4}e^2\cos(3nt - 2\omega) \pm \tfrac{1}{4}e^2\cos(nt - 2\omega) - \tfrac{1}{2}\gamma^2\cos(3nt - 2\Omega) \\
&+ \tfrac{1}{4}\gamma^2\cos(nt + 2\omega - 2\Omega) - \tfrac{1}{4}\gamma^2\cos(nt - 2\omega + 2\Omega)\}\,\delta r \\
+ andt\,\{&\mp 2\{1 - e^2 - \tfrac{1}{4}\gamma^2\}\sin nt \mp \tfrac{5}{2}e\sin(2nt - \omega) \pm \tfrac{1}{2}e\sin\omega \\
&\mp \tfrac{13}{4}e^2\sin(3nt - 2\omega) - \tfrac{1}{4}e^2\sin(nt - 2\omega) \pm \tfrac{1}{2}\gamma^2\sin(3nt - 2\Omega) \\
&\mp \tfrac{1}{4}\gamma^2\sin(nt + 2\omega - 2\Omega) \pm \tfrac{1}{4}\gamma^2\sin(nt - 2\omega + 2\Omega)\}\,\delta v \\
+ andt\,\{&\pm\gamma\sin\Omega \mp \gamma\sin(2nt - \Omega) \pm \tfrac{7}{4}e\gamma\sin(nt + \omega - \Omega) \\
&\mp \tfrac{7}{4}e\gamma\sin(3nt - \omega - \Omega) - \tfrac{1}{4}e\gamma\sin(nt - \omega - \Omega) \\
&\qquad\qquad \mp e\gamma\sin(nt - \omega + \Omega)\}\,\delta\theta \\
\pm \Big\{&\{1 - e^2 + \tfrac{1}{4}\gamma^2\}\sin nt + e\sin(2nt - \omega) - e\sin\omega \\
&+ \tfrac{3}{8}e^2\sin(3nt - 2\omega) \pm \tfrac{1}{8}e^2\sin(nt - 2\omega) - \tfrac{1}{4}\gamma^2\sin(3nt - 2\Omega) \\
&+ \tfrac{1}{8}\gamma^2\sin(nt + 2\omega - 2\Omega) - \tfrac{1}{8}\gamma^2\sin(nt - 2\omega + 2\Omega)\Big\}\,d\delta r \\
+ a\,\{&2\{1 - \tfrac{1}{2}e^2 - \tfrac{1}{4}\gamma^2\}\cos nt + e\cos(2nt - \omega) - 3e\cos\omega \\
&+ \tfrac{3}{4}e^2\cos(3nt - 2\omega) \pm \tfrac{1}{4}e^2\cos(nt - 2\omega) \pm \tfrac{1}{2}\gamma^2\cos(nt - 2\Omega) \\
&+ \tfrac{1}{4}\gamma^2\cos(nt + 2\omega - 2\Omega) - \tfrac{1}{4}\gamma^2\cos(nt - \omega + 2\Omega)\}\,d\delta v
\end{aligned}
\right\} \quad (325)
$$

$$
\left.
\begin{aligned}
\delta\,(c_4 \cos\beta) = ndt\,\{&\gamma\cos(2nt - \Omega) + \gamma\cos\Omega + 3e\gamma\cos(3nt - \omega - \Omega) \\
&+ e\gamma\cos(nt - \omega + \Omega) - e\gamma\cos(nt + \omega - \Omega) \\
&\qquad\qquad \pm e\gamma\cos(nt - \omega - \Omega)\}\,\delta r \\
\mp andt\,\{&\gamma\sin(2nt - \Omega) + \gamma\sin\Omega + \tfrac{9}{4}e\gamma\sin(3nt - \omega - \Omega) \\
&+ \tfrac{3}{4}e\gamma\sin(nt - \omega + \Omega) - \tfrac{5}{4}e\gamma\sin(nt + \omega - \Omega) \\
&\qquad\qquad \mp \tfrac{1}{4}e\gamma\sin(nt - \omega - \Omega)\}\,\delta v \\
+ andt\,\{&\pm \tfrac{1}{2}e\sin(2nt - \omega) \mp \tfrac{1}{2}e\sin\omega\}\,\delta\theta \\
&+ a\{2\cos nt + e\cos(2nt - \omega) - 3e\cos\omega\}\,d\delta\theta \\
&+ \{\pm \tfrac{1}{2}\gamma\sin(2nt - \Omega) \mp \tfrac{1}{2}\gamma\sin\Omega \pm e\gamma\sin(3nt - \omega - \Omega) \\
&\qquad\qquad \mp e\gamma\sin(nt + \omega - \Omega)\}\,d\delta r
\end{aligned}
\right\} \quad (326)
$$

The lower signs in equations (323–326) are to be used in finding $\delta\,(c_1 \sin\beta)$, $\delta\,(c_2 \sin\beta)$, $\delta\,(c_3 \sin\beta)$, and $\delta\,(c_4 \sin\beta)$, in which case we must change *sin* to *cos* and the reverse, in the second members of these equations.

28. We must now substitute the values of δr, δv, and $\delta\theta$, which are given by means of equations (303), (308), and (312); and also the values of their differentials, which are given by equations (302), (307), and (311). The necessary substitutions being made, we shall obtain the following values:

$$
\left.\begin{aligned}
\delta\left(\frac{dR}{dr}\right) = \frac{\overline{m}^4}{a^2\mu}\Big\{ & +\{3.793882 - 159.8570e^2 + 34.39014e'^2 \\
& \qquad - 10.47372\gamma^2\} + e\,(125.1089)\cos(nt-\omega) \\
& + e'\,(24.16916)\cos(n't-\omega') + e^2\,(251.6873)\cos 2(nt-\omega) \\
& + e'^2\,(59.89643)\cos 2(n't-\omega') + ee'\,(267.5236)\cos(nt+n't-\omega-\omega') \\
& + ee'\,(347.0538)\cos(nt-n't-\omega+\omega') + \gamma^2\,(51.75176)\cos 2(nt-\Omega) \\
& + \{0.893835 + 9.691075e^2 + 241.16936e'^2 - 9.206960\gamma^2\}\cos 2(nt-n't) \\
& - \{1.779044 + 191.4063e^2 - 15.23853e'^2 - 1.460510\gamma^2\}\cos 4(nt-n't) \\
& + e\,(1.642274)\cos(3nt-2n't-\omega) \\
& + e\,(12.23765)\cos(nt-2n't+\omega) \\
& - e\,(5.329553)\cos(5nt-4n't-\omega) - e\,(74.63955)\cos(3nt-4n't+\omega) \\
& + e'\,(59.92482)\cos(2nt-n't-\omega') \\
& - e'\,(57.19563)\cos(2nt-3n't+\omega') \\
& + e'\,(1.649042)\cos(4nt-3n't-\omega') \\
& - e'\,(13.59685)\cos(4nt-5n't+\omega') \\
& + e^2\,(3.157500)\cos(4nt-2n't-2\omega) \\
& - e^2\,(15.418506)\cos 2(n't-\omega) - e^2\,(11.188188)\cos(6nt-4n't-2\omega) \\
& + e^2\,(104.72245)\cos(2nt-4n't+2\omega) \\
& - e'^2\,(248.65007)\cos(2nt-4n't+2\omega') \\
& + e'^2\,(15.63891)\cos 2(nt-\omega') - e'^2\,(62.39036)\cos(4nt-6n't+2\omega') \\
& - e'^2\,(0.3797598)\cos(4nt-2n't-2\omega') \\
& + ee'\,(164.4105)\cos(3nt-n't-\omega-\omega') \\
& + ee'\,(77.39854)\cos(nt-3n't+\omega+\omega') \\
& - ee'\,(169.88794)\cos(3nt-3n't-\omega+\omega') \\
& - ee'\,(0.59817)\cos(nt-n't+\omega-\omega') \\
& + ee'\,(4.956323)\cos(5nt-3n't-\omega-\omega') \\
& - ee'\,(40.57257)\cos(5nt-5n't-\omega+\omega') \\
& - ee'\,(466.7718)\cos(3nt-5n't+\omega+\omega') \\
& + ee'\,(101.71433)\cos(3nt-3n't+\omega-\omega') \\
& - \gamma^2\,(1.1430438)\cos(4nt-2n't-2\Omega) + \gamma^2\,(8.880071)\cos 2(n't-\Omega) \\
& + \gamma^2\,(0.2234586)\cos(2nt-2n't+2\omega-2\Omega) \\
& - \gamma^2\,(0.2234586)\cos(2nt-2n't-2\omega+2\Omega)
\end{aligned}\right\}. \quad (327)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\gamma^2 (0.0191767) \cos (6nt - 4n't - 2\Omega) \\
&-\gamma^2 (47.87213) \cos (2nt - 4n't + 2\Omega) \\
&-\gamma^2 (0.889522) \cos (4nt - 4n't + 2\omega - 2\Omega) \\
&\left.+\gamma^2 (0.889522) \cos (4nt - 4n't - 2\omega + 2\Omega)\right\} \\
&+\frac{\overline{m}^4}{a^2\mu}\frac{a}{a'}\Big\{-(48.06837) \cos (nt - n't) + (34.13074) \cos 3(nt - n't) \\
&\qquad\qquad - (3.256396) \cos 5(nt - n't)\Big\}
\end{aligned}\right\} \quad (327)
$$

$$
\left.\begin{aligned}
\delta\left(\frac{dR}{dv}\right) = \frac{\overline{m}^4}{a\mu}\Big\{ &-e (157.4356) \sin (nt - \omega) - e' (2.36365) \sin (n't - \omega') \\
&-e^2 (194.8090) \sin 2(nt - \omega) - e'^2 (12.29321) \sin 2(n't - \omega') \\
&-ee' (360.2211) \sin (nt + n't - \omega - \omega') \\
&-ee' (422.2729) \sin (nt - n't - \omega + \omega') - \gamma^2 (45.58880) \sin 2(nt - \Omega) \\
&-\{\tfrac{1}{2} - 4.68750e^2 + 234.70365e'^2 - 1.21875\gamma^2\} \sin 2(nt - n't) \\
&+\{0.813293 + 102.95031e^2 - 6.98407e'^2 - 2.457382\gamma^2\} \sin 4(nt - n't) \\
&-e (0.5625) \sin (3nt - 2n't - \omega) + e (3.6875) \sin (nt - 2n't + \omega) \\
&-e' (57.98525) \sin (2nt - n't - \omega') + e' (61.01057) \sin (2nt - 3n't + \omega') \\
&+e (1.981119) \sin (5nt - 4n't - \omega) + e (54.38606) \sin (3nt - 4n't + \omega) \\
&-e' (0.742407) \sin (4nt - 3n't - \omega') + e' (6.32743) \sin (4nt - 5n't + \omega') \\
&-e^2 (1.000000) \sin (4nt - 2n't - 2\omega') + e^2 (5.06350) \sin 2(n't - \omega) \\
&+e^2 (3.504843) \sin (6nt - 4n't - 2\omega) \\
&-e^2 (62.8448) \sin (2nt - 4n't + 2\omega) \\
&+e'^2 (0.167880) \sin (4nt - 2n't - 2\omega') \\
&+e'^2 (29.57555) \sin (4nt - 6n't + 2\omega') \\
&+e'^2 (264.8727) \sin (2nt - 4n't + 2\omega') - e'^2 (13.45859) \sin 2(nt - \omega') \\
&-ee' (107.02778) \sin (3nt - n't - \omega - \omega') \\
&-ee' (22.06616) \sin (nt - 3n't + \omega + \omega') \\
&+ee' (118.41973) \sin (3nt - 3n't - \omega + \omega') \\
&+ee' (7.60013) \sin (nt - n't + \omega - \omega') \\
&-ee' (1.819882) \sin (5nt - 3n't - \omega - \omega') \\
&+ee' (346.7500) \sin (3nt - 5n't + \omega + \omega') \\
&+ee' (15.29698) \sin (5nt - 5n't - \omega + \omega') \\
&-ee' (82.08619) \sin (3nt - 3n't + \omega - \omega') \\
&-\gamma^2 (0.328125) \sin (4nt - 2n't - 2\Omega) + \gamma^2 (0.765625) \sin 2(n't - \Omega) \\
&-\gamma^2 (0.458653) \sin (6nt - 4n't - 2\Omega)
\end{aligned}\right\} \quad (328)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+\gamma^2(52.41108)\sin(2nt-4n't+2\Omega)\\
&+\gamma^2(0.406646)\sin(4nt-4n't+2\omega-2\Omega)\\
&-\gamma^2(0.406646)\sin(4nt-4n't-2\omega+2\Omega)\\
&-\gamma^2(0.125)\sin(2nt-2n't+2\omega-2\Omega)\\
&+\gamma^2(0.125)\sin(2nt-2n't-2\omega+2\Omega)\Big\}\\
+\frac{\overline{m}^4}{a\mu}\frac{a}{a'}&\Big\{+(56.80747)\sin(nt-n't)-(24.08358)\sin 3(nt-n't)\\
&\qquad+(1.688293)\sin 5(nt-n't)\Big\}
\end{aligned}\right\}. \quad (328)$$

$$\left.\begin{aligned}
\delta\left(\frac{dR}{d\theta}\right)=\frac{\overline{m}^4}{a\mu}&\Big\{-\gamma(8.177748)\sin(nt-\Omega)\\
&-\gamma(0.277856)\sin(3nt-2n't-\Omega)+\gamma(11.15055)\sin(nt-2n't+\Omega)\\
&+\gamma(1.092845)\sin(5nt-4n't-\Omega)\\
&+\gamma(4.343501)\sin(3nt-4n't+\Omega)\\
&-e\gamma(102.37081)\sin(2nt-\omega-\Omega)-e\gamma(31.23263)\sin(\omega-\Omega)\\
&-e\gamma(1.253002)\sin(4nt-2n't-\omega-\Omega)\\
&-e\gamma(1.542799)\sin(2nt-2n't-\omega+\Omega)\\
&+e\gamma(7.76993)\sin(2nt-2n't+\omega-\Omega)\\
&-e\gamma(75.55318)\sin(2n't-\omega-\Omega)\\
&+e\gamma(3.808140)\sin(6nt-4n't-\omega-\Omega)\\
&+e\gamma(49.37869)\sin(4nt-4n't+\omega-\Omega)\\
&+e\gamma(8.83333)\sin(4nt-4n't-\omega+\Omega)\\
&-e\gamma(21.75649)\sin(2nt-4n't+\omega+\Omega)\\
&-e'\gamma(29.13023)\sin(nt+n't-\omega'-\Omega)\\
&-e'\gamma(15.55053)\sin(nt-n't+\omega'-\Omega)\\
&-e'\gamma(34.09351)\sin(3nt-n't-\omega'-\Omega)\\
&+e'\gamma(34.74764)\sin(nt-n't-\omega'+\Omega)\\
&+e'\gamma(22.20663)\sin(nt-3n't+\omega'+\Omega)\\
&+e'\gamma(32.783489)\sin(3nt-3n't+\omega'-\Omega)\\
&-e'\gamma(1.010262)\sin(5nt-3n't-\omega'-\Omega)\\
&+e'\gamma(8.38028)\sin(5nt-5n't+\omega'-\Omega)\\
&-e'\gamma(6.206083)\sin(3nt-3n't-\omega'+\Omega)\\
&+e'\gamma(26.158691)\sin(3nt-5n't+\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (329)$$

$$
\left.
\begin{aligned}
\partial(c_1 \cos\beta) = \frac{\overline{m}^2}{\mu}\Big\{ & - \tfrac{3}{8} e \cos(2nt - \omega) + \tfrac{3}{8} e \cos\omega \\
& - e'(20.16597)\cos(nt + n't - \omega') + e'(20.16597)\cos(nt - n't + \omega') \\
& - e^2(0.90625)\cos(3nt - 2\omega) \pm e^2(0.16525)\cos(nt - 2\omega) \\
& + \{\tfrac{3}{4}e^2 - \tfrac{17}{48}\gamma^2\}\cos nt - e'^2(15.339485)\cos(nt + 2n't - 2\omega') \\
& + e'^2(15.339485)\cos(nt - 2n't + 2\omega') \\
& - ee'(53.89648)\cos(2nt + n't - \omega - \omega') \\
& + ee'(57.31426)\cos(2nt - n't - \omega + \omega') \\
& \pm ee'(13.56454)\cos(n't - \omega - \omega') - ee'(16.98232)\cos(n't + \omega - \omega') \\
& + \tfrac{5}{24}\gamma^2\cos(3nt - 2\Omega) \pm \tfrac{7}{48}\gamma^2\cos(nt - 2\Omega) \\
& + \{0.91493212 + 71.82326e^2 - 2.287330e'^2 - 0.083607\gamma^2\}\cos(3nt - 2n't) \\
& \mp \{0.91493212 + 7.02710e^2 - 2.287330e'^2 - 2.792905\gamma^2\}\cos(nt - 2n't) \\
& + e(2.299649)\cos(4nt - 2n't - \omega) \\
& \mp e(0.469784)\cos(2nt - 2n't - \omega) \\
& + e(32.86787)\cos(2nt - 2n't + \omega) - e(34.69773)\cos(2n't - \omega) \\
& - e'(0.3944260)\cos(3nt - n't - \omega') \\
& \pm e'(0.3944260)\cos(nt - n't - \omega') \\
& + e'(3.753160)\cos(3nt - 3n't + \omega') \mp e'(3.753160)\cos(nt - 3n't + \omega') \\
& + e^2(4.331852)\cos(5nt - 2n't - 2\omega) \\
& \mp e^2(0.418755)\cos(3nt - 2n't - 2\omega) \\
& - e^2(59.83115)\cos(nt + 2n't - 2\omega) - e^2(8.87812)\cos(nt - 2n't + 2\omega) \\
& + e'^2(10.82207)\cos(3nt - 4n't + 2\omega') \\
& \mp e'^2(10.82207)\cos(nt - 4n't + 2\omega') \\
& - ee'(0.998083)\cos(4nt - n't - \omega - \omega') \\
& \pm ee'(0.209231)\cos(2nt - n't - \omega - \omega') \\
& + ee'(89.93894)\cos(2nt - 3n't + \omega + \omega') \\
& - ee'(97.4453)\cos(3n't - \omega - \omega') \\
& + ee'(9.368335)\cos(4nt - 3n't - \omega + \omega') \\
& \mp ee'(1.862015)\cos(2nt - 3n't - \omega + \omega') \\
& - ee'(28.500959)\cos(2nt - n't + \omega - \omega') \\
& + ee'(29.289911)\cos(n't - \omega + \omega') \\
& - \gamma^2(0.0081532)\cos(5nt - 2n't - 2\Omega) \\
& \pm \gamma^2(0.2268861)\cos(3nt - 2n't - 2\Omega) \\
& - \gamma^2(18.078167)\cos(nt + 2n't - 2\Omega) \\
& + \gamma^2(15.14014)\cos(nt - 2n't + 2\Omega) \\
& + \gamma^2(0.3430995)\cos(3nt - 2n't + 2\omega - 2\Omega)
\end{aligned}
\right\} . \qquad (330)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\mp \gamma^2 (0.1143665) \cos (nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2 (0.3430995) \cos (3nt - 2n't - 2\omega + 2\Omega) \\
&\pm \gamma^2 (0.1143665) \cos (nt - 2n't - 2\omega + 2\Omega) \Big\} \\
&+ \frac{\overline{m}^2}{\mu}\frac{a}{a'}\Big\{ - (14.308899) \cos (2nt - n't) + (14.308899) \cos n't \\
&\qquad + (0.2950661) \cos (4nt - 3n't) \mp (0.2950661) \cos (2nt - 3n't) \Big\}
\end{aligned}\right\} . \quad (330)
$$

$$
\left.\begin{aligned}
\delta (c_2 \cos \beta) = {}& a^2 \frac{\overline{m}^2}{\mu} ndt \Big\{ \pm \{\tfrac{1}{3} - 2.7083333e^2 + \tfrac{1}{2}e'^2 - 0.2291667\gamma^2\} \cos nt \\
&\pm e (0.375) \cos (2nt - \omega) \mp e (1.0416667) \cos \omega \\
&\pm e' (20.16597) \cos (nt + n't - \omega') \mp e' (20.16597) \cos (nt - n't + \omega') \\
&\pm e^2 (0.8645833) \cos (3nt - 2\omega) + e^2 (0.0520833) \cos (nt - 2\omega) \\
&\pm e'^2 (15.339485) \cos (nt + 2n't - 2\omega') \\
&\mp e'^2 (15.339485) \cos (nt - 2n't + 2\omega') \\
&\mp \tfrac{1}{8}\gamma^2 \cos (3nt - 2\Omega) + \tfrac{9}{16}\gamma^2 \cos (nt - 2\Omega) \\
&\pm ee' (53.89648) \cos (2nt + n't - \omega - \omega') \\
&\mp ee' (57.31426) \cos (2nt - n't - \omega + \omega') \\
&\pm ee' (16.98232) \cos (n't + \omega - \omega') - ee' (13.56454) \cos (n't - \omega - \omega') \\
&\mp \{1.3202505 + 66.58077e^2 - 3.300626e'^2 - 0.743729\gamma^2\} \cos (3nt - 2n't) \\
&+ \{0.5096137 + 4.621185e^2 - 1.274034e'^2 + 1.965568\gamma^2\} \cos (nt - 2n't) \\
&\mp e (2.968087) \cos (4nt - 2n't - \omega) + e (0.611983) \cos (2nt - 2n't - \omega) \\
&\mp e (29.81672) \cos (2nt - 2n't + \omega) \pm e (36.93823) \cos (2n't - \omega) \\
&\pm e' (0.5887632) \cos (3nt - n't - \omega') - e' (0.2000888) \cos (nt - n't - \omega') \\
&\mp e' (5.231537) \cos (3nt - 3n't + \omega') \\
&+ e' (2.294783) \cos (nt - 3n't + \omega') \\
&\mp e^2 (5.245293) \cos (5nt - 2n't - 2\omega) \\
&+ e^2 (0.538204) \cos (3nt - 2n't - 2\omega) \\
&\pm e^2 (74.55418) \cos (nt + 2n't - 2\omega) \\
&\pm e^2 (18.81616) \cos (nt - 2n't + 2\omega) \\
&\mp e'^2 (14.57032) \cos (3nt - 4n't + 2\omega') \\
&+ e'^2 (7.07382) \cos (nt - 4n't + 2\omega') \\
&\pm ee' (1.320617) \cos (4nt - n't - \omega - \omega') \\
&- ee' (0\ 275371) \cos (2nt - n't - \omega - \omega') \\
&\pm ee' (78.30708) \cos (2nt - 3n't + \omega + \omega') \\
&\pm ee' (106.12037) \cos (3n't - \omega - \omega')
\end{aligned}\right\} . \quad (331)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\mp ee'(11.792498)\cos(4nt-3n't-\omega+\omega')\\
&+ee'(2.394606)\cos(2nt-3n't-\omega+\omega')\\
&\pm ee'(27.09015)\cos(2nt-n't+\omega-\omega')\\
&\mp ee'(30.31154)\cos(n't-\omega+\omega')\\
&\pm\tfrac{1}{24}\gamma^2\cos(nt+2\omega-2\Omega)\mp\tfrac{1}{24}\gamma^2\cos(nt-2\omega+2\Omega)\\
&\pm\gamma^2(0.0332834)\cos(5nt-2n't-2\Omega)\\
&+\gamma^2(0.3462578)\cos(3nt-2n't-2\Omega)\\
&\pm\gamma^2(17.451507)\cos(nt+2n't-2\Omega)\\
&\mp\gamma^2(16.52825)\cos(nt-2n't+2\Omega)\\
&\mp\gamma^2(0.4950940)\cos(3nt-2n't+2\omega-2\Omega)\\
&\pm\gamma^2(0.4950940)\cos(3nt-2n't-2\omega+2\Omega)\\
&+\gamma^2(0.0637018)\cos(nt-2n't+2\omega-2\Omega)\\
&-\gamma^2(0.0637018)\cos(nt-2n't-2\omega+2\Omega)\Big\}\\
&+a^2\frac{\overline{m}^2}{\mu}\frac{a}{a'}ndt\Big\{\pm(14.10625)\cos(2nt-n't)\mp(14.51155)\cos n't\\
&\qquad\mp(0.6328315)\cos(4nt-3n't)-(0.0426992)\cos(2nt-3n't)\Big\}
\end{aligned}\right\} \quad (331)
$$

$$
\left.\begin{aligned}
\delta(c_3\sin\beta)=a\frac{\overline{m}^2}{\mu}ndt\Big\{&-\{\tfrac{1}{3}-\tfrac{35}{12}e^2+\tfrac{1}{2}e'^2+\tfrac{1}{8}\gamma^2\}\sin nt\\
&-\tfrac{41}{48}e\sin(2nt-\omega)+\tfrac{7}{16}e\sin\omega-e'(41.78596)\sin(nt+n't-\omega')\\
&+e'(38.76708)\sin(nt-n't+\omega')-e^2(2.4375)\sin(3nt-2\omega)\\
&\mp e^2(0.2708333)\sin(nt-2\omega)-e'^2(32.80214)\sin(nt+2n't-2\omega')\\
&+e'^2(28.21249)\sin(nt-2n't+2\omega')\\
&-ee'(136.5606)\sin(2nt+n't-\omega-\omega')\\
&+ee'(141.3244)\sin(2nt-n't-\omega+\omega')\\
&-ee'(10.69179)\sin(n't+\omega-\omega')\mp ee'(5.17676)\sin(nt-\omega-\omega')\\
&-\tfrac{3}{8}\gamma^2\sin(3nt-2\Omega)\mp\tfrac{5}{12}\gamma^2\sin(nt-2\Omega)-\tfrac{1}{24}\gamma^2\sin(nt+2\omega-2\Omega)\\
&+\tfrac{1}{24}\gamma^2\sin(nt-2\omega+2\Omega)\\
&+\{2.7368203+210.48646e^2-6.84206e'^2-7.156149\gamma^2\}\sin(3nt-2n't)\\
&\mp\{1.4597925-16.10945e^2-3.64948e'^2+0.140473\gamma^2\}\sin(nt-2n't)\\
&+e(7.610186)\sin(4nt-2n't-\omega)\pm e(0.115536)\sin(2nt-2n't-\omega)\\
&+e(79.29330)\sin(2nt-2n't+\omega)-e(28.77728)\sin(2n't-\omega)\\
&-e'(1.198668)\sin(3nt-n't-\omega')\pm e'(0.7087031)\sin(nt-n't-\omega')\\
&+e'(11.044924)\sin(3nt-3n't+\omega')\\
&\mp e'(5.240024)\sin(nt-3n't+\omega')
\end{aligned}\right\} \quad (332)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ e^2 (15.561424) \sin (5nt - 2n't - 2\omega) \\
&\pm e^2 (0.081105) \sin (3nt - 2n't - 2\omega) \\
&- e^2 (125.5085) \sin (nt + 2n't - 2\omega) \\
&- e^2 (15.12901) \sin (nt - 2n't + 2\omega) \\
&+ e'^2 (31.33350) \sin (3nt - 4n't + 2\omega') \\
&\mp e'^2 (12.97524) \sin (nt - 4n't + 2\omega') \\
&- ee' (3.156742) \sin (4nt - n't - \omega - \omega') \\
&\pm ee' (0.161794) \sin (2nt - n't - \omega - \omega') \\
&+ ee' (213.3620) \sin (2nt - 3n't + \omega + \omega') \\
&- ee' (95.5720) \sin (3n't - \omega - \omega') \\
&+ ee' (30.62012) \sin (4nt - 3n't - \omega + \omega') \\
&\pm ee' (0.69161) \sin (2nt - 3n't - \omega + \omega') \\
&- ee' (69.97185) \sin (2nt - n't + \omega - \omega') \\
&+ ee' (19.59616) \sin (n't - \omega + \omega') \\
&- \gamma^2 (1.657622) \sin (5nt - 2n't - 2\Omega) \\
&\pm \gamma^2 (0.052996) \sin (3nt - 2n't - 2\Omega) \\
&- \gamma^2 (37.02972) \sin (nt + 2n't - 2\Omega) \\
&+ \gamma^2 (28.81600) \sin (nt - 2n't + 2\Omega) \\
&+ \gamma^2 (1.026307) \sin (3nt - 2n't + 2\omega - 2\Omega) \\
&\mp \gamma^2 (0.1824736) \sin (nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2 (1.026307) \sin (3nt - 2n't - 2\omega + 2\Omega) \\
&\pm \gamma^2 (0.1824736) \sin (nt - 2n't - 2\omega + 2\Omega) \Big\} \\
&+ a\frac{\overline{m}^2}{\mu}\frac{a}{a'}ndt \Big\{ -(35.435807) \sin (2nt - n't) + (9.363977) \sin n't \\
&\qquad + (1.0790470) \sin (4nt - 3n't) \mp (1.2344524) \sin (2nt - 3n't) \Big\}
\end{aligned}\right\} . \quad (332)
$$

$$
\left.\begin{aligned}
&\delta (c_4 \sin \beta) = a\frac{\overline{m}^2}{\mu} ndt \Big\{ + \gamma (0.5416667) \sin (2nt - \Omega) \\
&+ \gamma (0.5416667) \sin \Omega + \gamma (2.283342) \sin (4nt - 2n't - \Omega) \\
&+ \gamma (4.422928) \sin (2n't - \Omega) + \gamma (6.252792) \sin (2nt - 2n't + \Omega) \\
&\mp \gamma (0.453478) \sin (2nt - 2n't - \Omega) \Big\}
\end{aligned}\right\} . \quad (333)
$$

29. We have thus developed the variation of all the factors which enter into the formation of $\frac{d\delta_1 r}{dt}$, given by equation (259). It is evident that the whole variation of $\frac{d\delta_1 r}{dt}$ will be given, to terms of the order of the square of the disturbing force, by simply taking the differential of equation (259) relative to the functions and forces, and substituting their variations, which we have just computed, instead of the differentials of these quantities. We shall therefore have,

$$\left.\begin{aligned}\delta\frac{d\delta_1 r}{dt} = &\frac{1}{\sqrt{a\mu}}\,\delta(c_1\cos\beta)\int\left\{c_2\cos\beta\left(\frac{dR}{dr}\right)+c_3\sin\beta\left(\frac{dR}{dv}\right)+c_4\sin\beta\left(\frac{dR}{d\theta}\right)\right\}\\
&-\frac{1}{\sqrt{a\mu}}\,\delta(c_1\sin\beta)\int\left\{c_2\sin\beta\left(\frac{dR}{dr}\right)+c_3\cos\beta\left(\frac{dR}{dv}\right)+c_4\cos\beta\left(\frac{dR}{d\theta}\right)\right\}\\
&+\frac{1}{\sqrt{a\mu}}\,c_1\cos\beta\int\left\{c_2\cos\beta\,\delta\left(\frac{dR}{dr}\right)+\delta(c_2\cos\beta)\left(\frac{dR}{dr}\right)+c_3\sin\beta\,\delta\left(\frac{dR}{dv}\right)\right.\\
&\qquad\left.+\,\delta(c_3\sin\beta)\left(\frac{dR}{dv}\right)+c_4\sin\beta\,\delta\left(\frac{dR}{d\theta}\right)+\delta(c_4\sin\beta)\left(\frac{dR}{d\theta}\right)\right\}\\
&-\frac{1}{\sqrt{a\mu}}\,c_1\sin\beta\int\left\{c_2\sin\beta\,\delta\left(\frac{dR}{dr}\right)+\delta(c_2\sin\beta)\left(\frac{dR}{dr}\right)+c_3\cos\beta\,\delta\left(\frac{dR}{dv}\right)\right.\\
&\qquad\left.+\,\delta(c_3\cos\beta)\left(\frac{dR}{dv}\right)+c_4\cos\beta\,\delta\left(\frac{dR}{d\theta}\right)+\delta(c_4\cos\beta)\left(\frac{dR}{d\theta}\right)\right\}
\end{aligned}\right\}. \quad (334)$$

We shall now develop this equation.

If we multiply equation (327) by equation (228), we shall obtain,

$$c_2\cos\beta\,\delta\left(\frac{dR}{dr}\right) =$$

$$\left.\begin{aligned}\frac{\overline{m}^4}{\mu}\,ndt\Big\{&\mp\{3.7938818 - 165.5478e^2 + 34.39014e'^2 - 11.42219\gamma^2\}\cos nt\\
&\mp e\,(66.35333)\cos(2nt-\omega)\mp e\,(58.76557)\cos\omega\\
&\mp e'\,(12.08458)\cos(nt+n't-\omega')\mp e'\,(12.08458)\cos(nt-n't+\omega')\\
&\mp e^2\,(192.67122)\cos(3nt-2\omega) - e^2\,(62.80997)\cos(nt-2\omega)\\
&- e'^2\,(29.948215)\cos(nt+2n't-2\omega')\\
&\mp e'^2\,(29.948215)\cos(nt-2n't+2\omega')\\
&\mp ee'\,(145.8464)\cos(2nt+n't-\omega-\omega')\\
&\mp ee'\,(185.6115)\cos(2nt-n't-\omega+\omega')\\
&\mp ee'\,(161.4423)\cos(n't+\omega-\omega') - ee'\,(121.6772)\cos(n't-\omega-\omega')
\end{aligned}\right\}. \quad (335)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&\mp\gamma^2(25.87588)\cos(3nt-2\Omega)-\gamma^2(24.92741)\cos(nt-2\Omega)\\
&\mp\gamma^2(0.474235)\cos(nt+2\omega-2\Omega)\pm\gamma^2(0.474235)\cos(nt-2n't+2\Omega)\\
&\mp\{0.4469174+9.472849e^2+120.58468e'^2-4.715209\gamma^2\}\cos(3nt-2n't)\\
&-\{0.4469174-1.122527e^2+120.58468e'^2-4.715209\gamma^2\}\cos(nt-2n't)\\
&\pm\{0.8895220+129.02886e^2-7.61926e'^2-0.952636\gamma^2\}\cos(5nt-4n't)\\
&+\{0.8895220+59.70888e^2-7.61926e'^2-0.952636\gamma^2\}\cos(3nt-4n't)\\
&\mp e(1.268054)\cos(4nt-2n't-\omega)-e(0.374220)\cos(2nt-2n't-\omega)\\
&\mp e(5.671908)\cos(2nt-2n't+\omega)\mp e(6.565742)\cos(2n't-\omega)\\
&\pm e(3.554298)\cos(6nt-4n't-\omega)+e(1.775254)\cos(4nt-4n't-\omega)\\
&\pm e(36.43525)\cos(4nt-4n't+\omega)+e(38.21429)\cos(2nt-4n't+\omega)\\
&\mp e'(29.96241)\cos(3nt-n't-\omega')-e'(29.96241)\cos(nt-n't-\omega')\\
&\pm e'(28.59782)\cos(3nt-3n't+\omega')\\
&+e'(28.59782)\cos(nt-3n't+\omega')\\
&\mp e'(0.824521)\cos(5nt-3n't-\omega')\\
&-e'(0.824521)\cos(3nt-3n't-\omega')\\
&\pm e'(6.798425)\cos(5nt-5n't+\omega')\\
&+e'(6.798425)\cos(3nt-5n't+\omega')\\
&\mp e^2(2.902669)\cos(5nt-2n't-2\omega)\\
&-e^2(0.701748)\cos(3nt-2n't-2\omega)\\
&\pm e^2(1.087646)\cos(nt+2n't-2\omega)\\
&\pm e^2(13.883943)\cos(nt-2n't+2\omega)\\
&\pm e^2(9.259582)\cos(7nt-4n't-2\omega)\\
&+e^2(2.818128)\cos(5nt-4n't-2\omega)\\
&\mp e^2(89.79718)\cos(3nt-4n't+2\omega)\\
&-e^2(14.03574)\cos(nt-4n't+2\omega)\\
&\pm e'^2(124.32503)\cos(3nt-4n't+2\omega')\\
&+e'^2(124.32503)\cos(nt-4n't+2\omega')\\
&\mp e'^2(7.819455)\cos(3nt-2\omega')-e'^2(7.819455)\cos(nt-2\omega')\\
&\pm e'^2(31.19518)\cos(5nt-6n't+2\omega')\\
&+e'^2(31.19518)\cos(3nt-6n't+2\omega')\\
&\pm e'^2(0.1898799)\cos(5nt-2n't-2\omega')\\
&+e'^2(0.1898799)\cos(3nt-2n't-2\omega')\\
&\mp ee'(112.16766)\cos(4nt-n't-\omega-\omega')\\
&-ee'(52.24284)\cos(2nt-n't-\omega-\omega')\\
&\mp ee'(67.29708)\cos(2nt-3n't+\omega+\omega')\\
&\mp ee'(10.10146)\cos(3n't-\omega-\omega)
\end{aligned}\right\}\cdot\quad(335)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&\pm ee'\,(113.54178)\cos(4nt-3n't-\omega+\omega') \\
&+ee'\,(56.34616)\cos(2nt-3n't-\omega+\omega') \\
&\pm ee'\,(30.26149)\cos(2nt-n't+\omega-\omega') \\
&\mp ee'\,(29.66333)\cos(n't-\omega+\omega') \\
&\mp ee'\,(3.302682)\cos(6nt-3n't-\omega-\omega') \\
&-ee'\,(1.653640)\cos(4nt-3n't-\omega-\omega') \\
&\pm ee'\,(27.08471)\cos(6nt-5n't-\omega+\omega') \\
&+ee'\,(13.48786)\cos(4nt-5n't-\omega+\omega') \\
&\pm ee'\,(226.5877)\cos(4nt-5n't+\omega+\omega') \\
&+ee'\,(240.1843)\cos(2nt-5n't+\omega+\omega') \\
&\mp ee'\,(50.032644)\cos(4nt-3n't+\omega-\omega') \\
&-ee'\,(51.681686)\cos(2nt-3n't+\omega-\omega') \\
&\pm\gamma^2\,(0.5715219)\cos(5nt-2n't-2\Omega) \\
&+\gamma^2\,(0.6832513)\cos(3nt-2n't-2\Omega) \\
&\mp\gamma^2\,(4.440036)\cos(nt+2n't-2\Omega) \\
&\mp\gamma^2\,(4.328307)\cos(nt-2n't+2\Omega) \\
&\mp\gamma^2\,(0.1675940)\cos(3nt-2n't+2\omega-2\Omega) \\
&-\gamma^2\,(0.0558646)\cos(nt-2n't+2\omega-2\Omega) \\
&+\gamma^2\,(0.1675940)\cos(3nt-2n't-2\omega+2\Omega) \\
&\pm\gamma^2\,(0.0558646)\cos(nt-2n't-2\omega+2\Omega) \\
&\mp\gamma^2\,(0.00958835)\cos(7nt-4n't-2\Omega) \\
&-\gamma^2\,(0.231968)\cos(5nt-4n't-2\Omega) \\
&\pm\gamma^2\,(23.71368)\cos(3nt-4n't+2\Omega) \\
&+\gamma^2\,(23.93606)\cos(nt-4n't+2\Omega) \\
&\pm\gamma^2\,(0.555951)\cos(5nt-4n't+2\omega-2\Omega) \\
&+\gamma^2\,(0.333571)\cos(3nt-4n't+2\omega-2\Omega) \\
&\mp\gamma^2\,(0.555951)\cos(5nt-4n't-2\omega+2\Omega) \\
&-\gamma^2\,(0.333571)\cos(3nt-4n't-2\omega+2\Omega)\Big\} \\
&+\frac{\overline{m}^4}{\mu}\frac{a}{a'}ndt\Big\{\pm(24.034165)\cos(2nt-n't)\pm(24.034165)\cos n't \\
&\qquad\mp(17.06536)\cos(4nt-3n't)-(17.06536)\cos(2nt-3n't) \\
&\qquad\pm(1.6281983)\cos(6nt+5n't)+(1.6281983)\cos(4nt-5n't)\Big\}
\end{aligned}\right\} . \quad (335)
$$

In the same manner equations (234) and (328) will give

$$
\left.
\begin{aligned}
&c_3 \sin\beta\,\delta\left(\frac{dR}{dv}\right) = \\
&\frac{\overline{m}'}{\mu} ndt\Big\{ \pm e\,(157.4356)\cos(2nt-\omega) \mp e\,(157.4356)\cos\omega \\
&\quad \pm e'\,(2.36365)\cos(nt+n't-\omega') \mp e'\,(2.36365)\cos(nt-n't+\omega') \\
&\quad \pm e^2\,(391.60355)\cos(3nt-2\omega) - e^2\,(155.45015)\cos(nt-2\omega) \\
&\quad \mp e^2\,(236.1534)\cos nt \pm e'^2\,(12.29321)\cos(nt+2n't-2\omega') \\
&\quad \mp e'^2\,(12.29321)\cos(nt-2n't+2\omega') \\
&\quad \pm ee'\,(363.1757)\cos(2nt+n't-\omega-\omega') \\
&\quad \pm ee'\,(419.3183)\cos(2nt-n't-\omega+\omega') \\
&\quad \mp ee'\,(422.8638)\cos(n't+\omega-\omega') - ee'\,(359.6302)\cos(n't-\omega-\omega') \\
&\quad \pm \gamma^2\,(45.58880)\cos(3nt-2\Omega) - \gamma^2\,(45.58880)\cos(nt-2\Omega) \\
&\quad \pm\{\tfrac{1}{2} - 9.93250e^2 + 234.70365e'^2 - 1.34375\gamma^2\}\cos(3nt-2n't) \\
&\quad -\{\tfrac{1}{2} - 3.55750e^2 + 234.70365e'^2 - 1.34375\gamma^2\}\cos(nt-2n't) \\
&\quad \mp\{0.8132919 + 169.62431e^2 - 6.98407e'^2 - 2.660705\gamma^2\}\cos(5nt-4n't) \\
&\quad +\{0.8132919 + 91.01691e^2 - 6.98407e'^2 - 2.660705\gamma^2\}\cos(3nt-4n't) \\
&\quad \pm e\,(1.1875)\cos(4nt-2n't-\omega) - e\,(0.4375)\cos(2nt-2n't-\omega) \\
&\quad \mp e\,(3.8125)\cos(2nt-2n't+\omega) \pm e\,(3.0625)\cos(2n't-\omega) \\
&\quad \mp e\,(2.997735)\cos(6nt-4n't-\omega) + e\,(1.777796)\cos(4nt-4n't-\omega) \\
&\quad \mp e\,(54.18274)\cos(4nt-4n't+\omega) + e\,(55.40268)\cos(2nt-4n't+\omega) \\
&\quad \pm e'\,(57.98525)\cos(3nt-n't-\omega') - e'\,(57.98525)\cos(nt-n't-\omega') \\
&\quad \mp e'\,(61.01057)\cos(3nt-3n't+\omega') \\
&\quad + e'\,(61.01057)\cos(nt-3n't+\omega') \\
&\quad \pm e'\,(0.742407)\cos(5nt-3n't-\omega') \\
&\quad - e'\,(0.742407)\cos(3nt-3n't-\omega') \\
&\quad \mp e'\,(6.32743)\cos(5nt-5n't+\omega') + e'\,(6.32743)\cos(3nt-5n't+\omega') \\
&\quad \pm e^2\,(2.515625)\cos(5nt-2n't-2\omega) \\
&\quad - e^2\,(0.796875)\cos(3nt-2n't-2\omega) \\
&\quad \mp e^2\,(10.485375)\cos(nt+2n't-2\omega) \\
&\quad \pm e^2\,(5.922875)\cos(nt-2n't+2\omega) \\
&\quad \mp e^2\,(7.302843)\cos(7nt-4n't-2\omega) \\
&\quad + e^2\,(2.907901)\cos(5nt-4n't+2\omega) \\
&\quad \pm e^2\,(76.5430)\cos(3nt-4n't+2\omega) + e^2\,(6.4594)\cos(nt-4n't+2\omega) \\
&\quad \mp e'^2\,(0.167880)\cos(5nt-2n't-2\omega') \\
&\quad + e'^2\,(0.167880)\cos(3nt-2n't-2\omega')
\end{aligned}
\right\} . \quad (336)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&\mp e'^2 (29.57555) \cos (5nt - 6n't + 2\omega') \\
&+ e'^2 (29.57555) \cos (3nt - 6n't + 2\omega') \\
&\mp e'^2 (264.8727) \cos (3nt - 4n't + 2\omega') \\
&+ e'^2 (264.8727) \cos (nt - 4n't + 2\omega') \\
&\pm e'^2 (13.45859) \cos (3nt - 2\omega') - e'^2 (13.45859) \cos (nt - 2\omega') \\
&\pm ee' (179.50934) \cos (4nt - n't - \omega - \omega') \\
&- ee' (92.53147) \cos (2nt - n't - \omega - \omega') \\
&\pm ee' (37.31880) \cos (2nt - 3n't + \omega + \omega') \\
&\pm ee' (54.19705) \cos (3n't - \omega - \omega') \\
&\mp ee' (194.68294) \cos (4nt - 3n't - \omega + \omega') \\
&+ ee' (103.16709) \cos (2nt - 3n't - \omega + \omega') \\
&\mp ee' (22.09644) \cos (2nt - n't + \omega - \omega') \\
&\mp ee' (64.88143) \cos (n't - \omega + \omega') \\
&\pm ee' (2.747891) \cos (6nt - 3n't - \omega - \omega') \\
&\mp ee' (1.634280) \cos (4nt - 3n't - \omega - \omega') \\
&\mp ee' (345.1681) \cos (4nt - 5n't + \omega + \omega') \\
&+ ee' (354.6593) \cos (2nt - 5n't + \omega + \omega') \\
&\mp ee' (23.20627) \cos (6nt - 5n't - \omega + \omega') \\
&+ ee' (13.71512) \cos (4nt - 5n't - \omega + \omega') \\
&\pm ee' (81.90059) \cos (4nt - 3n't + \omega - \omega') \\
&- ee' (83.01420) \cos (2nt - 3n't + \omega - \omega') \\
&\pm \gamma^2 (0.203125) \cos (5nt - 2n't - 2\Omega) \\
&- \gamma^2 (0.328125) \cos (3nt - 2n't - 2\Omega) \\
&\mp \gamma^2 (0.640625) \cos (nt + 2n't - 2\Omega) \pm \gamma^2 (0.765625) \cos (nt - 2n't + 2\Omega) \\
&\pm \gamma^2 (0.661976) \cos (7nt - 4n't - 2\Omega) \\
&- \gamma^2 (0.458653) \cos (5nt - 4n't - 2\Omega) \\
&\mp \gamma^2 (52.41108) \cos (3nt - 4n't + 2\Omega) + \gamma^2 (52.20776) \cos (nt - 4n't + 2\Omega) \\
&\mp \gamma^2 (0.508308) \cos (5nt - 4n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (0.304984) \cos (3nt - 4n't + 2\omega - 2\Omega) \\
&\pm \gamma^2 (0.508308) \cos (5nt - 4n't - 2\omega + 2\Omega) \\
&- \gamma^2 (0.304984) \cos (3nt - 4n't - 2\omega + 2\Omega) \\
&\pm \gamma^2 (0.1875) \cos (3nt - 2n't + 2\omega - 2\Omega) \\
&\mp \gamma^2 (0.1875) \cos (3nt - 2n't - 2\omega + 2\Omega) \\
&- \gamma^2 (0.0625) \cos (nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (0.0625) \cos (nt - 2n't - 2\omega + 2\Omega) \Big\}
\end{aligned}\right\} . \quad (336)$$

(Continued on the next page.)

$$\left.\begin{array}{l} +\dfrac{\overline{m}^4}{\mu}\dfrac{a}{a'}ndt\Big\{\mp(56.80747)\cos(2nt-n't)\pm(56.80747)\cos n't \\ \qquad \pm(24.08356)\cos(4nt-3n't)-(24.08356)\cos(2nt-3n't) \\ \qquad \mp(1.688292)\cos(6nt-5n't)+(1.688292)\cos(4nt-5n't)\Big\} \end{array}\right\} . \quad (336)$$

Equations (239) and (329) will give

$$c^4 \sin\beta\,\delta\left(\frac{dR}{d\theta}\right)=$$

$$\left.\begin{array}{l} \dfrac{\overline{m}^4}{\mu}ndt\Big\{\pm\gamma^2(4.088874)\cos(3nt-2\Omega)-\gamma^2(4.088874)\cos(nt-2\Omega) \\ \quad\mp\gamma^2(5.436346)\cos(3nt-2n't)+\gamma^2(5.436346)\cos(nt-2n't) \\ \quad\mp\gamma^2(2.718172)\cos(5nt-4n't)+\gamma^2(2.718172)\cos(3nt-4n't) \\ \quad\pm\gamma^2(0.138928)\cos(5nt-2n't-2\Omega) \\ \quad-\gamma^2(0.138928)\cos(3nt-2n't-2\Omega) \\ \quad\pm\gamma^2(5.5752745)\cos(nt+2n't-2\Omega) \\ \quad\mp\gamma^2(5.5752745)\cos(nt-2n't+2\Omega) \\ \quad\mp\gamma^2(0.5464225)\cos(7nt-4n't-2\Omega) \\ \quad+\gamma^2(0.5464225)\cos(5nt-4n't-2\Omega) \\ \quad\mp\gamma^2(2.171750)\cos(3nt-4n't+2\Omega) \\ \quad+\gamma^2(2.171750)\cos(nt-4n't+2\Omega)\Big\} \end{array}\right\} . \quad (337)$$

Equations (218) and (331) will give

$$\delta(c_2\cos\beta)\left(\frac{dR}{dr}\right)=$$

$$\left.\begin{array}{l} \dfrac{\overline{m}^4}{\mu}ndt\Big\{\pm\{0.4413109+61.44888e^2+4.367345e'^2-1.971380\gamma^2\}\cos nt \\ \quad\mp e(28.63056)\cos(2nt-\omega)\pm e(24.94852)\cos\omega \\ \quad\mp e'(8.823881)\cos(nt+n't-\omega')\pm e'(12.072877)\cos(nt-n't+\omega') \\ \quad\mp e^2(98.67835)\cos(3nt-2\omega)+e^2(21.33779)\cos(nt-2\omega) \\ \quad\mp e'^2(20.75045)\cos(nt+2n't-2\omega') \\ \quad\pm e'^2(28.66158)\cos(nt-2n't+2\omega') \\ \quad\mp ee'(90.83653)\cos(2nt+n't-\omega-\omega') \\ \quad\mp ee'(53.39137)\cos(2nt-n't-\omega+\omega') \\ \quad\pm ee'(62.16500)\cos(n't+\omega-\omega')+ee'(56.90816)\cos(n't-\omega-\omega) \\ \quad\mp\gamma^2(12.68100)\cos(3nt-2\Omega)+\gamma^2(11.43920)\cos(nt-2\Omega) \end{array}\right\} . \quad (338)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\pm \gamma^2 (0.055165) \cos (nt + 2\omega - 2\Omega) \mp \gamma^2 (0.055165) \cos (nt - 2\omega + 2\Omega) \\
&\pm \{0.4101252 + 30.20549 e^2 - 57.42595 e'^2 - 1.098163 \gamma^2\} \cos (3nt - 2n't) \\
&- \{0.5048068 - 7.48049 e^2 - 59.44669 e'^2 + 0.336685 \gamma^2\} \cos (nt - 2n't) \\
&\pm \{0.9901879 + 74.44857 e^2 - 8.458268 e'^2 - 1.052891 \gamma^2\} \cos (5nt - 4n't) \\
&- \{0.3822103 + 42.53669 e^2 + 3.289328 e'^2 + 1.283072 \gamma^2\} \cos (3nt - 4n't) \\
&\pm e (0.4977308) \cos (4nt - 2n't - \omega) \\
&+ e (0.2276617) \cos (2nt - 2n't - \omega) \\
&\pm e (15.98455) \cos (2nt - 2n't + \omega) \mp e (17.997962) \cos (2n't - \omega) \\
&\pm e (3.711347) \cos (6nt - 4n't - \omega) - e (1.032302) \cos (4nt - 4n't - \omega) \\
&\pm e (19.88707) \cos (4nt - 4n't + \omega) - e (26.74815) \cos (2nt - 4n't + \omega) \\
&\mp e' (14.30367) \cos (3nt - n't - \omega') + e' (14.96731) \cos (nt - n't - \omega') \\
&\pm e' (17.85544) \cos (3nt - 3n't + \omega') \\
&- e' (17.51908) \cos (nt - 3n't + \omega') \\
&\mp e' (0.936666) \cos (5nt - 3n't - \omega') \\
&+ e' (0.3411716) \cos (3nt - 3n't - \omega') \\
&\pm e' (7.389311) \cos (5nt - 5n't + \omega') \\
&- e' (3.043821) \cos (3nt - 5n't + \omega') \\
&\pm e^2 (0.145281) \cos (5nt - 2n't - 2\omega) \\
&+ e^2 (0.580409) \cos (3nt - 2n't - 2\omega) \\
&\mp e^2 (28.29915) \cos (nt + 2n't - 2\omega) \\
&\mp e^2 (19.39447) \cos (nt - 2n't + 2\omega) \\
&\pm e^2 (9.253444) \cos (7nt - 4n't - 2\omega) \\
&- e^2 (1.856554) \cos (5nt - 4n't - 2\omega) \\
&\mp e^2 (68.53319) \cos (3nt - 4n't + 2\omega) \\
&+ e^2 (12.77023) \cos (nt - 4n't + 2\omega) \\
&\pm e'^2 (75.00937) \cos (3nt - 4n't + 2\omega') \\
&- e'^2 (72.38159) \cos (nt - 4n't + 2\omega') \\
&\mp e'^2 (2.89867) \cos (3nt - 2\omega') + e'^2 (3.51913) \cos (nt - 2\omega') \\
&\pm e' (33.07712) \cos (5nt - 6n't + 2\omega') \\
&- e'^2 (14.52545) \cos (3nt - 6n't + 2\omega') \\
&\pm e'^2 (0.2207862) \cos (5nt - 2n't - 2\omega') \\
&- e'^2 (0.0750333) \cos (3nt - 2n't - 2\omega') \\
&\mp ee' (61.56311) \cos (4nt - n't - \omega - \omega') \\
&+ ee' (32.47678) \cos (2nt - n't - \omega - \omega') \\
&\pm ee' (36.99719) \cos (2nt - 3n't + \omega + \omega') \\
&\mp ee' (81.41209) \cos (3n't - \omega - \omega')
\end{aligned}
\right\} . \qquad (338)
$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
\pm ee' (69.69489) \cos (4nt - 3n't - \omega + \omega') \\
- ee' (34.89808) \cos (2nt - 3n't - \omega + \omega') \\
\pm ee' (32.84110) \cos (2nt - n't + \omega - \omega') \\
\mp ee' (7.40419) \cos (n't - \omega + \omega') \\
\mp ee' (3.508496) \cos (6nt - 3n't - \omega - \omega') \\
+ ee' (0.947779) \cos (4nt - 3n't - \omega - \omega') \\
\pm ee' (118.52592) \cos (4nt - 5n't + \omega + \omega') \\
- ee' (168.94258) \cos (2nt - 5n't + \omega + \omega') \\
\pm ee' (27.72057) \cos (6nt - 5n't - \omega + \omega') \\
- ee' (7.968132) \cos (4nt - 5n't - \omega + \omega') \\
\mp ee' (29.15722) \cos (4nt - 3n't + \omega - \omega') \\
+ ee' (35.73257) \cos (2nt - 3n't + \omega - \omega') \\
\pm \gamma^2 (0.5065773) \cos (5nt - 2n't - 2\Omega) \\
- \gamma^2 (0.7579837) \cos (3nt - 2n't - 2\Omega) \\
\mp \gamma^2 (9.041858) \cos (nt + 2n't - 2\Omega) \\
\pm \gamma^2 (8.2123435) \cos (nt - 2n't + 2\Omega) \\
\pm \gamma^2 (0.1537970) \cos (3nt - 2n't + 2\omega - 2\Omega) \\
\mp \gamma^2 (0.1537970) \cos (3nt - 2n't - 2\omega + 2\Omega) \\
- \gamma^2 (0.0631009) \cos (nt - 2n't + 2\omega - 2\Omega) \\
+ \gamma^2 (0.0631009) \cos (nt - 2n't - 2\omega + 2\Omega) \\
\mp \gamma^2 (0.0249626) \cos (7nt - 4n't - 2\Omega) \\
- \gamma^2 (0.2596934) \cos (5nt - 4n't - 2\Omega) \\
\pm \gamma^2 (12.445696) \cos (3nt - 4n't + 2\Omega) \\
- \gamma^2 (13.279735) \cos (nt - 4n't + 2\Omega) \\
\pm \gamma^2 (0.6188675) \cos (5nt - 4n't + 2\omega - 2\Omega) \\
\mp \gamma^2 (0.6188675) \cos (5nt - 4n't - 2\omega + 2\Omega) \\
- \gamma^2 (0.1433288) \cos (3nt - 4n't + 2\omega - 2\Omega) \\
+ \gamma^2 (0.1433288) \cos (3nt - 4n't - 2\omega + 2\Omega) \Big\} \\
+ \frac{\overline{m}^4}{\mu} \frac{a}{a'} ndt \Big\{ \pm (4.382551) \cos (2nt - n't) \mp (2.527300) \cos n't \\
\qquad \mp (9.833124) \cos (4nt - 3n't) + (10.305861) \cos (2nt - 3n't) \\
\qquad \pm (1.7133584) \cos (6nt - 5n't) - (0.4457384) \cos (4nt - 5n't) \Big\}
\end{array}\right\} . \quad (338)
$$

Equations (219) and (332) will give

$$\delta\,(c_3 \sin\beta)\left(\frac{dR}{dv}\right) =$$

$$\left.\begin{aligned}
&\frac{\overline{m}^4}{\mu}\,ndt\Big\{\pm\{0.9577708 - 69.8255e^2 - 10.63274e'^2 - 5.711910\gamma^2\}\cos nt \\
&\pm\{0.25 - 1.21875e^2 + 123.9758e'^2 + 0.03125\gamma^2\}\cos(3nt - 2n't) \\
&-\{0.25 - 1.18750e^2 - 117.68332e'^2 + 0.03125\gamma^2\}\cos(nt - 2n't) \\
&\mp\{2.0526152 + 195.06687e^2 - 17.55144e'^2 - 5.880266\gamma^2\}\cos(5nt - 4n't) \\
&+\{1.0948444 - 36.14220e^2 - 9.29958e'^2 - 0.168356\gamma^2\}\cos(3nt - 4n't) \\
&\pm e\,(20.03791)\cos(2nt - \omega) \pm e\,(64.89378)\cos\omega \\
&\pm e\,(0.890625)\cos(4nt - 2n't - \omega) + e\,(0.109375)\cos(2n't - \omega) \\
&\mp e\,(1.078125)\cos(2nt - 2n't + \omega) + e\,(0.078125)\cos(2nt - 2n't - \omega) \\
&\mp e\,(7.76026)\cos(6nt - 4n't - \omega) + e\,(1.008193)\cos(4nt - 4n't - \omega) \\
&\mp e\,(53.31213)\cos(4nt - 4n't + \omega) \\
&- e\,(24.86749)\cos(2nt - 4n't + \omega) \\
&\pm e'\,(2.902557)\cos(nt + n't - \omega') \pm e'\,(3.956956)\cos(nt - n't + \omega') \\
&\pm e'\,(31.21447)\cos(3nt - n't - \omega') - e'\,(32.21447)\cos(nt - 3n't + \omega') \\
&\mp e'\,(28.20031)\cos(3nt - 3n't + \omega') + e'\,(29.20031)\cos(nt - n't - \omega') \\
&\pm e'\,(1.925309)\cos(5nt - 3n't - \omega') \\
&- e'\,(1.0789493)\cos(3nt - 3n't - \omega') \\
&\mp e'\,(15.467847)\cos(5nt - 5n't + \omega') \\
&+ e'\,(7.761974)\cos(3nt - 5n't + \omega') \\
&\pm e^2\,(114.19919)\cos(3nt - 2\omega) + e^2\,(47.23960)\cos(nt - 2\omega) \\
&\pm e^2\,(2.718750)\cos(5nt - 2n't - 2\omega) \\
&+ e^2\,(0.28125)\cos(3nt - 2n't - 2\omega) \\
&\mp e^2\,(0.53125)\cos(nt + 2n't - 2\omega) \pm e^2\,(1.40625)\cos(nt - 2n't + 2\omega) \\
&\mp e^2\,(19.431324)\cos(7nt - 4n't - 2\omega) \\
&+ e^2\,(0.947364)\cos(5nt - 4n't - 2\omega) \\
&\pm e^2\,(184.62514)\cos(3nt - 4n't + 2\omega) \\
&- e^2\,(26.64539)\cos(nt - 4n't + 2\omega) \\
&\pm e'^2\,(8.93186)\cos(3nt - 2\omega') + e'^2\,(6.62171)\cos(nt - 2\omega') \\
&\pm e'^2\,(6.53431)\cos(nt + 2n't - 2\omega') \\
&\pm e'^2\,(11.91245)\cos(nt - 2n't + 2\omega') \\
&\mp e'^2\,(120.79795)\cos(3nt - 4n't + 2\omega') \\
&- e'^2\,(136.41474)\cos(nt - 4n't + 2\omega') \\
&\mp e'^2\,(69.94028)\cos(5nt - 6n't + 2\omega')
\end{aligned}\right\} . \quad (339)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+ e'^2 (32.79267) \cos (3nt - 6n't + 2\omega') \\
&\mp e'^2 (0.449500) \cos (5nt - 2n't - 2\omega') \\
&+ e'^2 (0.2657637) \cos (3nt - 2n't - 2\omega') \\
&\pm ee' (56.2585) \cos (2nt + n't - \omega - \omega') \\
&\pm ee' (55.88192) \cos (2nt - n't - \omega + \omega') \\
&\pm ee' (172.5752) \cos (n't + \omega - \omega') + ee' (147.8698) \cos (n't - \omega + \omega') \\
&\pm ee' (133.3146) \cos (4nt - n't - \omega - \omega') \\
&+ ee' (32.91882) \cos (2nt - n't - \omega - \omega') \\
&\pm ee' (79.56992) \cos (2nt - 3n't + \omega + \omega') \\
&\mp ee' (8.01923) \cos (3n't - \omega - \omega') \\
&\mp ee' (131.95141) \cos (4nt - 3n't - \omega + \omega') \\
&- ee' (39.08487) \cos (2nt - 3n't - \omega + \omega') \\
&\mp ee' (85.45951) \cos (2nt - n't + \omega - \omega') \\
&\pm ee' (18.71267) \cos (n't - \omega + \omega') \\
&\pm ee' (7.146685) \cos (6nt - 3n't - \omega - \omega') \\
&- ee' (1.156969) \cos (4nt - 3n't - \omega - \omega') \\
&\mp ee' (321.7638) \cos (4nt - 5n't + \omega + \omega') \\
&- ee' (170.5054) \cos (2nt - 5n't + \omega + \omega') \\
&\mp ee' (58.40968) \cos (6nt - 5n't - \omega + \omega') \\
&+ ee' (6.94009) \cos (4nt - 5n't - \omega + \omega') \\
&\pm ee' (76.43796) \cos (4nt - 3n't + \omega - \omega') \\
&+ ee' (28.72545) \cos (2nt - 3n't + \omega - \omega') \\
&\pm \gamma^2 (27.43566) \cos (3nt - 2\Omega) + \gamma^2 (20.98460) \cos (nt - 2\Omega) \\
&\pm \gamma^2 (0.1197206) \cos (nt + 2\omega - 2\Omega) \\
&\mp \gamma^2 (0.1197206) \cos (nt - 2\omega + 2\Omega) \\
&\pm \gamma^2 (0.25) \cos (5nt - 2n't - 2\Omega) + \gamma^2 (0.34375) \cos (3nt - 2n't - 2\Omega) \\
&\mp \gamma^2 (0.375) \cos (nt + 2n't - 2\Omega) \mp \gamma^2 (0.21875) \cos (nt - 2n't + 2\Omega) \\
&\pm \gamma^2 (0.09375) \cos (3nt - 2n't + 2\omega - 2\Omega) \\
&\mp \gamma^2 (0.09375) \cos (3nt - 2n't - 2\omega + 2\Omega) \\
&- \gamma^2 (0.03125) \cos (nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2 (0.03125) \cos (nt - 2n't - 2\omega + 2\Omega) \\
&\pm \gamma^2 (1.499793) \cos (7nt - 4n't - 2\Omega) \\
&- \gamma^2 (0.176603) \cos (5nt - 4n't - 2\Omega) \\
&\mp \gamma^2 (22.38173) \cos (3nt - 4n't + 2\Omega) \\
&- \gamma^2 (27.36172) \cos (nt - 4n't + 2\Omega) \\
&\mp \gamma^2 (1.282885) \cos (5nt - 4n't + 2\omega - 2\Omega)
\end{aligned}
\right\} . \quad (339)
$$

(Continued on the next page.)

$$\left.\begin{array}{l}
\quad \pm \gamma^2 (1.282885) \cos (5nt - 4n't - 2\omega + 2\Omega) \\
\quad + \gamma^2 (0.4105664) \cos (3nt - 4n't + 2\omega - 2\Omega) \\
\quad - \gamma^2 (0.4105664) \cos (3nt - 4n't - 2\omega + 2\Omega) \Big\} \\
+ \frac{\overline{m}^4}{\mu} \frac{a}{a'} ndt \Big\{ \mp (7.006599) \cos (2nt - n't) \mp (25.273136) \cos n't \\
\qquad \pm (26.376201) \cos (4nt - 3n't) + (6.984194) \cos (2nt - 3n't) \\
\qquad \mp (3.3750543) \cos (6nt - 5n't) + (2.2943948) \cos (4nt - 5n't) \Big\}
\end{array}\right\} . \quad (339)$$

Lastly, equations (220) and (333) will give

$$\delta (c_4 \sin \beta) \left(\frac{dR}{d\theta}\right) =$$

$$\left.\begin{array}{l}
\frac{\overline{m}^4}{\mu} ndt \Big\{ \pm \gamma^2 (0.3960945) \cos (3nt - 2\Omega) + \gamma^2 (2.921101) \cos (nt - 2\Omega) \\
\quad \mp \gamma^2 (2.977087) \cos (3nt - 2n't) + \gamma^2 (2.977087) \cos (nt - 2n't) \\
\quad \mp \gamma^2 (1.488544) \cos (5nt - 4n't) + \gamma^2 (1.488544) \cos (3nt - 4n't) \\
\quad \mp \gamma^2 (1.915632) \cos (5nt - 2n't - 2\Omega) \\
\quad + \gamma^2 (0.543234) \cos (3nt - 2n't - 2\Omega) \\
\quad \mp \gamma^2 (3.520321) \cos (nt + 2n't - 2\Omega) \\
\quad \pm \gamma^2 (4.892719) \cos (nt - 2n't + 2\Omega) \\
\quad \mp \gamma^2 (0.8562535) \cos (7nt - 4n't - 2\Omega) \\
\quad + \gamma^2 (0.1700545) \cos (5nt - 4n't - 2\Omega) \\
\quad \pm \gamma^2 (2.344797) \cos (3nt - 4n't + 2\Omega) \\
\quad - \gamma^2 (1.658598) \cos (nt - 4n't + 2\Omega) \Big\}
\end{array}\right\} . \quad (340)$$

If we now take the sum of equations (335–340), we obtain

$$\Big\{ c_2 \cos \beta\, \delta\left(\frac{dR}{dr}\right) + \delta (c_2 \cos \beta) \left(\frac{dR}{dr}\right) + c_3 \sin \beta\, \delta\left(\frac{dR}{dv}\right) + \delta (c_3 \sin \beta) \left(\frac{dR}{dv}\right)$$
$$+ c_4 \sin \beta\, \delta\left(\frac{dR}{d\theta}\right) + \delta (c_4 \sin \beta) \left(\frac{dR}{d\theta}\right) \Big\} =$$

$$\left.\begin{array}{l}
\frac{\overline{m}^4}{\mu} ndt \Big\{ \mp \{2.3948001 + 78.9822e^2 + 19.39006e'^2 - 3.73890\gamma^2\} \cos nt \\
\quad \pm e (82.48962) \cos (2nt - \omega) \mp e (126.35887) \cos \omega \\
\quad \mp e' (15.64225) \cos (nt + n't - \omega') \pm e' (1.58161) \cos (nt - n't + \omega') \\
\quad \pm e^2 (214.4532) \cos (3nt - 2\omega) - e^2 (149.68273) \cos (nt - 2\omega)
\end{array}\right\} . \quad (341)$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
\mp e'^2 (31.87114) \cos (nt + 2n't - 2\omega') \\
\mp e'^2 (1.66740) \cos (nt - 2n't + 2\omega') \\
\pm ee' (182.7513) \cos (2nt + n't - \omega - \omega') \\
\pm ee' (236.1973) \cos (2nt - n't - \omega + \omega') \\
\mp ee' (349.5634) \cos (n't + \omega - \omega') - ee' (276.5294) \cos (n't - \omega - \omega') \\
\pm \gamma^2 (38.95254) \cos (3nt - 2\Omega) - \gamma^2 (39.26018) \cos (nt - 2\Omega) \\
\mp \gamma^2 (0.299349) \cos (nt + 2\omega - 2\Omega) \\
\pm \gamma^2 (0.299349) \cos (nt - 2\omega + 2\Omega) \\
\pm \{0.7132078 + 9.58139e^2 + 180.66882e'^2 - 6.108887\gamma^2\} \cos (3nt - 2n't) \\
-\{1.7017242 - 13.34802e^2 + 178.15832e'^2 - 14.104457\gamma^2\} \cos (nt - 2n't) \\
\mp \{0.9861972 + 161.21375e^2 - 8.45798e'^2 - 2.328728\gamma^2\} \cos (5nt - 4n't) \\
+ \{2.4154480 + 72.04690e^2 - 20.61358e'^2 - 0.858053\gamma^2\} \cos (3nt - 4n't) \\
\pm e (1.307802) \cos (4nt - 2n't - \omega) - e (0.505933) \cos (2nt - 2n't - \omega) \\
\pm e (5.42202) \cos (2nt - 2n't + \omega) \mp e (21.391829) \cos (2n't - \omega) \\
\mp e (3.49235) \cos (6nt - 4n't - \omega) + e (3.528941) \cos (4nt - 4n't - \omega) \\
\mp e (51.17255) \cos (4nt - 4n't + \omega) + e (42.00133) \cos (2nt - 4n't + \omega) \\
\pm e' (44.93364) \cos (3nt - n't - \omega') - e' (43.78004) \cos (nt - n't - \omega') \\
\mp e' (42.75762) \cos (3nt - 3n't + \omega') \\
+ e' (39.87484) \cos (nt - 3n't + \omega') \\
\pm e' (0.906529) \cos (5nt - 3n't - \omega') \\
- e' (2.304705) \cos (3nt - 3n't - \omega') \\
\mp e' (7.60754) \cos (5nt - 5n't + \omega') \\
+ e' (17.84401) \cos (3nt - 5n't + \omega') \\
\pm e^2 (2.476987) \cos (5nt - 2n't - 2\omega) \\
- e^2 (0.636964) \cos (3nt - 2n't - 2\omega) \\
\mp e^2 (38.22813) \cos (nt + 2n't - 2\omega) \pm e^2 (1.81860) \cos (nt - 2n't + 2\omega) \\
\mp e^2 (8.221141) \cos (7nt - 4n't - 2\omega) \\
+ e^2 (4.816839) \cos (5nt - 4n't - 2\omega) \\
\pm e^2 (102.8378) \cos (3nt - 4n't + 2\omega) - e^2 (21.4515) \cos (nt - 4n't + 2\omega) \\
\mp e'^2 (186.33625) \cos (3nt - 4n't + 2\omega') \\
+ e'^2 (180.40140) \cos (nt - 4n't + 2\omega') \\
\pm e'^2 (11.67233) \cos (3nt - 2\omega') - e'^2 (11.13720) \cos (nt - 2\omega') \\
\mp e'^2 (35.24353) \cos (5nt - 6n't + 2\omega') \\
+ e'^2 (79.03795) \cos (3nt - 6n't + 2\omega') \\
\mp e'^2 (0.2067144) \cos (5nt - 2n't - 2\omega') \\
+ e'^2 (0.5484903) \cos (3nt - 2n't - 2\omega')
\end{array}\right\} . \quad (341)
$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
\pm ee'(139.19318)\cos(4nt - n't - \omega - \omega') \\
- ee'(79.37871)\cos(2nt - n't - \omega - \omega') \\
\pm ee'(86.58883)\cos(2nt - 3n't + \omega + \omega') \\
\mp ee'(45.33573)\cos(3n't - \omega - \omega') \\
\mp ee'(143.39768)\cos(4nt - 3n't - \omega + \omega') \\
+ ee'(85.53030)\cos(2nt - 3n't - \omega + \omega') \\
\mp ee'(44.45336)\cos(2nt - n't + \omega - \omega') \\
\mp ee'(83.23628)\cos(n't - \omega + \omega') \\
\pm ee'(3.083398)\cos(6nt - 3n't - \omega - \omega') \\
- ee'(3.497110)\cos(4nt - 3n't - \omega - \omega') \\
\mp ee'(26.81067)\cos(6nt - 5n't - \omega + \omega') \\
+ ee'(26.17494)\cos(4nt - 5n't - \omega + \omega') \\
\mp ee'(321.8183)\cos(4nt - 5n't + \omega + \omega') \\
+ ee'(255.3926)\cos(2nt - 5n't + \omega + \omega') \\
\pm ee'(79.14869)\cos(4nt - 3n't + \omega - \omega') \\
- ee'(70.23787)\cos(2nt - 3n't + \omega - \omega') \\
\mp \gamma^2(0.345480)\cos(5nt - 2n't - 2\Omega) \\
+ \gamma^2(0.345199)\cos(3nt - 2n't - 2\Omega) \\
\mp \gamma^2(12.442566)\cos(nt + 2n't - 2\Omega) \pm \gamma^2(3.748356)\cos(nt - 2n't + 2\Omega) \\
\pm \gamma^2(0.2674530)\cos(3nt - 2n't + 2\omega - 2\Omega) \\
- \gamma^2(0.2127255)\cos(nt - 2n't + 2\omega - 2\Omega) \\
\mp \gamma^2(0.2674530)\cos(3nt - 2n't - 2\omega + 2\Omega) \\
+ \gamma^2(0.2127255)\cos(nt - 2n't - 2\omega + 2\Omega) \\
\pm \gamma^2(0.724543)\cos(7nt - 4n't - 2\Omega) \\
- \gamma^2(0.410439)\cos(5nt - 4n't - 2\Omega) \\
\mp \gamma^2(38.46038)\cos(3nt - 4n't + 2\Omega) \\
+ \gamma^2(36.01552)\cos(nt - 4n't + 2\Omega) \\
\mp \gamma^2(0.616375)\cos(5nt - 4n't + 2\omega - 2\Omega) \\
+ \gamma^2(0.905792)\cos(3nt - 4n't + 2\omega - 2\Omega) \\
\pm \gamma^2(0.616375)\cos(5nt - 4n't - 2\omega + 2\Omega) \\
- \gamma^2(0.905792)\cos(3nt - 4n't - 2\omega + 2\Omega) \Big\} \\
\\
+ \dfrac{\overline{m}^4}{\mu}\dfrac{a}{a'}ndt \Big\{ \mp (35.39736)\cos(2nt - n't) \pm (53.04120)\cos n't \\
\qquad\qquad \pm (32.56128)\cos(4nt - 3n't) - (23.85887)\cos(2nt - 3n't) \\
\qquad\qquad \mp (1.721789)\cos(6nt - 5n't) + (5.165146)\cos(4nt - 5n't) \Big\}
\end{array}
\right\} . \qquad (341)
$$

Taking the integral of equation (341), we obtain

$$
\begin{aligned}
\int\Bigg\{ c_2 \cos\beta\, \delta\left(\frac{dR}{dr}\right) + \delta\,(c_2\cos\beta)\left(\frac{dR}{dr}\right) + c_3 \sin\beta\, \delta\left(\frac{dR}{dv}\right) + \delta\,(c_3\sin\beta)\left(\frac{dR}{dv}\right) \\
+ c_4 \sin\beta\, \delta\left(\frac{dR}{d\theta}\right) + \delta\,(c_4\sin\beta)\left(\frac{dR}{d\theta}\right)\Bigg\} =
\end{aligned}
$$

$$
\left.
\begin{aligned}
&\frac{\overline{m}'}{\mu}\Big\{ -\{2.3948001 + 78.9822e^2 + 19.39006e'^2 - 3.73890\gamma^2\} \sin nt \\
&+ e\,(41.24481)\,(2nt - \omega) - e'\,(14.55362) \sin (nt + n't - \omega') \\
&+ e'\,(1.709482) \sin (nt - n't + \omega') + e^2\,(71.48439) \sin (3nt - 2\omega) \\
&\mp e^2\,(149.68273) \sin (nt - 2\omega) - e'^2\,(27.72361) \sin (nt + 2n't - 2\omega') \\
&- e'^2\,(1.960730) \sin (nt - 2n't + 2\omega') \\
&+ ee'\,(88.08138) \sin (2nt + n't - \omega - \omega') \\
&+ ee'\,(122.6872) \sin (2nt - n't - \omega + \omega') \\
&- ee'\,(4673.226) \sin (n't + \omega - \omega') \mp ee'\,(3696.854) \sin (n't - \omega - \omega') \\
&+ \gamma^2\,(12.98418) \sin (3nt - 2\Omega) \mp \gamma^2\,(39.26018) \sin (nt - 2\Omega) \\
&- \gamma^2\,(0.299349) \sin (nt + 2\omega - 2\Omega) + \gamma^2\,(0.299349) \sin (nt - 2\omega + 2\Omega) \\
&+ \{0.2502135 + 3.361423e^2 + 63.38372e'^2 - 2.143170\gamma^2\} \sin (3nt - n't) \\
&\mp \{2.001093 - 15.69621e^2 + 209.5000e'^2 - 16.58572\gamma^2\} \sin (nt - 2n't) \\
&- \{0.2097937 + 34.29500e^2 - 1.799267e'^2 - 0.4953902\gamma^2\} \sin (5nt - 4n't) \\
&\pm \{0.8943470 + 26.67618e^2 - 7.632412e'^2 - 0.3177040\gamma^2\} \sin (3nt - 4n't) \\
&+ e\,(0.3639456) \sin (4nt - 2n't - \omega) \\
&\mp e\,(0.2734186) \sin (2nt - 2n't - \omega) \\
&+ e\,(2.930145) \sin (2nt - 2n't + \omega) - e\,(142.9911) \sin (2n't - \omega) \\
&- e\,(0.6126074) \sin (6nt - 4n't - \omega) \\
&\pm e\,(0.9535630) \sin (4nt - 4n't - \omega) \\
&- e\,(13.82745) \sin (4nt - 4n't - \omega) \\
&\pm e\,(24.69512) \sin (2nt - 4n't + \omega) \\
&+ e'\,(15.36089) \sin (3nt - n't - \omega') \\
&\mp e'\,(47.31962) \sin (nt - n't - \omega') \\
&- e'\,(15.40485) \sin (3nt - 3n't + \omega') \\
&\pm e'\,(51.41187) \sin (nt - 3n't + \omega') \\
&+ e'\,(0.1898254) \sin (5nt - 3n't - \omega') \\
&\mp e'\,(0.8303476) \sin (3nt - 3n't - \omega') \\
&- e'\,(1.644542) \sin (5nt - 5n't + \omega') \\
&\pm e'\,(6.795146) \sin (3nt - 5n't + \omega') \\
&+ e^2\,(0.5106770) \sin (5nt - 2n't - 2\omega)
\end{aligned}
\right\} . \quad (342)
$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&\mp e^2(0.2234650)\sin(3nt-2n't-2\omega)\\
&-e^2(33.25335)\sin(nt+2n't-2\omega)\\
&+e^2(2.138530)\sin(nt-2n't+2\omega)\\
&-e^2(1.226890)\sin(7nt-4n't-2\omega)\\
&\pm e^2(1.024685)\sin(5nt-4n't-2\omega)\\
&+e^2(38.07688)\sin(3nt-4n't+2\omega)\\
&\mp e^2(30.61024)\sin(nt-4n't+2\omega)\\
&-e'^2(68.99310)\sin(3nt-4n't+2\omega')\\
&\pm e'^2(257.4240)\sin(nt-4n't+2\omega')\\
&+e'^2(3.890777)\sin(3nt-2\omega')\mp e'^2(11.13720)\sin(nt-2\omega')\\
&-e'^2(7.743804)\sin(5nt-6n't+2\omega')\\
&\pm e'^2(30.98079)\sin(3nt-6n't+2\omega')\\
&-e'^2(0.04261803)\sin(5nt-2n't-2\omega')\\
&\pm e'^2(0.1924259)\sin(3nt-2n't-2\omega')\\
&+ee'(35.46144)\sin(4nt-n't-\omega-\omega')\\
&\mp ee'(41.23091)\sin(2nt-n't-\omega-\omega')\\
&+ee'(48.76607)\sin(2nt-3n't+\omega+\omega')\\
&-ee'(202.0273)\sin(3n't-\omega-\omega')\\
&-ee'(37.98015)\sin(4nt-3n't-\omega+\omega')\\
&\pm ee'(48.16992)\sin(2nt-3n't-\omega+\omega')\\
&-ee'(23.09027)\sin(2nt-n't+\omega-\omega')\\
&-ee'(1112.766)\sin(n't-\omega+\omega')\\
&+ee'(0.5463020)\sin(6nt-3n't-\omega-\omega')\\
&\mp ee'(0.9262406)\sin(4nt-3n't-\omega-\omega')\\
&-ee'(4.765500)\sin(6nt-5n't-\omega+\omega')\\
&\pm ee'(7.218695)\sin(4nt-5n't-\omega+\omega')\\
&-ee'(88.75315)\sin(4nt-5n't+\omega+\omega')\\
&\pm ee'(157.0686)\sin(2nt-5n't+\omega+\omega')\\
&+ee'(20.96323)\sin(4nt-3n't+\omega-\omega')\\
&\mp ee'(39.55736)\sin(2nt-3n't+\omega-\omega')\\
&-\gamma^2(0.07122713)\sin(5nt-2n't-2\Omega)\\
&\pm\gamma^2(0.1211056)\sin(3nt-2n't-2\Omega)\\
&-\gamma^2(10.82336)\sin(nt+2n't-2\Omega)\\
&+\gamma^2(4.407769)\sin(nt-2n't+2\Omega)\\
&+\gamma^2(0.0938301)\sin(3nt-2n't+2\omega-2\Omega)\\
&\mp\gamma^2(0.2501483)\sin(nt-2n't+2\omega-2\Omega)
\end{aligned}
\right\} \quad (342)
$$

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$$\left.\begin{array}{l}
-\gamma^2(0.0938301)\sin(3nt-2n't-2\omega+2\Omega) \\
\pm\gamma^2(0.2501483)\sin(nt-2n't-2\omega+2\Omega) \\
+\gamma^2(0.1081279)\sin(7nt-4n't-2\Omega) \\
\mp\gamma^2(0.0873127)\sin(5nt-4n't-2\Omega) \\
-\gamma^2(14.24040)\sin(3nt-4n't+2\Omega) \\
\pm\gamma^2(51.39238)\sin(nt-4n't+2\Omega) \\
-\gamma^2(0.1311215)\sin(5nt-4n't+2\omega-2\Omega) \\
\pm\gamma^2(0.3353798)\sin(3nt-4n't+2\omega-2\Omega) \\
+\gamma^2(0.1311215)\sin(5nt-4n't-2\omega+2\Omega) \\
\mp\gamma^2(0.3353798)\sin(3nt-4n't-2\omega+2\Omega)\Big\} \\
+\dfrac{\overline{m}^4}{\mu}\dfrac{a}{a'}\Big\{-(18.42830)\sin(2nt-n't)+(709.0946)\sin n't \\
\qquad +(6.240414)\sin(4nt-3n't)\mp(13.43711)\sin(2nt-3n't) \\
\qquad -(0.3060419)\sin(6nt-5n't)\pm(1.424478)\sin(4nt-5n't)\Big\}
\end{array}\right\} . \quad (342)$$

Equation (386) will give the value of $\int\left\{c_2\sin\beta\,\delta\left(\dfrac{dR}{dr}\right)+\delta\,(c_2\sin\beta)\left(\dfrac{dR}{dr}\right)\right.$ $\left.+\,c_3\cos\beta\,\delta\left(\dfrac{dR}{dv}\right)+\delta\,(c_3\cos\beta)\left(\dfrac{dR}{dv}\right)+c_4\cos\beta\,\delta\left(\dfrac{dR}{d\theta}\right)+\delta\,(c_4\cos\beta)\left(\dfrac{dR}{d\theta}\right)\right\}$ by using the lower signs and changing *sin* to *cos* in the second member. We shall designate the equation so changed as equation (343).

If we now multiply equation (342) by $\dfrac{1}{\sqrt{a\mu}}\,c_1\cos\beta$, and equation (343) by $-\dfrac{1}{\sqrt{a\mu}}\,c_1\sin\beta$, and take the sum of the products, we shall obtain

$$\left.\begin{array}{l}
\overset{\text{1st term of}}{\delta\dfrac{d\delta_1 r}{dt}}=a\dfrac{\overline{m}^4}{\mu^2}n\Big\{+e(46.03441)\sin(nt-\omega)-e'(16.26310)\sin(n't-\omega') \\
-e^2(116.44965)\sin 2(nt-\omega)-e'^2(25.76288)\sin 2(n't-\omega') \\
-\gamma^2(26.87470)\sin 2(nt-\Omega)-ee'(3595.928)\sin(nt+n't-\omega-\omega') \\
+ee'(4779.650)\sin(nt-n't-\omega+\omega') \\
-\{1.750880-160.63314e^2+146.1163e'^2-14.00482\gamma^2\}\sin 2(nt-n't) \\
+\{0.6845533+28.99531e^2-5.833145e'^2+0.34882\gamma^2\}\sin 4(nt-n't)
\end{array}\right\} . \quad (344)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- e\,(2.160778)\sin(3nt - 2n't - \omega) + e\,(148.1725)\sin(nt - 2n't + \omega) \\
&+ e\,(1.4450977)\sin(5nt - 4n't - \omega) \\
&+ e\,(9.76353)\sin(3nt - 4n't + \omega) \\
&- e'\,(31.95873)\sin(2nt - n't - \omega') + e'\,(36.00702)\sin(2nt - 3n't + \omega') \\
&- e'\,(0.6405222)\sin(4nt - 3n't - \omega') \\
&+ e'\,(7.305823)\sin(4nt - 5n't + \omega') \\
&- e^2\,(2.632657)\sin(4nt - 2n't - 2\omega) + e^2\,(108.4145)\sin 2(n't - \omega) \\
&+ e^2\,(2.396330)\sin(6nt - 4n't - 2\omega) \\
&- e^2\,(31.40374)\sin(2nt - 4n't + 2\omega) \\
&+ e'^2\,(188.4309)\sin(2nt - 4n't + 2\omega') - e'^2\,(7.24642)\sin 2(nt - \omega') \\
&+ e'^2\,(23.23699)\sin(4nt - 6n't + 2\omega') \\
&+ e'^2\,(0.1498079)\sin(4nt - 2n't - 2\omega') \\
&- ee'\,(67.44998)\sin(3nt - n't - \omega - \omega') \\
&+ ee'\,(183.9767)\sin(nt - 3n't + \omega + \omega') \\
&+ ee'\,(77.00649)\sin(3nt - 3n't - \omega + \omega') \\
&+ ee'\,(1152.357)\sin(nt - n't + \omega - \omega') \\
&- ee'\,(1.4001116)\sin(5nt - 3n't - \omega - \omega') \\
&+ ee'\,(10.892883)\sin(5nt - 5n't - \omega + \omega') \\
&+ ee'\,(59.8759)\sin(3nt - 5n't + \omega + \omega') \\
&- ee'\,(17.57395)\sin(3nt - 3n't + \omega - \omega') \\
&+ \gamma^2\,(0.1124319)\sin(4nt - 2n't - 2\Omega) \\
&- \gamma^2\,(14.73086)\sin 2(n't - \Omega) \\
&- \gamma^2\,(0.437731)\sin(2nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2\,(0.437731)\sin(2nt - 2n't - 2\omega + 2\Omega) \\
&- \gamma^2\,(0.0316335)\sin(6nt - 4n't - 2\Omega) \\
&+ \gamma^2\,(37.37557)\sin(2nt - 4n't + 2\Omega) \\
&- \gamma^2\,(0.2042583)\sin(4nt - 4n't + 2\omega - 2\Omega) \\
&- \gamma^2\,(0.2042583)\sin(4nt - 4n't - 2\omega + 2\Omega) \Big\} \\
&+ a\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}n\Big\{ -(727.5229)\sin(nt - n't) - (7.19670)\sin 3(nt - n't) \\
&\qquad\qquad + (1.118436)\sin 5(nt - n't) \Big\}
\end{aligned}\right\} . \quad (344)$$

If we now multiply equations (272) and (273) by $\frac{1}{\sqrt{a\mu}}\,\delta\,(c_1 \cos\beta)$ and $-\frac{1}{\sqrt{a\mu}}\,\delta\,(c_1 \sin\beta)$ respectively, which last quantities are given by equation (330), and call the sum of the products the 2nd term of $\delta\frac{d\delta_1 r}{dt}$, we shall have

$$
\left.
\begin{aligned}
&\text{2nd term of } \delta\frac{d\delta_1 r}{dt} = a\frac{\overline{m}^4}{\mu^2}n\Big\{-e(77.60574)\sin(nt-\omega)+e'(19.48574)\sin(n't-\omega') \\
&-e^2(177.2048)\sin 2(nt-\omega)+e'^2(42.07338)\sin 2(n't-\omega') \\
&-\gamma^2(52.56284)\sin 2(nt-\Omega)-ee'(156.3663)\sin(nt+n't-\omega-\omega') \\
&-ee'(266.0054)\sin(nt-n't-\omega+\omega') \\
&-\{0.9149326+49.84074e^2+255.0854e'^2+2.195514\gamma^2\}\sin 2(nt-n't) \\
&+\{2.661486+284.5021e^2-22.73007e'^2-2.428388\gamma^2\}\sin 4(nt-4n't) \\
&-e(2.589937)\sin(3nt-2n't-\omega)-e(33.80632)\sin(nt-2n't+\omega) \\
&+e(7.306180)\sin(5nt-4n't-\omega)+e(117.91380)\sin(3nt-4n't+\omega) \\
&-e'(59.64737)\sin(2nt-n't-\omega')+e'(53.52839)\sin(2nt-3n't+\omega') \\
&-e'(2.377170)\sin(4nt-3n't-\omega') \\
&+e'(21.072769)\sin(4nt-5n't+\omega') \\
&-e^2(5.323014)\sin(4nt-2n't-2\omega)+e^2(49.76307)\sin 2(n't-\omega) \\
&+e^2(14.45090)\sin(6nt-4n't-2\omega) \\
&+e^2(775.3358)\sin(2nt-4n't+2\omega) \\
&+e'^2(249.8591)\sin(2nt-4n't+2\omega')-e'^2(19.02629)\sin 2(nt-\omega') \\
&+e'^2(100.26648)\sin(4nt-6n't+2\omega') \\
&+e'^2(0.5301682)\sin(4nt-2n't-2\omega') \\
&-ee'(185.3336)\sin(3nt-n't-\omega-\omega') \\
&-ee'(549.8135)\sin(nt-n't+\omega-\omega') \\
&+ee'(181.6653)\sin(3nt-3n't-\omega+\omega') \\
&+ee'(376.0509)\sin(nt-3n't+\omega+\omega') \\
&-ee'(6.556939)\sin(5nt-3n't-\omega-\omega') \\
&+ee'(769.1103)\sin(3nt-5n't+\omega+\omega') \\
&+ee'(57.55293)\sin(5nt-5n't-\omega+\omega') \\
&-ee'(155.53598)\sin(3nt-3n't+\omega-\omega') \\
&+\gamma^2(1.6596596)\sin(4nt-2n't-2\Omega) \\
&+\gamma^2(16.020524)\sin 2(n't-\Omega) \\
&-\gamma^2(0.2287331)\sin(2nt-2n't+2\omega-2\Omega)
\end{aligned}
\right\} . \quad (345)
$$

(Continued on the next page.)

$$\left.\begin{array}{l}
+\gamma^2(0.2287331)\sin(2nt-2n't-2\omega+2\Omega) \\
+\gamma^2(44.66562)\sin(2nt-4n't+2\Omega) \\
-\gamma^2(0.1366386)\sin(6nt-4n't-2\Omega) \\
+\gamma^2(1.3307423)\sin(4nt-4n't+2\omega-2\Omega) \\
-\gamma^2(1.3307423)\sin(4nt-4n't-2\omega+2\Omega\Big\} \\
+a\dfrac{\overline{m}^4}{\mu^2}\dfrac{a}{a'}n\Big\{+(43.55802)\sin(nt-n't)-(53.47606)\sin3(nt-n't) \\
\qquad\qquad +(1.0849904)\sin5(nt-n't)\Big\}
\end{array}\right\} \quad . \quad (345)$$

The complete analytical value of $\left(\frac{d\delta_1 r}{dt}\right)$ is given by equation (252). In the development of that equation we have neglected the disturbing function in the denominator, since it produces terms depending on the square of the disturbing force. And if we now put

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\frac{d\delta_1 r}{dt}\int\left(\frac{dR}{dv}\right)dt=\delta\frac{d\delta_4 r}{dt}, \qquad (345')$$

we shall have by means of equations (265) and (274),

$$\left.\begin{array}{l}
\delta\dfrac{d\delta_4 r}{dt}=a\dfrac{\overline{m}^4}{\mu^2}n\Big\{+e(15.05511)\sin(nt-\omega)+e'(0.186312)\sin(n't-\omega') \\
+e^2(32.60251)\sin2(nt-\omega)+e'^2(0.98814)\sin2(n't-\omega') \\
+\gamma^2(4.634403)\sin2(nt-\Omega)+ee'(35.50842)\sin(nt+n't-\omega-\omega') \\
+ee'(43.65001)\sin(nt-n't-\omega+\omega') \\
-\{2.545325e^2-0.1887884e'^2\}\sin2(nt-n't) \\
-\{0.9657522+3.315648e^2-8.23041e'^2-0.1160284\gamma^2\}\sin4(nt-n't) \\
+e(0.3546536)\sin(3nt-2n't-\omega)-e(0.3546536)\sin(nt-2n't+\omega) \\
-e(1.976652)\sin(5nt-4n't-\omega) \\
-e(3.169484)\sin(3nt-4n't+\omega) \\
+e'(0.04573342)\sin(2nt-n't-\omega') \\
-e'(0.04573342)\sin(2nt-3n't+\omega') \\
+e'(0.9050032)\sin(4nt-3n't-\omega') \\
-e'(7.254597)\sin(4nt-5n't+\omega') \\
+e^2(0.4835551)\sin(4nt-2n't-2\omega)-e^2(2.061770)\sin2(n't-\omega) \\
-e^2(3.024008)\sin(6nt-4n't-2\omega) \\
+e^2(102.70623)\sin(2nt-4n't+2\omega)
\end{array}\right\} \quad . \quad (346)$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
- e'^2 (32.64806) \sin (4nt - 6n't + 2\omega') \\
- e'^2 (0.2118793) \sin (4nt - 2n't - 2\omega') \\
+ e'^2 (0.1171725) \sin 2(nt - \omega') \\
- e'^2 (0.3059609) \sin (2nt - 4n't + 2\omega') \\
+ ee' (6.49332) \sin (3nt - n't - \omega - \omega') \\
- ee' (7.629112) \sin (nt - 3n't + \omega + \omega') \\
- ee' (4.299549) \sin (3nt - 3n't - \omega + \omega') \\
+ ee' (5.435341) \sin (nt - n't + \omega - \omega') \\
+ ee' (1.8655655) \sin (5nt - 3n't - \omega - \omega') \\
- ee' (6.95700) \sin (3nt - 5n't + \omega + \omega') \\
- ee' (14.732823) \sin (5nt - 5n't - \omega + \omega') \\
+ ee' (8.599446) \sin (3nt - 3n't + \omega - \omega') \\
- \gamma^2 (0.4053184) \sin (4nt - 2n't - 2\Omega) \\
- \gamma^2 (0.4053184) \sin 2(n't - \Omega) \\
+ \gamma^2 (0.0690081) \sin (6nt - 4n't - 2\Omega) \\
+ \gamma^2 (4.277438) \sin (2nt - 4n't + 2\Omega) \\
- \gamma^2 (0.482876) \sin (4nt - 4n't + 2\omega - 2\Omega) \\
+ \gamma^2 (0.482876) \sin (4nt - 4n't - 2\omega + 2\Omega) \Big\} \\
\\
+ a \dfrac{\overline{m}^4}{\mu^2} \dfrac{a}{a'} n \Big\{ - (5.259921) \sin (nt - n't) + (4.557590) \sin 3(nt - n't) \\
\qquad\qquad\qquad\qquad - (1.3461649) \sin 5(nt - n't) \Big\}
\end{array}\right\} . \quad (346)
$$

If we now take the sum of equations (344), (345), and (346), we shall obtain the complete value of $\delta\left(\dfrac{d\delta_1 r}{dt}\right)$, as follows:

$$
\left.\begin{array}{l}
\delta\left(\dfrac{d\delta_1 r}{dt}\right) = a \dfrac{\overline{m}^4}{\mu^2} n \Big\{ - e(16.51622) \sin (nt - \omega) + e' (3.40895) \sin (n't - \omega') \\
- e^2 (261.0520) \sin 2(nt - \omega) + e'^2 (17.29864) \sin 2(n't - \omega') \\
- \gamma^2 (74.80314) \sin 2(nt - \Omega) - ee' (3716.786) \sin (nt + n't - \omega - \omega') \\
+ ee' (4557.295) \sin (nt - n't - \omega + \omega') \\
- \{2.665812 - 108.2454 e^2 + 401.0129 e'^2 - 11.80931 \gamma^2\} \sin 2(nt - n't) \\
+ \{2.380288 + 310.1818 e^2 - 20.33280 e'^2 - 1.963547 \gamma^2\} \sin 4(nt - n't) \\
- e (4.396061) \sin (3nt - 2n't - \omega) + e (114.0115) \sin (nt - 2n't + \omega)
\end{array}\right\} . \quad (347)
$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
+ e\,(6.774626) \sin (5nt - 4n't - \omega) \\
+ e\,(124.50785) \sin (3nt - 4n't + \omega) \\
- e'\,(91.56037) \sin (2nt - n't - \omega') \\
+ e'\,(89.48968) \sin (2nt - 3n't + \omega') \\
- e'\,(2.112689) \sin (4nt - 3n't - \omega') \\
+ e'\,(21.123995) \sin (4nt - 5n't - \omega') \\
- e^2\,(7.472116) \sin (4nt - 2n't - 2\omega) + e^2\,(156.1158) \sin 2\,(n't - \omega) \\
+ e^2\,(13.82322) \sin (6nt - 4n't - 2\omega) \\
+ e^2\,(846.6383) \sin (2nt - 4n't + 2\omega) \\
+ e'^2\,(437.9840) \sin (2nt - 4n't + 2\omega') - e'^2\,(26.15554) \sin 2\,(nt - \omega') \\
+ e'^2\,(90.85541) \sin (4nt - 6n't + 2\omega') \\
+ e'^2\,(0.4680968) \sin (4nt - 2n't - 2\omega') \\
- ee'\,(246.2903) \sin (3nt - n't - \omega - \omega') \\
+ ee'\,(552.3985) \sin (nt - 3n't + \omega + \omega') \\
+ ee'\,(254.3723) \sin (3nt - 3n't - \omega + \omega') \\
+ ee'\,(607.979) \sin (nt - n't + \omega - \omega') \\
- ee'\,(6.091485) \sin (5nt - 3n't - \omega - \omega') \\
+ ee'\,(53.71299) \sin (5nt - 5n't - \omega + \omega') \\
+ ee'\,(822.0292) \sin (3nt - 5n't + \omega + \omega') \\
- ee'\,(164.51048) \sin (3nt - 3n't + \omega - \omega') \\
+ \gamma^2\,(1.3667731) \sin (4nt - 2n't - 2\Omega) + \gamma^2\,(0.88434) \sin 2\,(n't - \Omega) \\
- \gamma^2\,(0.666464) \sin (2nt - 2n't + 2\omega - 2\Omega) \\
+ \gamma^2\,(0.666464) \sin (2nt - 2n't - 2\omega + 2\Omega) \\
- \gamma^2\,(0.0992640) \sin (6nt - 4n't - 2\Omega) \\
+ \gamma^2\,(86.31863) \sin (2nt - 4n't + 2\omega) \\
+ \gamma^2\,(1.0521246) \sin (4nt - 4n't + 2\omega - 2\Omega) \\
- \gamma^2\,(1.0521246) \sin (4nt - 4n't - 2\omega + 2\Omega) \Big\} \\
\\
+ a\,\dfrac{\overline{m}^4}{\mu^2}\,\dfrac{a}{a'}\,n \Big\{ - (689.2248) \sin (nt - n't) - (56.11517) \sin 3\,(nt - n't) \\
\qquad\qquad\qquad\qquad\qquad\qquad + (0.857261) \sin 5\,(nt - n't) \Big\}
\end{array}
\right\} . \quad (347)
$$

30. We shall now determine the value of $\delta \dfrac{d\delta_0\theta}{dt}$. For this purpose we must first find the values of $\delta \sin v$, $\delta (\tan \theta \sin v)$, and $\delta \dfrac{\sin v}{r^2}$. Now we have

$$\left.\begin{aligned} \delta \sin v = \pm \cos v\, \delta v, \qquad \delta (\tan \theta \sin v) &= \pm \tan \theta \cos v\, \delta v + \sin v\, \delta \theta\,; \\ \delta \frac{\sin v}{r^2} &= \pm \frac{\cos v}{r^2}\, \delta v - 2 \frac{\sin v}{r^3}\, \delta r. \end{aligned}\right\} \quad (348)$$

These equations will give the values of $\delta \cos v$, $\delta (\tan \theta \cos v)$, and $\delta \dfrac{\cos v}{r^2}$, by using the lower signs and interchanging *cos* and *sin* in the second member. By substituting the values of δr, δv, and $\delta \theta$, together with the elliptical values of r, v, and θ, we shall obtain the following values:

$$\left.\begin{aligned} \mp \delta \sin v = \frac{\overline{m}^2}{\mu} \Big\{ & \pm \tfrac{3}{8} e \sin (2nt - \omega) \mp \tfrac{3}{8} e \sin \omega \\ & \pm e' (20.16597) \sin (nt + n't - \omega') \mp e' (20.16597) \sin (nt - n't + \omega') \\ & \mp (0.914932) \sin (3nt - 2n't) - (0.914932) \sin (nt - 2n't) \\ & \mp e (2.299648) \sin (4nt - 2n't - \omega) - e (0.469784) \sin (2nt - 2n't - \omega) \\ & \mp e (32.86787) \sin (2nt - 2n't + \omega) \pm e (34.69773) \sin (2n't - \omega) \\ & \pm e' (0.3944260) \sin (3nt - n't - \omega') \\ & + e' (0.3944260) \sin (nt - n't - \omega') \\ & \mp e' (3.753160) \sin (3nt - 3n't + \omega') \\ & - e' (3.753160) \sin (nt - 3n't + \omega') \Big\} \end{aligned}\right\} . \quad (349)$$

$$\left.\begin{aligned} \delta (\tan \theta \cos v) = & \\ \frac{\overline{m}^2}{\mu} \Big\{ & \pm \gamma (0.3541667) \sin (2nt - \Omega) \\ & \mp \gamma (0.3541667) \sin \Omega \mp e\gamma (0.0625) \sin (3nt - \omega - \Omega) \\ & - e\gamma (0.3958333) \sin (nt - \omega - \Omega) \mp \tfrac{1}{8} e\gamma \sin (nt + \omega - \Omega) \\ & \mp \tfrac{1}{8} e\gamma \sin (nt - \omega + \Omega) \mp e'\gamma (12.917155) \sin (2nt + n't - \omega' - \Omega) \\ & \pm e'\gamma (12.353865) \sin (2nt - n't + \omega' - \Omega) \\ & \pm e'\gamma (7.812105) \sin (nt - \omega' + \Omega) + e'\gamma (7.248815) \sin (n't - \omega' - \Omega) \\ & \pm \gamma (0.914932) \sin (4nt - 2n't - \Omega) \mp \gamma (3.624231) \sin (2n't - \Omega) \\ & \pm \gamma (2.709299) \sin (2nt - 2n't + \Omega) \\ & \pm e\gamma (5.585800) \sin (3nt - 2n't - \omega + \Omega) \\ & \mp e\gamma (6.66241) \sin (nt - 2n't + \omega + \Omega) \\ & \pm e\gamma (3.214580) \sin (5nt - 2n't - \omega - \Omega) \end{aligned}\right\} . \quad (350)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- e\gamma\,(0.277946)\sin(nt - 2n't - \omega + \Omega)\\
&\pm e\gamma\,(31.95293)\sin(3nt - 2n't + \omega - \Omega)\\
&- e\gamma\,(34.36884)\sin(nt + 2n't - \omega - \Omega)\\
&\mp e'\gamma\,(2.6042755)\sin(2nt - n't - \omega' + \Omega)\\
&\pm e'\gamma\,(6.58801)\sin(2nt - 3n't + \omega' + \Omega)\\
&\mp e'\gamma\,(0.394426)\sin(4nt - n't - \omega' - \Omega)\\
&\pm e'\gamma\,(2.998701)\sin(n't + \omega' - \Omega)\\
&\pm e'\gamma\,(3.753160)\sin(4nt - 3n't + \omega' - \Omega)\\
&\mp e'\gamma\,(10.34117)\sin(3n't - \omega' - \Omega)\Big\}
\end{aligned}\right\} \quad (350)$$

$$\left.\begin{aligned}
&\delta\left(\frac{\sin v}{r^2}\right) = \frac{\overline{m}^2}{a^2\mu}\Big\{ + \tfrac{1}{8}\sin nt - \tfrac{5}{12}e\sin(2nt - \omega) - \tfrac{1}{8}e\sin\omega\\
&\quad - e'\,(21.67441)\sin(nt + n't - \omega') + e'\,(18.65753)\sin(nt - n't + \omega')\\
&\quad + (2.202601)\sin(3nt - 2n't) \mp (0.372737)\sin(nt - 2n't)\\
&\quad + e\,(7.780800)\sin(4nt - 2n't - \omega) - e\,(3.35568)\sin(2n't - \omega)\\
&\quad \mp e\,(0.606166)\sin(2nt - 2n't - \omega) + e\,(63.46444)\sin(2nt - 2n't + \omega)\\
&\quad - e'\,(0.9594373)\sin(3nt - n't - \omega')\\
&\quad \mp e'\,(0.1705853)\sin(nt - n't - \omega')\\
&\quad + e'\,(8.938880)\sin(3nt - 3n't + \omega')\\
&\quad \mp e'\,(1.432560)\sin(nt - 3n't + \omega')\Big\}
\end{aligned}\right\} \quad (351)$$

Now by means of equations (241) and (328) we obtain

$$\tan\theta\cos v\,\delta\left(\frac{dR}{dv}\right) =$$

$$\left.\begin{aligned}
&\frac{\overline{m}^4}{a\mu}\Big\{ + \tfrac{1}{8}\gamma\cos(4nt - 2n't - \Omega) - \tfrac{1}{8}\gamma\cos(2n't - \Omega)\\
&\quad - \tfrac{1}{8}\gamma\cos(2nt - 2n't + \Omega) \pm \gamma\cos(2nt - 2n't - \Omega)\\
&\quad + \gamma\,(0.2033232)\cos(4nt - 4n't + \Omega)\\
&\quad \mp \gamma\,(0.2033232)\cos(4nt - 4n't - \Omega)\\
&\quad - \gamma\,(0.2033232)\cos(6nt - 4n't - \Omega)\\
&\quad \pm \gamma\,(0.2033232)\cos(2nt - 4n't + \Omega)\\
&\quad - e\gamma\,(39.35890)\cos(nt - \omega + \Omega) \pm e\gamma\,(39.35890)\cos(nt - \omega - \Omega)\\
&\quad + e\gamma\,(39.35890)\cos(3nt - \omega - \Omega) - e\gamma\,(39.35890)\cos(nt + \omega - \Omega)\\
&\quad + e\gamma\,(0.390625)\cos(5nt - 2n't - \omega - \Omega)
\end{aligned}\right\} \quad (352)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- e\gamma\,(1.171875)\cos(3nt - 2n't + \omega - \Omega)\\
&+ e\gamma\,(0.671875)\cos(nt + 2n't - \omega - \Omega)\\
&\pm e\gamma\,(0.109375)\cos(nt - 2n't - \omega + \Omega)\\
&- e\gamma\,(0.9019262)\cos(7nt - 4n't - \omega - \Omega)\\
&- e\gamma\,(13.189869)\cos(5nt - 4n't + \omega - \Omega)\\
&\pm e\gamma\,(0.0886332)\cos(3nt - 4n't - \omega + \Omega)\\
&\pm e\gamma\,(14.003161)\cos(nt - 4n't + \omega + \Omega)\\
&- e\gamma\,(0.140625)\cos(3nt - 2n't - \omega + \Omega)\\
&\pm e\gamma\,(0.140625)\cos(3nt - 2n't - \omega - \Omega)\\
&+ e\gamma\,(0.921875)\cos(nt - 2n't + \omega + \Omega)\\
&\mp e\gamma\,(0.921875)\cos(nt - 2n't + \omega - \Omega)\\
&+ e\gamma\,(0.4952797)\cos(5nt - 4n't - \omega + \Omega)\\
&\mp e\gamma\,(0.4952797)\cos(5nt - 4n't - \omega - \Omega)\\
&+ e\gamma\,(13.59651)\cos(3nt - 4n't + \omega + \Omega)\\
&\mp e\gamma\,(13.59651)\cos(3nt - 4n't + \omega - \Omega)\\
&- e'\gamma\,(0.5909125)\cos(n't - \omega' + \Omega)\\
&\pm e'\gamma\,(0.5909125)\cos(n't - \omega' - \Omega)\\
&+ e'\gamma\,(0.5909125)\cos(2nt + n't - \omega' - \Omega)\\
&- e'\gamma\,(0.5909125)\cos(2nt - n't + \omega' - \Omega)\\
&- e'\gamma\,(14.49631)\cos(2nt - n't - \omega' + \Omega)\\
&\pm e'\gamma\,(14.49631)\cos(2nt - n't - \omega' - \Omega)\\
&+ e'\gamma\,(15.25264)\cos(2nt - 3n't + \omega' + \Omega)\\
&\mp e'\gamma\,(15.25264)\cos(2nt - 3n't + \omega' - \Omega)\\
&- e'\gamma\,(0.1856018)\cos(4nt - 3n't - \omega' + \Omega)\\
&\pm e'\gamma\,(0.1856018)\cos(4nt - 3n't - \omega' - \Omega)\\
&+ e'\gamma\,(1.5818575)\cos(4nt - 5n't + \omega' + \Omega)\\
&\mp e'\gamma\,(1.5818575)\cos(4nt - 5n't + \omega' - \Omega)\\
&+ e'\gamma\,(14.49631)\cos(4nt - n't - \omega' - \Omega)\\
&- e'\gamma\,(14.49631)\cos(n't + \omega' - \Omega)\\
&- e'\gamma\,(15.25264)\cos(4nt - 3n't + \omega' - \Omega)\\
&+ e'\gamma\,(15.25264)\cos(3n't - \omega' - \Omega)\\
&+ e'\gamma\,(0.1856018)\cos(6nt - 3n't - \omega' - \Omega)\\
&\mp e'\gamma\,(0.1856018)\cos(2nt - 3n't - \omega' + \Omega)\\
&- e'\gamma\,(1.5818575)\cos(6nt - 5n't + \omega' - \Omega)\\
&\pm e'\gamma\,(1.5818575)\cos(2nt - 5n't + \omega' + \Omega)
\end{aligned}\right\} \quad (352)$$

Equations (221) and (329) will give

$$
\begin{aligned}
&-\sin v\,\delta\left(\frac{dR}{d\theta}\right)= \\
&\frac{\overline{m}^4}{a\mu}\Big\{-\gamma\,(4.088874)\cos(2nt-\Omega)+\gamma\,(4.088874)\cos\Omega \\
&\quad-\gamma\,(0.138928)\cos(4nt-2n't-\Omega)\pm\gamma\,(0.138928)\cos(2nt-2n't-\Omega) \\
&\quad+\gamma\,(5.575274)\cos(2nt-2n't+\Omega)-\gamma\,(5.575274)\cos(2n't-\Omega) \\
&\quad+\gamma\,(0.5464225)\cos(6nt-4n't-\Omega) \\
&\quad\mp\gamma\,(0.5464225)\cos(4nt-4n't-\Omega) \\
&\quad+\gamma\,(2.1717505)\cos(4nt-4n't+\Omega) \\
&\quad\mp\gamma\,(2.1717505)\cos(2nt-4n't+\Omega) \\
&\quad-e\gamma\,(55.274279)\cos(3nt-\omega-\Omega)+e\gamma\,(19.705189)\cos(nt-\omega+\Omega) \\
&\quad-e\gamma\,(11.52744)\cos(nt+\omega-\Omega)\pm e\,(47.09653)\cos(nt-\omega-\Omega) \\
&\quad-e\gamma\,(0.765429)\cos(5nt-2n't-\omega-\Omega) \\
&\quad\mp e\gamma\,(3.746037)\cos(nt-2n't+\omega-\Omega) \\
&\quad+e\gamma\,(4.803875)\cos(3nt-2n't-\omega+\Omega) \\
&\quad-e\gamma\,(43.35186)\cos(nt+2n't-\omega-\Omega) \\
&\quad+e\gamma\,(4.023893)\cos(3nt-2n't+\omega-\Omega) \\
&\quad\pm e\gamma\,(0.487573)\cos(3nt-2n't-\omega-\Omega) \\
&\quad+e\gamma\,(32.20132)\cos(nt-2n't+\omega+\Omega) \\
&\quad\pm e\gamma\,(6.346674)\cos(nt-2n't-\omega+\Omega) \\
&\quad+e\gamma\,(2.450492)\cos(7nt-4n't-\omega-\Omega) \\
&\quad\mp e\gamma\,(1.357648)\cos(5nt-4n't-\omega-\Omega) \\
&\quad+e\gamma\,(24.142923)\cos(5nt-4n't+\omega-\Omega) \\
&\quad\mp e\gamma\,(25.23577)\cos(3nt-4n't+\omega-\Omega) \\
&\quad+e\gamma\,(6.588415)\cos(5nt-4n't-\omega+\Omega) \\
&\quad\mp e\gamma\,(2.244915)\cos(3nt-4n't-\omega+\Omega) \\
&\quad-e\gamma\,(13.05000)\cos(3nt-4n't+\omega+\Omega) \\
&\quad\pm e\gamma\,(8.706495)\cos(nt-4n't+\omega+\Omega) \\
&\quad-e'\gamma\,(14.56536)\cos(2nt+n't-\omega'-\Omega) \\
&\quad\pm e'\gamma\,(14.56536)\cos(n't-\omega'-\Omega) \\
&\quad-e'\gamma\,(7.77527)\cos(2nt-n't+\omega'-\Omega) \\
&\quad+e'\gamma\,(7.77527)\cos(n't-\omega'+\Omega) \\
&\quad-e'\gamma\,(17.04655)\cos(4nt-n't-\omega'-\Omega) \\
&\quad\pm e'\gamma\,(17.04655)\cos(2nt-n't-\omega'-\Omega) \\
&\quad+e'\gamma\,(17.37382)\cos(2nt-n't-\omega'+\Omega)
\end{aligned}
\quad (353)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- e'\gamma\,(17.37382)\cos\,(n't + \omega' - \Omega')\\
&+ e'\gamma\,(11.103315)\cos\,(2nt - 3n't + \omega' + \Omega)\\
&- e'\gamma\,(11.103315)\cos\,(3n't - \omega' - \Omega)\\
&+ e'\gamma\,(16.391744)\cos\,(4nt - 2n't + \omega' - \Omega)\\
&\mp e'\gamma\,(16.391744)\cos\,(2nt - 3n't + \omega' - \Omega)\\
&- e'\gamma\,(0.5053685)\cos\,(6nt - 3n't - \omega' - \Omega)\\
&\pm e'\gamma\,(0.5053685)\cos\,(4nt - 3n't - \omega' - \Omega)\\
&+ e'\gamma\,(4.19014)\cos\,(6nt - 5n't + \omega' - \Omega)\\
&\mp e'\gamma\,(4.19014)\cos\,(4nt - 5n't + \omega' - \Omega)\\
&- e'\gamma\,(3.103054)\cos\,(4nt - 3n't - \omega' + \Omega)\\
&\pm e'\gamma\,(3.103054)\cos\,(2nt - 3n't - \omega' + \Omega)\\
&+ e'\gamma\,(13.079346)\cos\,(4nt - 5n't + \omega' + \Omega)\\
&\mp e'\gamma\,(13.079346)\cos\,(2nt - 5n't + \omega' + \Omega)\Big\}
\end{aligned}\right\} . \quad (353)$$

Equations (219) and (350) will give

$$\delta\,(\tan\theta\cos v)\left(\frac{dR}{dv}\right) =$$

$$\left.\begin{aligned}
&\frac{\overline{m}^4}{\mu}\Big\{ + \gamma\,(3.404372)\cos\,(2nt - \Omega) + \gamma\,(2.031974)\cos\Omega\\
&\quad - \gamma\,(0.265625)\cos\,(4nt - 2n't - \Omega) + \gamma\,(0.265625)\cos\,(2n't - \Omega)\\
&\quad + \gamma\,(0.265625)\cos\,(2nt - 2n't + \Omega)\\
&\quad \mp \gamma\,(0.265625)\cos\,(2nt - 2n't + \Omega)\\
&\quad - \gamma\,(0.686199)\cos\,(6nt - 4n't - \Omega)\\
&\quad \mp \gamma\,(2.718173)\cos\,(2nt - 4n't + \Omega)\\
&\quad - \gamma\,(2.0319743)\cos\,(4nt - 4n't + \Omega)\\
&\quad + e\gamma\,(28.84714)\cos\,(3nt - \omega - \Omega) + e\gamma\,(14.91552)\cos\,(nt + \omega - \Omega)\\
&\quad \mp e\gamma\,(2.964833)\cos\,(nt - \omega - \Omega) - e\gamma\,(1.90657)\cos\,(nt - \omega + \Omega)\\
&\quad - e\gamma\,(0.218750)\cos\,(5nt - 2n't - \omega - \Omega)\\
&\quad - e\gamma\,(0.843750)\cos\,(nt + 2n't - \omega - \Omega)\\
&\quad \pm e\gamma\,(0.031250)\cos\,(3nt - 2n't - \omega - \Omega)\\
&\quad - e\gamma\,(1.093750)\cos\,(nt - 2n't + \omega + \Omega)\\
&\quad + e\gamma\,(0.921875)\cos\,(3nt - 2n't + \omega - \Omega)\\
&\quad \pm e\gamma\,(0.140625)\cos\,(nt - 2n't - \omega + \Omega)\\
&\quad + e\gamma\,(0.390625)\cos\,(3nt - 2n't - \omega + \Omega)\\
&\quad \pm e\gamma\,(0.671875)\cos\,(nt - 2n't + \omega - \Omega)
\end{aligned}\right\} . \quad (354)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e\gamma\,(6.221324)\cos(5nt - 4n't - \omega + \Omega) \\
&+ e\gamma\,(11.09273)\cos(3nt - 4n't + \omega + \Omega) \\
&- e\gamma\,(3.097134)\cos(7nt - 4n't - \omega - \Omega) \\
&\mp e\gamma\,(2.509713)\cos(3nt - 4n't - \omega + \Omega) \\
&- e\gamma\,(21.90610)\cos(5nt - 4n't + \omega - \Omega) \\
&\mp e\gamma\,(17.62211)\cos(nt - 4n't + \omega + \Omega) \\
&+ e'\gamma\,(8.502668)\cos(2nt + n't - \omega' - \Omega) \\
&+ e'\gamma\,(9.736351)\cos(2nt - n't + \omega' - \Omega) \\
&+ e'\gamma\,(9.820678)\cos(4nt - n't - \omega' - \Omega) \\
&- e'\gamma\,(8.758179)\cos(3n't - \omega' - \Omega) \\
&- e'\gamma\,(10.195086)\cos(4nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma\,(9.132586)\cos(n't + \omega' - \Omega) \\
&- e'\gamma\,(5.991891)\cos(2nt - n't - \omega' + \Omega) \\
&\pm e'\gamma\,(4.929391)\cos(2nt - 3n't + \omega' - \Omega) \\
&\mp e'\gamma\,(5.303799)\cos(2nt - n't - \omega' - \Omega) \\
&+ e'\gamma\,(6.366299)\cos(2nt - 3n't + \omega' + \Omega) \\
&+ e'\gamma\,(2.969194)\cos(4nt - 3n't - \omega' + \Omega) \\
&+ e'\gamma\,(5.158703)\cos(n't - \omega' + \Omega) \\
&- e'\gamma\,(12.05292)\cos(4nt - 5n't + \omega' + \Omega) \\
&\pm e'\gamma\,(3.925020)\cos(n't - \omega' - \Omega) \\
&+ e'\gamma\,(0.638919)\cos(6nt - 3n't - \omega' - \Omega) \\
&\pm e'\gamma\,(3.608112)\cos(2nt - 3n't - \omega' + \Omega) \\
&- e'\gamma\,(5.216578)\cos(6nt - 5n't + \omega' - \Omega) \\
&\mp e'\gamma\,(17.26948)\cos(2nt - 5n't + \omega' + \Omega)\ \Big\}
\end{aligned}\right\} . \qquad (354)
$$

Equations (220) and (349) will give

$$
\mp\,\delta\,(\sin v)\left(\frac{dR}{d\theta}\right) =
$$

$$
\left.\begin{aligned}
&\frac{\overline{m}^4}{a\mu}\Big\{ + \gamma\,(0.686199)\cos(4nt - 2n't - \Omega) \\
&\quad - \gamma\,(0.686199)\cos(2nt - 2n't + \Omega) \pm \gamma\,(0.686199)\cos(2nt - 2n't - \Omega) \\
&\quad - \gamma\,(0.686199)\cos(2n't - \Omega) + \gamma\,(0.3430995)\cos(6nt - 4n't - \Omega) \\
&\quad \pm \gamma\,(0.3430995)\cos(4nt - 4n't - \Omega) \\
&\quad - \gamma\,(0.3430995)\cos(4nt - 4n't + \Omega) \\
&\quad \pm \gamma\,(0.3430995)\cos(2nt - 4n't + \Omega)
\end{aligned}\right\} . \qquad (355)
$$

(Continued on the next page.)

$$\left.\begin{array}{l}
- e\gamma\,(13.80293)\cos(3nt-\omega-\Omega) + e\gamma\,(13.80293)\cos(nt-\omega+\Omega) \\
+ e\gamma\,(13.80293)\cos(nt+\omega-\Omega) \mp e\gamma\,(13.80293)\cos(nt-\omega-\Omega) \\
+ e\gamma\,(1.584111)\cos(5nt-2n't-\omega-\Omega) \\
- e\gamma\,(0.211713)\cos(3nt-2n't-\omega+\Omega) \\
\pm e\gamma\,(0.211713)\cos(3nt-2n't-\omega-\Omega) \\
\pm e\gamma\,(1.160685)\cos(nt-2n't-\omega+\Omega) \\
+ e\gamma\,(23.41913)\cos(3nt-2n't+\omega-\Omega) \\
- e\gamma\,(24.791527)\cos(nt-2n't+\omega+\Omega) \\
- e\gamma\,(26.16392)\cos(nt+2n't-\omega-\Omega) \\
\pm e\gamma\,(24.79152)\cos(nt-2n't+\omega-\Omega) \\
+ e\gamma\,(1.548567)\cos(7nt-4n't-\omega-\Omega) \\
\pm e\gamma\,(0.862368)\cos(5nt-4n't-\omega-\Omega) \\
+ e\gamma\,(10.95305)\cos(5nt-4n't+\omega-\Omega) \\
\pm e\gamma\,(11.63925)\cos(3nt-4n't+\omega-\Omega) \\
- e\gamma\,(11.63925)\cos(3nt-4n't+\omega+\Omega) \\
\mp e\gamma\,(12.32545)\cos(nt-4n't+\omega+\Omega) \\
- e\gamma\,(0.862368)\cos(5nt-4n't-\omega+\Omega) \\
\mp e\gamma\,(0.176169)\cos(3nt-4n't-\omega+\Omega) \\
- e'\gamma\,(15.307424)\cos(2nt+n't-\omega'-\Omega) \\
+ e'\gamma\,(15.307424)\cos(2nt-n't+\omega'-\Omega) \\
+ e'\gamma\,(15.307424)\cos(n't-\omega'+\Omega) \mp e'\gamma\,(15.307424)\cos(nt-\omega'-\Omega) \\
- e'\gamma\,(6.828758)\cos(4nt-n't-\omega'-\Omega) \\
+ e'\gamma\,(6.828758)\cos(2nt-n't-\omega'+\Omega) \\
\mp e'\gamma\,(6.828758)\cos(2nt-n't-\omega'-\Omega) + e'\gamma\,(6.828758)\cos(n't+\omega'-\Omega) \\
+ e'\gamma\,(11.406407)\cos(4nt-3n't+\omega'-\Omega) \\
- e'\gamma\,(11.406407)\cos(2nt-3n't+\omega'+\Omega) \\
\pm e'\gamma\,(11.406407)\cos(2nt-3n't+\omega'-\Omega) \\
- e'\gamma\,(11.406407)\cos(3n't-\omega'-\Omega) \\
- e'\gamma\,(0.3194596)\cos(6nt-3n't-\omega'-\Omega) \\
\mp e'\gamma\,(0.3194596)\cos(4nt-3n't-\omega'-\Omega) \\
+ e'\gamma\,(0.3194596)\cos(4nt-3n't-\omega'+\Omega) \\
\pm e'\gamma\,(0.3194596)\cos(2nt-3n't-\omega'+\Omega) \\
+ e'\gamma\,(2.608283)\cos(6nt-5n't+\omega'-\Omega) \\
\pm e'\gamma\,(2.608283)\cos(4nt-5n't+\omega'-\Omega) \\
- e'\gamma\,(2.608283)\cos(4nt-5n't+\omega'+\Omega) \\
\mp e'\gamma\,(2.608283)\cos(2nt-5n't+\omega'+\Omega) \Big\}
\end{array}\right\} \quad (355)$$

If we now take the sum of equations (352–355), we shall obtain

$$\tan\theta\cos v\,\delta\left(\frac{dR}{dv}\right)+\delta(\tan\theta\cos v)\left(\frac{dR}{dv}\right)-\sin v\,\delta\left(\frac{dR}{d\theta}\right)-\delta(\sin v)\left(\frac{dR}{d\theta}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^4}{a\mu}\Big\{-\gamma(0.684502)\cos(2nt-\Omega)+\gamma(6.120848)\cos\Omega\\
&\quad+\gamma(0.4066462)\cos(4nt-2n't-\Omega)\\
&\quad+\gamma(5.029700)\cos(2nt-2n't+\Omega)\\
&\quad\pm\gamma(0.684502)\cos(2nt-2n't-\Omega)\\
&\quad-\gamma(6.120848)\cos(2n't-\Omega)\mp\gamma(0.4066462)\cos(4nt-4n't-\Omega)\\
&\quad\mp\gamma(5.029700)\cos(2nt-4n't+\Omega)-e\gamma(0.87116)\cos(3nt-\omega-\Omega)\\
&\quad-e\gamma(7.75735)\cos(nt-\omega+\Omega)-e\gamma(22.16789)\cos(nt+\omega-\Omega)\\
&\quad\pm e\gamma(69.68767)\cos(nt-\omega-\Omega)\\
&\quad+e\gamma(0.990557)\cos(5nt-2n't-\omega-\Omega)\\
&\quad+e\gamma(4.842162)\cos(3nt-2n't-\omega+\Omega)\\
&\quad\pm e\gamma(0.871161)\cos(3nt-2n't-\omega-\Omega)\\
&\quad\pm e\gamma(7.757359)\cos(nt-2n't-\omega+\Omega)\\
&\quad+e\gamma(27.19302)\cos(3nt-2n't+\omega-\Omega)\\
&\quad+e\gamma(7.23791)\cos(nt-2n't+\omega+\Omega)\\
&\quad-e\gamma(69.68766)\cos(nt+2n't-\omega-\Omega)\\
&\quad\pm e\gamma(20.79549)\cos(nt-2n't+\omega-\Omega)\\
&\quad\mp e\gamma(0.990560)\cos(5nt-4n't-\omega-\Omega)\\
&\quad\mp e\gamma(27.19304)\cos(3nt-4n't+\omega-\Omega)\\
&\quad\mp e\gamma(7.23791)\cos(nt-4n't+\omega+\Omega)\\
&\quad\mp e\gamma(4.842164)\cos(3nt-4n't-\omega+\Omega)\\
&\quad-e'\gamma(20.77921)\cos(2nt+n't-\omega'-\Omega)\\
&\quad+e'\gamma(16.67759)\cos(2nt-n't+\omega'-\Omega)\\
&\quad+e'\gamma(27.65049)\cos(n't-\omega'+\Omega)\pm e'\gamma(3.773873)\cos(n't-\omega'-\Omega)\\
&\quad+e'\gamma(0.44168)\cos(4nt-n't-\omega'-\Omega)\\
&\quad+e'\gamma(3.71438)\cos(2nt-n't-\omega'+\Omega)\\
&\quad\pm e'\gamma(19.41030)\cos(2nt-n't-\omega'-\Omega)\\
&\quad-e'\gamma(15.90879)\cos(n't+\omega'-\Omega)\\
&\quad+e'\gamma(2.35042)\cos(4nt-3n't+\omega'-\Omega)\\
&\quad+e'\gamma(21.31585)\cos(2nt-3n't+\omega'+\Omega)\\
&\quad\mp e'\gamma(15.30858)\cos(2nt-3n't+\omega'-\Omega)\\
&\quad-e'\gamma(16.01526)\cos(3n't-\omega'-\Omega)
\end{aligned}\right\}.\quad(356)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&\pm e'\gamma\,(0.371274)\cos(4nt-3n't-\omega'-\Omega)\\
&\pm e'\gamma\,(6.845024)\cos(2nt-3n't-\omega'+\Omega)\\
&\mp e'\gamma\,(3.16372)\cos(4nt-5n't+\omega'-\Omega)\\
&\mp e'\gamma\,(31.37525)\cos(2nt-5n't+\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (356)$$

If we take the integral of equation (356), and put $a\mu n = \frac{\mu^2}{a^2 n}$, we shall obtain

$$\int\left\{\tan\theta\cos v\,\delta\left(\frac{dR}{dv}\right)+\delta\,(\tan\theta\cos v)\left(\frac{dR}{dv}\right)-\sin v\,\delta\left(\frac{dR}{d\theta}\right)-\delta\,(\sin v)\left(\frac{dR}{d\theta}\right)\right\}dt$$

$$\left.\begin{aligned}
=a^2\frac{\overline{m}^4}{\mu^2}n\Big\{&\mp\gamma\,(0.342251)\sin(2nt-\Omega)\\
&\pm\gamma(0.1056115)\sin(4nt-2n't-\Omega)\pm\gamma\,(2.718173)\sin(2nt-2n't+\Omega)\\
&+\gamma\,(0.3699217)\sin(2nt-2n't-\Omega)\\
&\mp\gamma\,(40.91407)\sin(2n't-\Omega)-\gamma\,(0.1098808)\sin(4nt-4n't-\Omega)\\
&-\gamma\,(2.957265)\sin(2nt-4n't+\Omega)\mp e\gamma\,(0.290387)\sin(3nt-\omega-\Omega)\\
&\mp\gamma\,(7.75735)\sin(nt-\omega+\Omega)\mp e\gamma\,(22.16789)\sin(nt+\omega-\Omega)\\
&+e\gamma\,(69.68767)\sin(nt-\omega-\Omega)\\
&\pm e\gamma\,(0.2042218)\sin(5nt-2n't-\omega-\Omega)\\
&\pm e\gamma\,(1.744548)\sin(3nt-2n't-\omega+\Omega)\\
&+e\gamma\,(0.3138644)\sin(3nt-2n't-\omega-\Omega)\\
&+e\gamma\,(9.12204)\sin(nt-2n't-\omega+\Omega)\\
&\pm e\gamma\,(9.797187)\sin(3nt-2n't+\omega-\Omega)\\
&\pm e\gamma\,(8.511208)\sin(nt-2n't+\omega+\Omega)\\
&\mp e\gamma\,(60.61890)\sin(nt+2n't-\omega-\Omega)\\
&+e\gamma\,(24.45385)\sin(nt-2n't+\omega-\Omega)\\
&-e\gamma\,(0.2107217)\sin(5nt-4n't-\omega-\Omega)\\
&-e\gamma\,(10.09174)\sin(3nt-4n't+\omega-\Omega)\\
&-e\gamma\,(10.32814)\sin(nt-4n't+\omega+\Omega)\\
&-e\gamma\,(1.792867)\sin(3nt-4n't-\omega+\Omega)\\
&\mp e'\gamma\,(10.01504)\sin(2nt+n't-\omega'-\Omega)\\
&\pm e'\gamma\,(8.662786)\sin(2nt-n't+\omega'-\Omega)\\
&\pm e'\gamma\,(369.6527)\sin(n't-\omega'+\Omega)+e'\gamma\,(50.45198)\sin(n't-\omega'-\Omega)\\
&\pm e'\gamma\,(0.1125243)\sin(4nt-n't-\omega'-\Omega)\\
&\pm e'\gamma\,(1.929350)\sin(2nt-n't-\omega'+\Omega)\\
&+e'\gamma\,(10.08223)\sin(2nt-n't-\omega'-\Omega)
\end{aligned}\right\}. \quad (357)$$

(Continued on the next page.)

$$\left.\begin{array}{l}
\mp e'\gamma\,(212.6806)\sin(n't+\omega'-\Omega) \\
\pm e'\gamma\,(0.6225296)\sin(4nt-3n't+\omega'-\Omega) \\
\pm e'\gamma\,(12.00490)\sin(2nt-3n't+\omega'+\Omega) \\
-e'\gamma\,(8.621659)\sin(2nt-3n't+\omega'-\Omega) \\
\mp e'\gamma\,(71.36800)\sin(3n't-\omega'-\Omega) \\
+e'\gamma\,(0.09833787)\sin(4nt-3n't-\omega'-\Omega) \\
+e'\gamma\,(3.855058)\sin(2nt-3n't-\omega'+\Omega) \\
-e'\gamma\,(0.8725112)\sin(4nt-5n't+\omega'-\Omega) \\
-e'\gamma\,(19.29604)\sin(2nt-5n't+\omega'+\Omega)\Big\}
\end{array}\right\} \quad (357)$$

By means of equations (242) and (357) we obtain the first term of $\delta\dfrac{d\delta_0\theta}{dt}$, as follows:

$$\left.\begin{array}{l}
\text{1st term of } \delta\dfrac{d\delta_0\theta}{dt}=\dfrac{\overline{m}^4}{\mu^2}n\Big\{-\gamma\,(0.342251)\cos(nt-\Omega) \\
-\gamma\,(0.2643102)\cos(3nt-2n't-\Omega)-\gamma(38.19590)\cos(nt-2nt+\Omega) \\
+\gamma\,(0.1098808)\cos(5nt-4n't-\Omega) \\
+\gamma\,(2.957265)\cos(3nt-4n't+\Omega)-e\gamma\,(64.54171)\cos(2nt-\omega-\Omega) \\
-e\gamma\,(30.60974)\cos(\omega-\Omega)-e\gamma\,(0.8494860)\cos(4nt-2n't-\omega-\Omega) \\
-e\gamma\,(89.20563)\cos(2nt-2n't-\omega+\Omega) \\
-e\gamma\,(24.44544)\cos(2nt-2n't+\omega-\Omega)-e\gamma\,(52.10769)\cos(2n't-\omega-\Omega) \\
+e\gamma\,(0.4304833)\cos(6nt-4n't-\omega-\Omega) \\
+e\gamma\,(10.09174)\cos(4nt-4n't+\omega-\Omega) \\
+e\gamma\,(7.707397)\cos(4nt-4n't-\omega+\Omega) \\
+e\gamma\,(10.32814)\cos(2nt-4n't+\omega+\Omega) \\
-e'\gamma\,(60.46702)\cos(nt+n't-\omega'-\Omega) \\
+e'\gamma\,(378.3155)\cos(nt-n't+\omega'-\Omega) \\
-e'\gamma\,(9.96971)\cos(3nt-n't-\omega'-\Omega) \\
-e'\gamma\,(210.7513)\cos(nt-n't-\omega'+\Omega) \\
+e'\gamma\,(9.244189)\cos(3nt-3n't+\omega'-\Omega) \\
-e'\gamma\,(59.3631)\cos(nt-3n't+\omega'+\Omega) \\
-e'\gamma\,(0.09833787)\cos(5nt-3n't-\omega'-\Omega) \\
+e'\gamma\,(19.29604)\cos(3nt-5n't+\omega'+\Omega) \\
+e'\gamma\,(0.8725112)\cos(5nt-5n't+\omega'-\Omega) \\
-e'\gamma\,(3.855058)\cos(3nt-3n't-\omega'+\Omega)\Big\}
\end{array}\right\} \quad (358)$$

Equations (280) and (351) will also give

$$
\left.
\begin{aligned}
\overset{\text{2nd term of}}{\delta\frac{d\delta_0\theta}{dt}} = \frac{\overline{m}^4}{\mu^2} n \Big\{ & + \gamma\,(2.144713)\cos(nt - \Omega) \\
& + \gamma\,(0.2748825)\cos(3nt - 2n't - \Omega) + \gamma\,(2.497069)\cos(nt - 2n't + \Omega) \\
& + \gamma\,(0.8927546)\cos(5nt - 4n't - \Omega) \\
& + \gamma\,(11.04226)\cos(3nt - 4n't + \Omega) - e\gamma\,(17.89099)\cos(\omega - \Omega) \\
& + e\gamma\,(1.241998)\cos(2nt - \omega - \Omega) - e\gamma\,(1.412630)\cos(4nt - 2n't - \omega - \Omega) \\
& - e\gamma\,(1.462135)\cos(2nt - 2n't + \omega - \Omega) \\
& + e\gamma\,(22.18171)\cos(2n't - \omega - \Omega) \\
& - e\gamma\,(3.563386)\cos(2nt - 2n't - \omega + \Omega) \\
& + e\gamma\,(3.733252)\cos(6nt - 4n't - \omega - \Omega) \\
& + e\gamma\,(19.89561)\cos(4nt - 4n't + \omega - \Omega) \\
& + e\gamma\,(40.94991)\cos(4nt - 4n't - \omega + \Omega) \\
& + e\gamma\,(316.7281)\cos(2nt - 4n't + \omega + \Omega) \\
& + e'\gamma\,(16.20359)\cos(nt + n't - \omega' - \Omega) \\
& - e'\gamma\,(7.460775)\cos(nt - n't + \omega' - \Omega) \\
& + e'\gamma\,(24.41495)\cos(3nt - n't - \omega' - \Omega) \\
& - e'\gamma\,(115.0097)\cos(nt - n't - \omega' + \Omega) \\
& - e'\gamma\,(19.47984)\cos(3nt - 3n't + \omega' - \Omega) \\
& + e'\gamma\,(107.89619)\cos(nt - 3n't + \omega' + \Omega) \\
& - e'\gamma\,(0.8179115)\cos(5nt - 3n't - \omega' - \Omega) \\
& - e'\gamma\,(15.85219)\cos(3nt - 3n't - \omega' + \Omega) \\
& + e'\gamma\,(6.879363)\cos(5nt - 5n't + \omega' - \Omega) \\
& + e'\gamma\,(70.57841)\cos(3nt - 5n't + \omega' + \Omega) \Big\}
\end{aligned}
\right\} . \quad (359)
$$

If we now take the sum of equations (358) and (359), we get

$$
\left.
\begin{aligned}
\delta\frac{d\delta_0\theta}{dt} = \frac{\overline{m}^4}{\mu^2} n \Big\{ & + \gamma\,(1.802462)\cos(nt - \Omega) \\
& + \gamma\,(0.0105723)\cos(3nt - 2n't - \Omega) - \gamma\,(35.69883)\cos(nt - 2n't + \Omega) \\
& + \gamma\,(1.0026354)\cos(5nt - 4n't - \Omega) \\
& + \gamma\,(13.99952)\cos(3nt - 4n't + \Omega) - e\gamma\,(63.29971)\cos(2nt - \omega - \Omega) \\
& - e\gamma\,(48.50073)\cos(\omega - \Omega) - e\gamma\,(2.262116)\cos(4nt - 2n't - \omega - \Omega) \\
& - e\gamma\,(92.76902)\cos(2nt - 2n't - \omega + \Omega) \\
& - e\gamma\,(25.90757)\cos(2nt - 2n't + \omega - \Omega)
\end{aligned}
\right\} . \quad (360)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- e\gamma\,(29.92598)\cos(2n't-\omega-\Omega)\\
&+ e\gamma\,(4.163735)\cos(6nt-4n't-\omega-\Omega)\\
&+ e\gamma\,(29.98735)\cos(4nt-4n't+\omega-\Omega)\\
&+ e\gamma\,(48.65731)\cos(4nt-4n't-\omega+\Omega)\\
&+ e\gamma\,(327.0562)\cos(2nt-4n't+\omega+\Omega)\\
&- e'\gamma\,(44.26343)\cos(nt+n't-\omega'-\Omega)\\
&+ e'\gamma\,(370.8547)\cos(nt-n't+\omega'-\Omega)\\
&+ e'\gamma\,(14.44524)\cos(3nt-n't-\omega'-\Omega)\\
&- e'\gamma\,(325.7610)\cos(nt-n't-\omega'+\Omega)\\
&- e'\gamma\,(10.23565)\cos(3nt-3n't+\omega'-\Omega)\\
&+ e'\gamma\,(48.5331)\cos(nt-3n't+\omega'+\Omega)\\
&- e'\gamma\,(0.9162494)\cos(5nt-3n't-\omega'-\Omega)\\
&+ e'\gamma\,(89.87445)\cos(3nt-5n't+\omega'+\Omega)\\
&+ e'\gamma\,(7.751874)\cos(5nt-5n't+\omega'-\Omega)\\
&- e'\gamma\,(19.70725)\cos(3nt-3n't-\omega'+\Omega)\Big\}
\end{aligned}\right\} \quad . \quad (360)$$

31. We must now find the value of $\delta\dfrac{d\delta_0 v}{dt}$. It is evident that

$$\left.\begin{aligned}
\delta\frac{d\delta_0 v}{dt} &= -\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\frac{dv_1}{dt}\int\delta\left(\frac{dR}{dv}\right)dt\\
&\quad -\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\left\{\frac{d\delta_1 v}{dt}+\frac{d\delta_2 v}{dt}\right\}\int\left(\frac{dR}{dv}\right)dt
\end{aligned}\right\} \quad . \quad (361)$$

If we multiply equation (328) by dt, and take the integral, we shall obtain the following equation:

$$\left.\begin{aligned}
&\int\delta\left(\frac{dR}{d\theta}\right)dt =\\
&a^2\frac{\overline{m}^4}{\mu^2}n\Big\{+e\,(157.4350)\cos(nt-\omega)+e'\,(31.59906)\cos(n't-\omega')\\
&\quad + e^2\,(97.40455)\cos 2(nt-\omega)+e'^2\,(82.17244)\cos 2(n't-\omega')\\
&\quad + \gamma^2\,(22.79440)\cos 2(nt-\Omega)+ee'\,(335.1513)\cos(nt+n't-\omega-\omega')\\
&\quad + ee'\,(456.4132)\cos(nt-n't-\omega+\omega')\\
&\quad + \{0.2702122-2.533240e^2+126.8396e'^2-0.6586423\gamma^2\}\cos 2(nt-n't)\\
&\quad - \{0.2197617+27.81844e^2-1.887180e'^2-0.6640148\gamma^2\}\cos 4(nt-n't)\\
&\quad + e\,(0.1973409)\cos(3nt-2n't-\omega)-e\,(4.336207)\cos(nt-2n't+\omega)
\end{aligned}\right\} \quad . \quad (362)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e\,(0.4214434)\cos(5nt-4n't-\omega) - e\,(20.13706)\cos(3nt-4n't+\omega)\\
&+ e'\,(30.11910)\cos(2nt-n't-\omega') - e'\,(34.36062)\cos(2nt-3n't+\omega')\\
&+ e'\,(0.1966331)\cos(4nt-3n't-\omega')\\
&- e'\,(1.745020)\cos(4nt-5n't+\omega')\\
&+ e^2\,(0.2597135)\cos(4nt-2n't-2\omega)\\
&- e^2\,(33.84634)\cos 2(n't-\omega) - e^2\,(0.6147990)\cos(6nt-4n't-2\omega)\\
&+ e^2\,(36.95026)\cos(2nt-4n't+2\omega)\\
&- e'^2\,(0.04565554)\cos(4nt-2n't-2\omega')\\
&- e'^2\,(8.328346)\cos(4nt-6n't+2\omega')\\
&- e'^2\,(155.7347)\cos(2nt-4n't+2\omega') + e'^2\,(6.729297)\cos 2(nt-\omega')\\
&+ ee'\,(36.58821)\cos(3nt-n't-\omega-\omega')\\
&+ ee'\,(28.45058)\cos(nt-3n't+\omega+\omega')\\
&- ee'\,(42.66461)\cos(3nt-3n't-\omega+\omega')\\
&- ee'\,(8.214592)\cos(nt-n't+\omega-\omega')\\
&+ ee'\,(0.3810796)\cos(5nt-3n't-\omega-\omega')\\
&- ee'\,(132.0453)\cos(3nt-5n't+\omega+\omega')\\
&- ee'\,(3.306745)\cos(5nt-5n't-\omega+\omega')\\
&+ ee'\,(29.57425)\cos(3nt-3n't+\omega-\omega')\\
&+ \gamma^2\,(0.08521848)\cos(4nt-2n't-2\Omega) - \gamma^2\,(5.117726)\cos 2(n't-\Omega)\\
&+ \gamma^2\,(0.06755306)\cos(2nt-2n't+2\omega-2\Omega)\\
&- \gamma^2\,(0.06755306)\cos(2nt-2n't-2\omega+2\Omega)\\
&+ \gamma^2\,(0.08045422)\cos(6nt-4n't-2\Omega)\\
&- \gamma^2\,(30.81564)\cos(2nt-4n't+2\Omega)\\
&- \gamma^2\,(0.1098807)\cos(4nt-4n't+2\omega-2\Omega)\\
&+ \gamma^2\,(0.1098807)\cos(4nt-4n't-2\omega+2\Omega)\Big\}\\
&+ a^2\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}n\Big\{-(61.40028)\cos(nt-n't) + (8.676904)\cos 3(nt-n't)\\
&\qquad\qquad - (0.364958)\cos 5(nt-n't)\Big\}
\end{aligned}\right\} . \quad (362)
$$

Now we have

$$
\left.\begin{aligned}
\frac{\sqrt{1-\gamma^2}}{\sqrt{a\mu}\,(1-e^2)}\frac{dv_1}{dt} = \frac{ndt}{\sqrt{a\mu}}\Big\{ 1 + \tfrac{1}{2}e^2 + \tfrac{1}{2}\gamma^2 + 2e\cos(nt-\omega)\\
+ \tfrac{5}{2}e^2\cos 2(nt-\omega) - \tfrac{1}{2}\gamma^2\cos 2(nt-\Omega)\Big\}
\end{aligned}\right\} . \quad (363)
$$

If we multiply equations (362) and (363) together, we shall obtain

$$
\left.
\begin{aligned}
&\text{1st term of}\\
&\delta\frac{d\delta_0 v}{dt}=\frac{\overline{m}^4}{\mu^2}n\Big\{-e\,(157.4350)\cos(nt-\omega)-e'\,(31.59906)\cos(n't-\omega')\\
&-e^2\,(254.83955)\cos 2\,(nt-\omega)-e'^2\,(82.17244)\cos 2\,(n't-\omega')\\
&-\gamma^2\,(22.7944)\cos 2\,(nt-\Omega)-ee'\,(366.7504)\cos(nt+n't-\omega-\omega')\\
&-ee'\,(488.0123)\cos(nt-n't-\omega+\omega')\\
&-\{0.2702122-6.537001e^2+126.8396e'^2-0.5235362\gamma^2\}\cos 2\,(nt-n't)\\
&+\{0.2197617+48.48782e^2-1.887180e'^2-0.5541347\gamma^2\}\cos 4\,(nt-n't)\\
&-e\,(0.4675531)\cos(3nt-2n't-\omega)+e\,(4.065995)\cos(nt-2n't+\omega)\\
&+e\,(0.6412051)\cos(5nt-4n't-\omega)\\
&+e\,(20.35682)\cos(3nt-4n't+\omega)\\
&-e'\,(30.11910)\cos(2nt-n't-\omega')+e'\,(34.36062)\cos(2nt-3n't+\omega')\\
&-e'\,(0.1966331)\cos(4nt-3n't-\omega')\\
&+e'\,(1.745020)\cos(4nt-5n't+\omega')\\
&-e^2\,(0.7948190)\cos(4nt-2n't-2\omega)\\
&+e^2\,(37.84479)\cos 2\,(n't-\omega)+e^2\,(1.310944)\cos(6nt-4n't-2\omega)\\
&-e^2\,(16.53850)\cos(2nt-4n't+2\omega)\\
&+e'^2\,(0.04565554)\cos(4nt-2n't-2\omega')\\
&+e'^2\,(8.328346)\cos(4nt-6n't+2\omega')\\
&+e'^2\,(155.7347)\cos(2nt-4n't+2\omega')-e'^2\,(6.729297)\cos 2\,(nt-\omega')\\
&-ee'\,(66.70731)\cos(3nt-n't-\omega-\omega')\\
&+ee'\,(5.91004)\cos(nt-3n't+\omega+\omega')\\
&+ee'\,(77.02523)\cos(3nt-3n't-\omega+\omega')\\
&-ee'\,(21.90451)\cos(nt-n't+\omega-\omega')\\
&-ee'\,(0.5777127)\cos(5nt-3n't-\omega-\omega')\\
&+ee'\,(133.7903)\cos(3nt-5n't+\omega+\omega')\\
&+ee'\,(5.051765)\cos(5nt-5n't-\omega+\omega')\\
&-ee'\,(29.77088)\cos(3nt-3n't+\omega-\omega')\\
&-\gamma^2\,(0.017665)\cos(4nt-2n't-2\Omega)+\gamma^2\,(5.050173)\cos 2\,(n't-\Omega)\\
&-\gamma^2\,(0.06755306)\cos(2nt-2n't+2\omega-2\Omega)\\
&+\gamma^2\,(0.06755306)\cos(2nt-2n't-2\omega+2\Omega)\\
&-\gamma^2\,(0.1353946)\cos(6nt-4n't-2\Omega)\\
&+\gamma^2\,(30.76170)\cos(2nt-4n't+2\Omega)\\
&+\gamma^2\,(0.1098807)\cos(4nt-4n't+2\omega-2\Omega)\\
&-\gamma^2\,(0.1098807)\cos(4nt-4n't-2\omega+2\Omega)\Big\}
\end{aligned}
\right\}\quad (364)
$$

(*Continued on the next page.*)

$$\left.\begin{aligned}&+\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}n\Big\{+(61.40028)\cos(nt-n't)-(8.676904)\cos 3(nt-n't)\\&\qquad\qquad+(0.364958)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (364)$$

Equations (265), (304), and (305) will give

$$\left.\begin{aligned}
&\delta\frac{d\delta_0 v}{dt}_{\text{2nd term of}}=\frac{\overline{m}^4}{\mu^2}n\Big\{+e(21.62634)\cos(nt-\omega)+e'(7.058980)\cos(n't-\omega')\\
&-e^2(18.76290)\cos 2(nt-\omega)+e'^2(17.84443)\cos 2(n't-\omega')\\
&-\gamma^2(5.936059)\cos 2(nt-\Omega)+ee'(41.63824)\cos(nt+n't-\omega-\omega')\\
&+ee'(56.28987)\cos(nt-n't-\omega+\omega')\\
&+\{0.2702122+2.969203e^2-4.142658e'^2-0.194215\gamma^2\}\cos 2(nt-n't)\\
&+\{1.043833+59.2774e^2-8.90994e'^2-2.321666\gamma^2\}\cos 4(nt-n't)\\
&-e(0.1285747)\cos(3nt-2n't-\omega)\\
&-e(2.067870)\cos(nt-2n't+\omega)+e(3.335361)\cos(5nt-4n't-\omega)\\
&+e(18.29098)\cos(3nt-4n't+\omega)\\
&-e'(1.092941)\cos(2nt-n't-\omega')\\
&-e'(0.237214)\cos(2nt-3n't+\omega')\\
&-e'(0.9607124)\cos(4nt-3n't-\omega')\\
&+e'(8.019791)\cos(4nt-5n't+\omega')\\
&-e^2(0.9277858)\cos(4nt-2n't-2\omega)-e^2(7.232408)\cos 2(n't-\omega)\\
&+e^2(7.531547)\cos(6nt-4n't-2\omega)\\
&-e^2(187.2960)\cos(2nt-4n't+2\omega)\\
&-e'^2(1.272627)\cos 2(nt-\omega')-e'^2(3.821528)\cos(2nt-4n't+2\omega')\\
&+e'^2(36.86795)\cos(4nt-6n't+2\omega')\\
&+e'^2(0.2201114)\cos(4nt-2n't-2\omega')\\
&-ee'(30.12169)\cos(3nt-n't-\omega-\omega')\\
&-ee'(29.28415)\cos(nt-3n't+\omega+\omega')\\
&+ee'(26.58914)\cos(3nt-3n't-\omega+\omega')\\
&+ee'(36.80007)\cos(nt-n't+\omega-\omega')\\
&-ee'(3.074696)\cos(5nt-3n't-\omega-\omega')\\
&-ee'(28.44315)\cos(3nt-3n't+\omega-\omega')\\
&+ee'(25.72761)\cos(5nt-5n't-\omega+\omega')\\
&+ee'(103.91834)\cos(3nt-5n't+\omega+\omega')\\
&+\gamma^2(0.0344328)\cos(4nt-2n't-2\Omega)-\gamma^2(1.202656)\cos 2(n't-\Omega)
\end{aligned}\right\}. \quad (365)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&-\gamma^2(0.7002119)\cos(6nt-4n't-2\Omega)\\
&-\gamma^2(5.235846)\cos(2nt-4n't+2\Omega)\\
&+\gamma^2(0.06755306)\cos(2nt-2n't+2\omega-2\Omega)\\
&-\gamma^2(0.06755306)\cos(2nt-2n't-2\omega+2\Omega)\\
&+\gamma^2(0.5219160)\cos(4nt-4n't+2\omega-2\Omega)\\
&-\gamma^2(0.5219160)\cos(4nt-4n't-2\omega+2\Omega)\Big\}\\
+\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}n\Big\{&-(10.62902)\cos(nt-n't)-(10.37404)\cos 3(nt-n't)\\
&\qquad-(0.3900940)\cos 5(nt-n't)\Big\}
\end{aligned}\right\} \quad (365)
$$

If we take the sum of equations (364) and (365), we shall obtain

$$
\left.\begin{aligned}
\delta\frac{d\delta_0 v}{dt}=\frac{\overline{m}^4}{\mu^2}n\Big\{&-e(135.8087)\cos(nt-\omega)-e'(24.54008)\cos(n't-\omega')\\
&-e^2(273.60245)\cos 2(nt-\omega)-e'^2(64.32801)\cos 2(n't-\omega')\\
&-\gamma^2(28.73046)\cos 2(nt-\Omega)-ee'(325.1122)\cos(nt+n't-\omega-\omega')\\
&-ee'(431.7224)\cos(nt-n't-\omega+\omega')\\
&+\{9.506204e^2-130.9823e'^2+0.329321\gamma^2\}\cos 2(nt-n't)\\
&+\{1.263595+107.7652e^2-10.79712e'^2-2.875800\gamma^2\}\cos 4(nt-n't)\\
&-e(0.5961278)\cos(3nt-2n't-\omega)+e(1.998125)\cos(nt-2n't+\omega)\\
&+e(3.976566)\cos(5nt-4n't-\omega)\\
&+e(38.64780)\cos(3nt-4n't+\omega)\\
&-e'(31.21204)\cos(2nt-n't-\omega')+e'(34.12341)\cos(2nt-3n't+\omega')\\
&-e'(1.1573455)\cos(4nt-3n't-\omega')\\
&+e'(9.764811)\cos(4nt-5n't+\omega')\\
&-e^2(1.722605)\cos(4nt-2n't-2\omega)+e^2(30.61238)\cos 2(n't-\omega)\\
&+e^2(8.842491)\cos(6nt-4n't-2\omega)\\
&-e^2(203.8345)\cos(2nt-4n't+2\omega)\\
&+e'^2(0.2657669)\cos(4nt-2n't-2\omega')\\
&+e'^2(45.19630)\cos(4nt-6n't+2\omega')\\
&+e'^2(151.9132)\cos(2nt-4n't+2\omega')-e'^2(8.001924)\cos 2(nt-\omega')\\
&-ee'(96.82900)\cos(3nt-n't-\omega-\omega')\\
&-ee'(23.37411)\cos(nt-3n't+\omega+\omega')\\
&+ee'(103.61437)\cos(3nt-3n't-\omega+\omega')\\
&+ee'(14.89556)\cos(nt-n't+\omega-\omega')
\end{aligned}\right\} \quad (366)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&- ee' (3.652409) \cos (5nt - 3n't - \omega - \omega') \\
&+ ee' (237.7086) \cos (3nt - 5n't + \omega + \omega') \\
&+ ee' (30.77937) \cos (5nt - 5n't - \omega + \omega') \\
&- ee' (58.21403) \cos (3nt - 3n't + \omega - \omega') \\
&+ \gamma^2 (0.016768) \cos (4nt - 2n't - 2\Omega) + \gamma^2 (3.847517) \cos 2(n't - \Omega) \\
&- \gamma^2 (0.8356065) \cos (6nt - 4n't - 2\Omega) \\
&+ \gamma^2 (25.52586) \cos (2nt - 4n't + 2\Omega) \\
&+ \gamma^2 (0.6317967) \cos (4nt - 4n't + 2\omega - 2\Omega) \\
&- \gamma^2 (0.6317967) \cos (4nt - 4n't - 2\omega + 2\Omega) \Big\} \\
&+ \frac{\overline{m}^4}{\mu^2} \frac{a}{a'} n \Big\{ + (50.77126) \cos (nt - n't) - (19.05094) \cos 3 (nt - n't) \\
&\qquad\qquad - (0.025136) \cos 5 (nt - n't) \Big\}
\end{aligned}\right\} . \quad (366)$$

32. Lastly, we must determine the value of $\delta \dfrac{d\delta_0 r}{dt}$. It is evident by development of the first term of the second member of equation (B) that we shall have

$$\left.\begin{aligned}
\delta \frac{d\delta_0 r}{dt} = \frac{dr_1}{dt} \Bigg\{ - \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}} \int \delta \left(\frac{dR}{dv} \right) dt + \frac{1+\gamma^2}{a\mu(1-e^2)} \left[\int \left(\frac{dR}{dv} \right) dt \right]^2 \Bigg\} \\
+ \frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}} \frac{d\delta_2 r}{dt} \int \left(\frac{dR}{dv} \right) dt
\end{aligned}\right\} . \quad (367)$$

Equation (265) gives

$$\left\{ \int \left(\frac{dR}{dv} \right) dt \right\}^2 =$$

$$\left.\begin{aligned}
&\frac{\overline{m}^4}{a^2 n^2} \Big\{ + (0.3285660) + (0.3285660) \cos 4 (nt - n't) \\
&- e (3.863009) \cos (nt - \omega) + e' (2.081054) \cos (n't - \omega') \\
&+ e (0.4265915) \cos (5nt - 4n't - \omega) \\
&- e (4.289600) \cos (3nt - 4n't + \omega) \\
&- e' (0.3158000) \cos (4nt - 3n't - \omega') \\
&+ e' (2.396854) \cos (4nt - 5n't + \omega') \Big\}
\end{aligned}\right\} . \quad (368)$$

By means of equations (362) and (368) we obtain

$$
\left.
\begin{aligned}
&\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\int\delta\left(\frac{dR}{dv}\right)dt+\frac{1+\gamma^2}{a\mu(1-e^2)}\left[\int\left(\frac{dR}{dv}\right)dt\right]^2=\\
&\frac{\overline{m}^4}{\mu^2}\Big\{+(0.3285660)+e(153.5720)\cos(nt-\omega)\\
&\quad+e'(33.68011)\cos(n't-\omega')+(0.2702122)\cos 2(nt-n't)\\
&\quad+(0.1088043)\cos 4(nt-n't)+e(0.1973409)\cos(3nt-2n't-\omega)\\
&\quad-e(4.336207)\cos(nt-2n't+\omega)+e(0.0051481)\cos(5nt-4n't-\omega)\\
&\quad-e(24.42666)\cos(3nt-4n't+\omega)+e'(30.11910)\cos(2nt-n't-\omega')\\
&\quad-e'(34.36062)\cos(2nt-3n't+\omega')\\
&\quad-e'(0.1191669)\cos(4nt-3n't-\omega')\\
&\quad-e'(0.651834)\cos(4nt-5n't+\omega')\Big\}
\end{aligned}
\right\} . \quad (369)
$$

Whence we get

$$
\left.
\begin{aligned}
&\overset{\text{1st term of}}{\delta\frac{d\delta_0 r}{dt}}=a\,\frac{\overline{m}^4}{\mu^2}n\Big\{+e(0.3285660)\sin(nt-\omega)+e^2(77.1146)\sin 2(nt-\omega)\\
&\quad+ee'(16.840055)\sin(nt+n't-\omega-\omega')\\
&\quad+ee'(16.840055)\sin(nt-n't-\omega+\omega')\\
&\quad-e^2(2.266774)\sin 2(nt-n't)-e^2(12.21590)\sin 4(nt-n't)\\
&\quad+e(0.1351061)\sin(3nt-2n't-\omega)-e(0.1351061)\sin(nt-2n't+\omega)\\
&\quad+e(0.05440215)\sin(5nt-4n't-\omega)\\
&\quad-e(0.05440215)\sin(3nt-4n't+\omega)\\
&\quad+e^2(0.2337765)\sin(4nt-2n't-2\omega)-e^2(2.0329974)\sin 2(n't-\omega)\\
&\quad+e^2(0.0569762)\sin(6nt-4n't-2\omega)\\
&\quad+e^2(12.15893)\sin(2nt-4n't+2\omega)\\
&\quad+ee'(15.05955)\sin(3nt-n't-\omega-\omega')\\
&\quad-ee'(15.05955)\sin(nt-n't+\omega-\omega')\\
&\quad-ee'(17.18031)\sin(3nt-3n't-\omega+\omega')\\
&\quad+ee'(17.18031)\sin(nt-3n't+\omega+\omega')\\
&\quad-ee'(0.05958345)\sin(5nt-3n't-\omega-\omega')\\
&\quad+ee'(0.05958345)\sin(3nt-3n't+\omega-\omega')\\
&\quad-ee'(0.3259170)\sin(5nt-5n't-\omega+\omega')\\
&\quad+ee'(0.3259170)\sin(3nt-5n't+\omega+\omega')\Big\}
\end{aligned}
\right\} . \quad (370)
$$

Equations (265) and (300) will give

$$
\left.
\begin{aligned}
&\overset{\text{2nd term of}}{\delta \frac{d\delta_0 r}{dt}} = a \frac{\overline{m}^4}{\mu^2} n \Big\{ + e^2 (15.79294) \sin 2(nt-\omega) \\
&+ ee' (0.150279) \cos (nt + n't - \omega - \omega') \\
&- ee' (0.150279) \sin (nt - n't - \omega + \omega') \\
&- e^2 (11.33243) \sin 4(nt - n't) - e (0.3708346) \sin (5nt - 4n't - \omega) \\
&- e (0.3708346) \sin (3nt - 4n't + \omega) \\
&+ e^2 (0.1519944) \sin (4nt - 2n't - 2\omega) + e^2 (0.1519944) \sin 2(n't - \omega) \\
&- e^2 (1.1728858) \sin (6nt - 4n't - 2\omega) \\
&- e^2 (11.64291) \sin (2nt - 4n't + 2\omega) \\
&+ ee' (8.173634) \sin (3nt - n't - \omega - \omega') \\
&- ee' (8.173634) \sin (nt - 3n't + \omega + \omega') \\
&- ee' (8.173634) \sin (3nt - 3n't - \omega + \omega') \\
&+ ee' (8.173634) \sin (nt - n't + \omega - \omega') \\
&+ ee' (0.3380812) \sin (5nt - 3n't - \omega - \omega') \\
&+ ee' (0.3380812) \sin (3nt - 3n't + \omega - \omega') \\
&- ee' (2.873824) \sin (5nt - 5n't - \omega + \omega') \\
&- ee' (2.873824) \sin (3nt - 5n't + \omega + \omega') \Big\}
\end{aligned}
\right\} . \quad (371)
$$

The sum of equations (370) and (371) is

$$
\left.
\begin{aligned}
&\delta \frac{d\delta_0 r}{dt} = a \frac{\overline{m}^4}{\mu^2} n \Big\{ + e (0.3285660) \sin (nt - \omega) + e^2 (92.9075) \sin 2(nt - \omega) \\
&+ ee' (16.990334) \sin (nt + n't - \omega - \omega') \\
&+ ee' (16.689776) \sin (nt - n't - \omega + \omega') \\
&- e^2 (2.266774) \sin 2(nt - n't) - e^2 (23.54833) \sin 4(nt - n't) \\
&+ e (0.1351061) \sin (3nt - 2n't - \omega) - e (0.1351061) \sin (nt - 2n't + \omega) \\
&- e (0.3164325) \sin (5nt - 4n't - \omega) \\
&- e (0.4252367) \sin (3nt - 4n't + \omega) \\
&+ e^2 (0.3857709) \sin (4nt - 2n't - 2\omega) - e^2 (1.881003) \sin 2(n't - \omega) \\
&- e^2 (1.115910) \sin (6nt - 4n't - 2\omega) \\
&+ e^2 (0.51602) \sin (2nt - 4n't + 2\omega) \\
&+ ee' (23.23318) \sin (3nt - n't - \omega - \omega') \\
&- ee' (6.88592) \sin (nt - n't + \omega - \omega') \\
&- ee' (25.35394) \sin (3nt - 3n't - \omega + \omega')
\end{aligned}
\right\} . \quad (372)
$$

(Continued on the next page.)

$$\left.\begin{array}{l} + ee'\,(9.00668)\sin(nt - 3n't + \omega + \omega') \\ + ee'\,(0.2784978)\sin(5nt - 3n't - \omega - \omega') \\ + ee'\,(0.3976646)\sin(3nt - 3n't + \omega - \omega') \\ - ee'\,(3.199741)\sin(5nt - 5n't - \omega + \omega') \\ - ee'\,(2.547907)\sin(3nt - 5n't + \omega + \omega')\Big\} \end{array}\right\} . \quad (372)$$

33. We have thus found the differential variations of $\delta_0 r$, $\delta_1 r$, $\delta_0 v$, and $\delta_0\theta$ arising from the square of the disturbing force; and we must now determine the variations of $\delta_2 r$, $\delta_3 r$, $\delta_1 v$, $\delta_2 v$, $\delta_1\theta$, and $\delta_2\theta$, depending on the same cause. Now, it is easy to see that these quantities are given by the following equations:

$$\left.\begin{array}{l} \delta\dfrac{d\delta_2 r}{dt} = \dfrac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e\cos\theta_0\Big\{\cos\theta_1\cos(v_1-\omega)\,\delta^2 v \\ \qquad\qquad\qquad\qquad\qquad\qquad - \frac{1}{2}\cos\theta_1\sin(v_1-\omega)\,\delta v^2\Big\} \\ \delta\dfrac{d\delta_3 r}{dt} = 0; \end{array}\right\} . \quad (373)$$

$$\left.\begin{array}{l} \delta\dfrac{d\delta_1 v}{dt} = -2\dfrac{dv_1}{r_1 dt}\,\delta^2 r + 3\dfrac{dv_1}{r_1^2 dt}\,\delta r^2, \\ \delta\dfrac{d\delta_2 v}{dt} = +2\dfrac{dv_1}{dt}\tan\theta_1\,\delta^2\theta + \dfrac{dv_1}{dt}\,\delta\theta^2 - 4\dfrac{dv_1}{r_1 dt}\tan\theta_1\,\delta r\delta\theta; \end{array}\right\} . \quad (374)$$

$$\left.\begin{array}{l} \delta\dfrac{d\delta_1\theta}{dt} = -2\dfrac{d\theta_1}{r_1 dt}\,\delta^2 r + 3\dfrac{d\theta_1}{r_1^2 dt}\,\delta r^2, \\ \delta\dfrac{d\delta_2\theta}{dt} = -\dfrac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\dfrac{\tan\theta_1}{r_1^2}\,\delta^2 v - \frac{1}{2}\dfrac{d\theta_1}{dt}\,\delta v^2 + 2\dfrac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\dfrac{\tan\theta_1}{r_1^3}\,\delta r\delta v. \end{array}\right\} . \quad (375)$$

The parts of these values which depend on $\delta^2 r$, $\delta^2 v$, and $\delta^2\theta$ must be found by successive approximations, in the same manner as $\delta_2 r$, $\delta_3 r$, $\delta_1 v$, etc. were found in Chapter III. Now, equations (303), (307), and (311), will give

$$\left.\begin{array}{l} \delta r^2 = a^2\dfrac{\overline{m}^4}{\mu^2}\Big\{+\{0.8568242 + 461.7152e^2 + 10.68127e'^2 - 0.6970603\gamma^2\} \\ \qquad\qquad + e\,(38.83398)\cos(nt-\omega) + e'\,(5.447131)\cos(n't-\omega') \\ + \{0.4292231 - 23.29815e^2 - 0.4292231e'^2 - 0.8027084\gamma^2\}\cos 2(nt-n't) \\ + \{0.8290464 + 77.91558e^2 - 7.075322e'^2 - 0.6137270\gamma^2\}\cos 4(nt-n't) \\ - e\,(0.677695)\cos(3nt-2n't-\omega) + e\,(8.552557)\cos(nt-2n't+\omega) \\ + e\,(1.734553)\cos(5nt-4n't-\omega) \\ + e\,(37.39110)\cos(3nt-4n't+\omega) \end{array}\right\} . \quad (376)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e'(2.130710)\cos(2nt - n't - \omega') \\
&- e'(0.213796)\cos(2nt - 3n't + \omega') \\
&- e'(0.7275478)\cos(4nt - 3n't - \omega') \\
&+ e'(6.677492)\cos(4nt - 5n't + \omega') - e^2(4.47778)\cos 2(nt - \omega) \\
&+ e'^2(16.31725)\cos 2(n't - \omega') - \gamma^2(4.131889)\cos 2(nt - \Omega) \\
&+ ee'(84.52691)\cos(nt + n't - \omega - \omega') \\
&+ ee'(135.6338)\cos(nt - n't - \omega + \omega') \\
&- e^2(1.321669)\cos(4nt - 2n't - 2\omega) - e^2(37.89342)\cos 2(n't - \omega) \\
&+ e^2(2.841891)\cos(6nt - 4n't - 2\omega) \\
&+ e^2(375.8532)\cos(2nt - 4n't + 2\omega) \\
&- e'^2(5.891373)\cos(2nt - 4n't + 2\omega') - e'^2(2.102692)\cos 2(nt - \omega') \\
&+ e'^2(32.32033)\cos(4nt - 6n't + 2\omega') \\
&+ e'^2(0.1596189)\cos(4nt - 2n't - 2\omega') \\
&- ee'(45.50526)\cos(3nt - n't - \omega - \omega') \\
&- ee'(67.58717)\cos(nt - 3n't + \omega + \omega') \\
&+ ee'(42.39502)\cos(3nt - 3n't - \omega + \omega') \\
&- ee'(5.10662)\cos(nt - n't + \omega - \omega') \\
&- ee'(1.5282482)\cos(5nt - 3n't - \omega - \omega') \\
&+ ee'(245.3077)\cos(3nt - 5n't + \omega + \omega') \\
&+ ee'(13.916573)\cos(5nt - 5n't - \omega + \omega') \\
&- ee'(51.05551)\cos(3nt - 3n't + \omega - \omega') \\
&+ \gamma^2(0.6414867)\cos(4nt - 2n't - 2\Omega) - \gamma^2(0.466565)\cos 2(n't - \Omega) \\
&+ \gamma^2(0.1073058)\cos(2nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2(0.1073058)\cos(2nt - 2n't - 2\omega + 2\Omega) \\
&- \gamma^2(0.00907088)\cos(6nt - 4n't - 2\Omega) \\
&- \gamma^2(4.289485)\cos(2nt - 4n't + 2\Omega) \\
&+ \gamma^2(0.4145231)\cos(4nt - 4n't + 2\omega - 2\Omega) \\
&- \gamma^2(0.4145231)\cos(4nt - 4n't - 2\omega + 2\Omega) \Big\} \\
&+ a^2\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}\Big\{ -(21.16864)\cos(nt - n't) - (17.14747)\cos 3(nt - n't) \\
&\qquad\qquad + (0.6196511)\cos 5(nt - n't) \Big\}
\end{aligned}\right\} \quad (376)
$$

$$\left.\begin{aligned}\delta v^2 = \frac{\overline{m}^4}{\mu^2}\Big\{ &+ (1.674202) - (1.674202)\cos 4(nt - n't) \\ &+ e\,(128.7036)\cos(nt - \omega) + e'\,(12.29207)\cos(n't - \omega') \\ &+ e\,(1.372398)\cos(3nt - 2n't - \omega) - e\,(1.372398)\cos(nt - 2n't + \omega) \\ &+ e'\,(73.80200)\cos(2nt - n't - \omega') \\ &- e'\,(73.80200)\cos(2nt - 3n't + \omega') \\ &+ e\,(5.067686)\cos(5nt - 4n't - \omega) \\ &- e\,(123.6359)\cos(3nt - 4n't + \omega) \\ &- e'\,(1.443492)\cos(4nt - 3n't - \omega') \\ &- e'\,(13.73556)\cos(4nt - 5n't + \omega')\Big\}\end{aligned}\right\} . \quad (377)$$

$$\left.\begin{aligned}\delta\theta^2 = \frac{\overline{m}^4}{\mu^2}\Big\{ &+ \gamma^2(20.72622) + \gamma^2(5.543879)\cos 2(nt - \Omega) \\ &- \gamma^2(3.838173)\cos 2(nt - n't) - \gamma^2(5.794747)\cos 4(nt - n't) \\ &- \gamma^2(0.6480766)\cos(4nt - 2n't - 2\Omega) + \gamma^2(4.486250)\cos 2(n't - \Omega) \\ &- \gamma^2(0.4185500)\cos(6nt - 4n't - 2\Omega) \\ &- \gamma^2(20.05680)\cos(2nt - 4n't + 2\Omega)\Big\}\end{aligned}\right\} . \quad (378)$$

$$\left.\begin{aligned}\delta r\delta v = a\frac{\overline{m}^4}{\mu^2}\Big\{ &+ e\,(16.50786)\sin(nt - \omega) + e'\,(6.801184)\sin(n't - \omega') \\ &- (0.304977)\sin 2(nt - n't) - (1.178130)\sin 4(nt - n't) \\ &+ e\,(0.8218698)\sin(3nt - 2n't - \omega) - e\,(10.94324)\sin(nt - 2n't + \omega) \\ &- e\,(3.015515)\sin(5nt - 4n't - \omega) \\ &- e(70.06870)\sin(3nt - 4n't + \omega) + e'\,(27.47871)\sin(2nt - n't - \omega') \\ &- e'\,(25.83804)\sin(2nt - 3n't + \omega') \\ &+ e'\,(1.0248372)\sin(4nt - 3n't - \omega') \\ &- e'\,(9.577413)\sin(4nt - 5n't + \omega')\Big\}\end{aligned}\right\} . \quad (379)$$

$$\left.\begin{aligned}\delta r\delta\theta = a\frac{\overline{m}^4}{\mu^2}\Big\{ &+ \gamma\,(3.370626)\sin(nt - \Omega) \\ &- \gamma\,(0.6085382)\sin(3nt - 2n't - \Omega) - \gamma\,(0.5995387)\sin(nt - 2n't + \Omega) \\ &- \gamma\,(0.5890650)\sin(5nt - 4n't - \Omega) - \gamma\,(4.077747)\sin(3nt - 4n't + \Omega)\Big\}\end{aligned}\right\} . \quad (380)$$

Now we have, with sufficient accuracy,

$$\left.\begin{aligned}&\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\,\mu e\cos\theta_0\cos\theta_1\sin(v_1-\omega)\\&\qquad\qquad=\frac{\mu}{\sqrt{a\mu}}\{e\sin(nt-\omega)+e^2\sin 2(nt-\omega)\}\end{aligned}\right\}. \quad (381)$$

Substituting equations (377) and (381) in equation (373), we get

$$\left.\begin{aligned}&\text{2nd term of } \delta\frac{d\delta_2 r}{dt}=a\frac{\overline{m}^4}{\mu^2}n\Big\{-e(0.837101)\sin(nt-\omega)\\
&+e(0.4185505)\sin(5nt-4n't-\omega)\\
&-e(0.4185505)\sin(3nt-4n't+\omega)-e^2(33.0130)\sin 2(nt-\omega)\\
&-ee'(3.07302)\sin(nt+n't-\omega-\omega')\\
&-ee'(3.07302)\sin(nt-n't-\omega+\omega')\\
&+e^2(0.686200)\sin 2(nt-n't)+e^2(32.1759)\sin 4(nt-n't)\\
&-e^2(0.343100)\sin(4nt-2n't-2\omega)+e^2(0.343100)\sin 2(n't-\omega)\\
&-e^2(0.848370)\sin(6nt-4n't-2\omega)\\
&-e^2(31.3276)\sin(2nt-4n't+2\omega)\\
&-ee'(18.4505)\sin(3nt-n't-\omega-\omega')\\
&+ee'(18.4505)\sin(nt-n't+\omega-\omega')\\
&+ee'(18.4505)\sin(3nt-3n't-\omega+\omega')\\
&+ee'(18.4505)\sin(nt-3n't+\omega+\omega')\\
&-ee'(0.360873)\sin(5nt-3n't-\omega-\omega')\\
&+ee'(0.360873)\sin(3nt-3n't+\omega-\omega')\\
&+ee'(3.43389)\sin(5nt-5n't-\omega+\omega')\\
&-ee'(3.43389)\sin(3nt-5n't+\omega+\omega')\Big\}\end{aligned}\right\}. \quad (382)$$

We also have

$$\left.\begin{aligned}\frac{dv_1}{r_1^2dt}=\frac{n}{a^2}\Big\{&1+\tfrac{5}{2}e^2+4e\cos(nt-\omega)+7e^2\cos 2(nt-\omega)\\&\qquad-\tfrac{1}{2}\gamma^2\cos 2(nt-\Omega)\Big\}\end{aligned}\right\}. \quad (383)$$

Equations (376) and (383) will give

$$
\left.
\begin{aligned}
&\overset{\text{2nd term of}}{\delta\frac{d\delta_1 v}{dt}} = \frac{\overline{m}^4}{\mu^2}\Big\{ +\{1.2876693 - 19.4282e^2 - 1.2876693e'^2 \\
&\qquad\qquad\qquad\qquad - 2.4081252\gamma^2\}\cos 2(nt-n't) \\
&+\{2.4871392 + 474.7185e^2 - 21.225966e'^2 - 1.8411181\gamma^2\}\cos 4(nt-n't) \\
&+ e\,(126.78383)\cos(nt-\omega) + e\,(0.542254)\cos(3nt-2n't-\omega) \\
&+ e\,(28.23401)\cos(nt-2n't+\omega) + e\,(10.177937)\cos(5nt-4n't-\omega) \\
&+ e\,(117.14758)\cos(3nt-4n't+\omega) + e'\,(16.341393)\cos(n't-\omega') \\
&- e'\,(6.392130)\cos(2nt-n't-\omega') \\
&- e'\,(0.641388)\cos(2nt-3n't+\omega') \\
&- e'\,(2.1826434)\cos(4nt-3n't-\omega') \\
&+ e'\,(20.032476)\cos(4nt-5n't+\omega') \\
&+ e^2\,(237.56385)\cos 2(nt-\omega) - e^2\,(3.524334)\cos(4nt-2n't-2\omega) \\
&- e^2\,(57.86008)\cos 2(n't-\omega) + e^2\,(27.637978)\cos(6nt-4n't-2\omega) \\
&+ e^2\,(1.3606112)\cos(2nt-4n't+2\omega) + e'^2\,(48.95175)\cos 2(n't-\omega') \\
&- e'^2\,(17.674119)\cos(2nt-4n't+2\omega') - e'^2\,(6.308076)\cos 2(nt-\omega') \\
&+ e'^2\,(96.96099)\cos(4nt-6n't+2\omega') \\
&+ e'^2\,(0.4788567)\cos(4nt-2n't-2\omega') \\
&+ ee'\,(286.26352)\cos(nt+n't-\omega-\omega') \\
&+ ee'\,(439.5842)\cos(nt-n't-\omega+\omega') \\
&- ee'\,(149.30004)\cos(3nt-n't-\omega-\omega') \\
&- ee'\,(204.04429)\cos(nt-3n't+\omega+\omega') \\
&+ ee'\,(125.90228)\cos(3nt-3n't-\omega+\omega') \\
&- ee'\,(28.10412)\cos(nt-n't+\omega-\omega') \\
&- ee'\,(8.9500314)\cos(5nt-3n't-\omega-\omega') \\
&+ ee'\,(775.9881)\cos(3nt-5n't+\omega+\omega') \\
&+ ee'\,(81.814671)\cos(5nt-5n't-\omega+\omega') \\
&- ee'\,(157.53182)\cos(3nt-3n't+\omega-\omega') \\
&- \gamma^2\,(13.680903)\cos 2(nt-\Omega) + \gamma^2\,(1.6025428)\cos(4nt-2n't-2\Omega) \\
&- \gamma^2\,(1.721612)\cos 2(n't-\Omega) - \gamma^2\,(0.6439974)\cos(6nt-4n't-2\Omega) \\
&- \gamma^2\,(13.48524)\cos(2nt-4n't+2\Omega) \\
&+ \gamma^2\,(0.3219174)\cos(2nt-2n't+2\omega-2\Omega)
\end{aligned}
\right\} . \quad (384)
$$

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$$\left.\begin{aligned}
&-\gamma^2(0.3219174)\cos(2nt-2n't-2\omega+2\Omega)\\
&+\gamma^2(1.2435693)\cos(4nt-4n't+2\omega-2\Omega)\\
&\left.-\gamma^2(1.2435693)\cos(4nt-4n't-2\omega+2\Omega)\right\}\\
&+\frac{\overline{m}^4}{\mu^2}\frac{a}{a'}\Big\{-(63.50592)\cos(nt-n't)-(51.44241)\cos 3(nt-n't)\\
&\qquad\qquad\qquad\qquad+(1.8589533)\cos 5(nt-n't)\Big\}
\end{aligned}\right\}. \quad (384)$$

The 2nd term of $\delta\frac{d\delta_2 v}{dt}=n\delta\theta^2$ with sufficient accuracy. We therefore have

$$\left.\begin{aligned}
&\overset{\text{2nd term of}}{\delta\frac{d\delta_2 v}{dt}}=\frac{\overline{m}^4}{\mu^2}n\Big\{+\gamma^2(20.72622)+\gamma^2(5.543879)\cos 2(nt-\Omega)\\
&-\gamma^2(3.838173)\cos 2(nt-n't)-\gamma^2(5.794747)\cos 4(nt-n't)\\
&-\gamma^2(0.6480766)\cos(4nt-2n't-2\Omega)+\gamma^2(4.486250)\cos 2(n't-\Omega)\\
&-\gamma^2(0.4185500)\cos(6nt-4n't-2\Omega)\\
&-\gamma^2(20.05680)\cos(2nt-4n't+2\Omega)\Big\}
\end{aligned}\right\}. \quad (385)$$

To find the 3rd term of $\delta\frac{d\delta_2 v}{dt}$ we have, with sufficient accuracy,

$$\left.\frac{dv_1}{r_1 dt}\tan\theta_1=\frac{n}{a}\gamma\sin(nt-\Omega).\right\}. \quad (386)$$

Whence we get

$$\left.\begin{aligned}
&\overset{\text{3rd term of}}{\delta\frac{d\delta_2 v}{dt}}=\frac{\overline{m}^4}{\mu^2}n\Big\{-\gamma^2(6.741252)+\gamma^2(6.741252)\cos 2(nt-\Omega)\\
&+\gamma^2(0.0179990)\cos 2(nt-n't)-\gamma^2(6.977364)\cos 4(nt-n't)\\
&-\gamma^2(1.2170764)\cos(4nt-2n't-2\Omega)+\gamma^2(1.1990774)\cos 2(n't-\Omega)\\
&-\gamma^2(1.178130)\cos(6nt-4n't-2\Omega)\\
&+\gamma^2(8.155494)\cos(2nt-4n't+2\Omega)\Big\}
\end{aligned}\right\} \quad (387)$$

In order to find the 2nd term of $\delta \frac{d\delta_1\theta}{dt}$, we have

$$\frac{d\theta_1}{r_1^2 dt} = \frac{n}{a^2}\left\{ \gamma \cos(nt - \Omega) + 3e\gamma \cos(2nt - \omega - \Omega) + e\gamma \cos(\omega - \Omega) \right\}. \quad (388)$$

With this and δr^2 we find

$$\begin{aligned}
&\text{2nd term of} \\
&\delta \frac{d\delta_1\theta}{dt} = \frac{\overline{m}^4}{\mu^2} n \Big\{ + \gamma\,(2.570473) \cos(nt - \Omega) \\
&+ \gamma\,(0.6438347) \cos(3nt - 2n't - \Omega) \\
&+ \gamma\,(0.6438347) \cos(nt - 2n't + \Omega) \\
&+ \gamma\,(1.2435696) \cos(5nt - 4n't - \Omega) \\
&+ \gamma\,(1.2435696) \cos(3nt - 4n't + \Omega) \\
&+ e\gamma\,(65.96239) \cos(2nt - \omega - \Omega) \\
&+ e\gamma\,(60.82144) \cos(\omega - \Omega) + e\gamma\,(0.914961) \cos(4nt - 2n't - \omega - \Omega) \\
&- e\gamma\,(0.372708) \cos(2nt - 2n't - \omega + \Omega) \\
&+ e\gamma\,(13.472671) \cos(2nt - 2n't + \omega - \Omega) \\
&+ e\gamma\,(14.76034) \cos(2n't - \omega - \Omega) \\
&+ e\gamma\,(6.332539) \cos(6nt - 4n't - \omega - \Omega) \\
&+ e\gamma\,(3.845400) \cos(4nt - 4n't - \omega + \Omega) \\
&+ e\gamma\,(57.33022) \cos(4nt - 4n't + \omega - \Omega) \\
&+ e\gamma\,(59.81736) \cos(2nt - 4n't + \omega + \Omega) \\
&+ e'\gamma\,(8.170697) \cos(nt + n't - \omega' - \Omega) \\
&+ e'\gamma\,(8.170697) \cos(nt - n't + \omega' - \Omega) \\
&- e'\gamma\,(3.196065) \cos(3nt - n't - \omega' - \Omega) \\
&- e'\gamma\,(3.196065) \cos(nt - n't - \omega' + \Omega) \\
&- e'\gamma\,(0.320694) \cos(3nt - 3n't + \omega' - \Omega) \\
&- e'\gamma\,(0.320694) \cos(nt - 3n't + \omega' + \Omega) \\
&- e'\gamma\,(1.0913217) \cos(5nt - 3n't - \omega' - \Omega) \\
&- e'\gamma\,(1.0913217) \cos(3nt - 3n't - \omega' + \Omega) \\
&+ e'\gamma\,(1.0016238) \cos(5nt - 5n't + \omega' - \Omega) \\
&+ e'\gamma\,(1.0016238) \cos(3nt - 5n't + \omega' + \Omega) \Big\}
\end{aligned} \quad (389)$$

The value of $\frac{d\theta_1}{dt}$ and δv^2 gives the second term of $\delta\frac{d\delta_2\theta}{dt}$, as follows:

$$\left.\begin{aligned}
&\text{2nd term of } \delta\frac{d\delta_2\theta}{dt}=\frac{\overline{m}^4}{\mu^2}n\Big\{-\gamma\,(0.837101)\cos(nt-\Omega)\\
&+\gamma\,(0.4185505)\cos(5nt-4n't-\Omega)\\
&+\gamma\,(0.4185505)\cos(3nt-4n't+\Omega)-e\gamma\,(33.8501)\cos(2nt-\omega-\Omega)\\
&-e\gamma\,(32.1759)\cos(\omega-\Omega)-e\gamma\,(0.3430995)\cos(4nt-2n't-\omega-\Omega)\\
&-e\gamma\,(0.3430995)\cos(2nt-2n't-\omega+\Omega)\\
&+e\gamma\,(0.3430995)\cos(2nt-2n't+\omega-\Omega)\\
&+e\gamma\,(0.3430995)\cos(2n't-\omega-\Omega)\\
&-e\gamma\,(0.429820)\cos(6nt-4n't-\omega-\Omega)\\
&-e\gamma\,(1.2669215)\cos(4nt-4n't-\omega+\Omega)\\
&+e\gamma\,(30.90898)\cos(4nt-4n't+\omega-\Omega)\\
&+e\gamma\,(31.74608)\cos(2nt-4n't+\omega+\Omega)\\
&-e'\gamma\,(3.073018)\cos(nt+n't-\omega'-\Omega)\\
&-e'\gamma\,(3.073018)\cos(nt-n't+\omega'-\Omega)\\
&-e'\gamma\,(18.45050)\cos(3nt-n't-\omega'-\Omega)\\
&-e'\gamma\,(18.45050)\cos(nt-n't-\omega'+\Omega)\\
&+e'\gamma\,(18.45050)\cos(3nt-3n't+\omega'-\Omega)\\
&+e'\gamma\,(18.45050)\cos(nt-3n't+\omega'+\Omega)\\
&-e'\gamma\,(0.360873)\cos(5nt-3n't-\omega'-\Omega)\\
&-e'\gamma\,(0.360873)\cos(3nt-3n't-\omega'+\Omega)\\
&+e'\gamma\,(3.43389)\cos(5nt-5n't+\omega'-\Omega)\\
&+e'\gamma\,(3.43389)\cos(3nt-5n't+\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (390)$$

In order to find the 3rd term of $\delta\frac{d\delta_2\theta}{dt}$ we have

$$\left.\begin{aligned}
\frac{\sqrt{a\mu(1-e^2)}}{\sqrt{1+\gamma^2}}\frac{\tan\theta_1}{r_1^3}=\frac{\sqrt{a\mu}}{a^3}\Big\{\dot\gamma\sin(nt-\Omega)+\tfrac{3}{2}e\gamma\sin(2nt-\omega-\Omega)\\
+\tfrac{1}{2}e\gamma\sin(\omega-\Omega)\Big\}
\end{aligned}\right\}. \quad (391)$$

Then we get

$$\left.\begin{aligned}
&\overset{\text{3rd term of}}{\delta\frac{d\delta_2\theta}{dt}}=\frac{\overline{m}^4}{\mu^2}n\Big\{+\gamma\,(0.304977)\cos(3nt-n't-\Omega)\\
&-\gamma\,(0.304977)\cos(nt-2nt+\Omega)+\gamma\,(1.178130)\cos(5nt-4n't-\Omega)\\
&-\gamma\,(1.178130)\cos(3nt-4n't+\Omega)-e\gamma\,(16.50786)\cos(2nt-\omega-\Omega)\\
&-e\gamma\,(16.50786)\cos(\omega-\Omega)-e\gamma\,(0.0594273)\cos(4nt-2n't-\omega-\Omega)\\
&+e\gamma\,(0.6693813)\cos(2nt-2n't-\omega+\Omega)\\
&+e\gamma\,(11.09573)\cos(2nt-2n't+\omega-\Omega)-e\gamma\,(11.70568)\cos(2n't-\omega-\Omega)\\
&+e\gamma\,(5.960840)\cos(6nt-4n't-\omega-\Omega)\\
&-e\gamma\,(3.604580)\cos(4nt-4n't-\omega+\Omega)\\
&+e\gamma\,(70.65776)\cos(4nt-4n't+\omega-\Omega)\\
&-e\gamma\,(73.01402)\cos(2nt-4n't+\omega+\Omega)\\
&-e'\gamma\,(6.801184)\cos(nt+n't-\omega'-\Omega)\\
&+e'\gamma\,(6.801184)\cos(nt-n't+\omega'-\Omega)\\
&-e'\gamma\,(27.47871)\cos(3nt-n't-\omega'-\Omega)\\
&+e'\gamma\,(27.47871)\cos(nt-n't-\omega'+\Omega)\\
&+e'\gamma\,(25.83804)\cos(3nt-3n't+\omega'-\Omega)\\
&-e'\gamma\,(25.83804)\cos(nt-3n't+\omega'+\Omega)\\
&-e'\gamma\,(1.0248372)\cos(5nt-3n't-\omega'-\Omega)\\
&+e'\gamma\,(1.0248372)\cos(3nt-3n't-\omega'+\Omega)\\
&+e'\gamma\,(9.577413)\cos(5nt-5n't+\omega'-\Omega)\\
&-e'\gamma\,(9.577413)\cos(3nt-5n't+\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (392)$$

By means of successive approximations, as explained in Chapter III., we find

$$\left.\begin{aligned}
&\overset{\text{1st term of}}{\delta\frac{d\delta_2 r}{dt}}=a\frac{\overline{m}^4}{\mu^2}n\Big\{-e\,(0.4306294)\sin(3nt-2n't-\omega)\\
&-e\,(0.4306294)\sin(nt-2n't+\omega)+e\,(0.680543)\sin(5nt-4n't-\omega)\\
&+e\,(0.680543)\sin(3nt-4n't+\omega)+e^2\,(171.9037)\sin 2(nt-n't)\\
&+e^2\,(46.93824)\sin 4(nt-n't)-e^2\,(24.87275)\sin 2(nt-\omega)\\
&-e^2\,(1.499028)\sin(4nt-2n't-2\omega)-e^2\,(171.6802)\sin 2(n't-\omega)\\
&+e^2\,(2.733320)\sin(6nt-4n't-2\omega)\\
&+e^2\,(46.92709)\sin(2nt-4n't+2\omega)\\
&+ee'\,(554.4565)\sin(nt+n't-\omega-\omega')\\
&-ee'\,(554.4565)\sin(nt-n't-\omega+\omega')
\end{aligned}\right\}. \quad (393)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- ee' (34.46968) \sin (3nt - n't - \omega - \omega') \\
&- ee' (34.46968) \sin (nt - n't + \omega - \omega') \\
&+ ee' (37.80811) \sin (3nt - 3n't - \omega + \omega') \\
&+ ee' (37.80811) \sin (nt - 3n't + \omega + \omega') \\
&- ee' (0.5905185) \sin (5nt - 3n't - \omega - \omega') \\
&- ee' (0.5905185) \sin (3nt - 3n't + \omega - \omega') \\
&+ ee' (5.71550) \sin (5nt - 5n't - \omega + \omega') \\
&+ ee' (5.71550) \sin (3nt - 5n't + \omega + \omega') \Big\}
\end{aligned}\right\} . \quad (393)
$$

If we now take the sum of equations (347), (372), (382), and (393), we shall obtain the complete value of $\delta \dfrac{d\delta r}{dt}$, as follows:

$$
\left.\begin{aligned}
\delta \frac{d\delta r}{dt} = a \frac{\overline{m}'}{\mu^2} n \Big\{ &- e (17.02475) \sin (nt - \omega) + e' (3.40895) \sin (n't - \omega') \\
&- e^2 (226.0302) \sin 2 (nt - \omega) + e'^2 (17.29864) \sin 2 (n't - \omega') \\
&- \gamma^2 (74.80314) \sin 2 (nt - \Omega) - ee' (3148.413) \sin (nt + n't - \omega - \omega') \\
&+ ee' (4016.456) \sin (nt - n't - \omega + \omega') \\
&- \{2.665812 - 278.5685 e^2 + 401.0129 e'^2 - 11.80931 \gamma^2\} \sin 2 (nt - n't) \\
&+ \{2.380288 + 365.7476 e^2 - 20.33280 e'^2 - 1.963547 \gamma^2\} \sin 4 (nt - n't) \\
&- e (4.691584) \sin (3nt - 2n't - \omega) + e (113.4458) \sin (nt - 2n't + \omega) \\
&+ e (7.557287) \sin (5nt - 4n't - \omega) \\
&+ e (124.3446) \sin (3nt - 4n't + \omega) \\
&- e' (91.56037) \sin (2nt - n't - \omega') + e' (89.48968) \sin (2nt - 3n't + \omega') \\
&- e' (2.112689) \sin (4nt - 3n't - \omega') \\
&+ e' (21.123995) \sin (4nt - 5n't + \omega') \\
&- e^2 (8.928473) \sin (4nt - 2n't - 2\omega) - e^2 (17.1023) \sin 2 (n't - \omega) \\
&+ e^2 (14.59226) \sin (6nt - 4n't - 2\omega) \\
&+ e^2 (862.7543) \sin (2nt - 4n't + 2\omega) \\
&+ e'^2 (437.9840) \sin (2nt - 4n't + 2\omega') - e'^2 (26.15554) \sin 2 (nt - \omega') \\
&+ e'^2 (90.85541) \sin (4nt - 6n't + 2\omega') \\
&+ e'^2 (0.4680968) \sin (4nt - 2n't - 2\omega') \\
&- ee' (275.9773) \sin (3nt - n't - \omega - \omega') \\
&+ ee' (617.6638) \sin (nt - 3n't + \omega + \omega') \\
&+ ee' (285.2770) \sin (3nt - 3n't - \omega + \omega') \\
&+ ee' (585.073) \sin (nt - n't + \omega - \omega')
\end{aligned}\right\} . \quad (394)
$$

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$$\left.\begin{array}{l}
- ee' (6.764378) \sin (5nt - 3n't - \omega - \omega') \\
+ ee' (59.66264) \sin (5nt - 5n't - \omega + \omega') \\
+ ee' (821.7629) \sin (3nt - 5n't + \omega + \omega') \\
- ee' (164.34247) \sin (3nt - 3n't + \omega - \omega') \\
+ \gamma^2 (1.3667731) \sin (4nt - 2n't - 2\Omega) + \gamma^2 (0.88434) \sin 2 (n't - \Omega) \\
- \gamma^2 (0.666464) \sin (2nt - 2n't + 2\omega - 2\Omega) \\
+ \gamma^2 (0.666464) \sin (2nt - 2n't - 2\omega + 2\Omega) \\
- \gamma^2 (0.0992640) \sin (6nt - 4n't - 2\Omega) \\
+ \gamma^2 (86.31863) \sin (2nt - 4n't + 2\Omega) \\
+ \gamma^2 (1.052125) \sin (4nt - 4n't + 2\omega - 2\Omega) \\
- \gamma^2 (1.052125) \sin (4nt - 4n't - 2\omega + 2\Omega) \\
- \dfrac{a}{a'} (689.2248) \sin (nt - n't) - \dfrac{a}{a'} (56.11517) \sin 3 (nt - n't) \\
\qquad\qquad + \dfrac{a}{a'} (0.857261) \sin 5 (nt - n't) \Big\}
\end{array}\right\} . \quad (394)$$

Equation (394) gives by integration, the numbers in brackets being logarithms,

$$\left.\begin{array}{l}
\delta^2 r = a \dfrac{\overline{m}^4}{\mu^2} \Big\{ + [0.1514939] + e [1.2310807] \cos (nt - \omega) \\
- e' [1.6587116] \cos (n't - \omega') + e^2 [2.0531365] \cos 2 (nt - \omega) \\
- e'^2 [2.0630728] \cos 2 (n't - \omega') + \gamma^2 [1.5728898] \cos 2 (nt - \Omega) \\
+ ee' [3.4667635] \cos (nt + n't - \omega - \omega') \\
+ ee' [3.6376080] \cos (nt - n't - \omega + \omega') \\
+ \{[0.1585646] - [2.1776670] e^2 + [2.3358934] e'^2 \\
\qquad\qquad - [0.8049595] \gamma^2\} \cos 2 (nt - n't) \\
- \{[9.8083345] + [1.9948864 e^2] - [0.7399023] e'^2 \\
\qquad\qquad - [9.7247448] \gamma^2\} \cos 4 (nt - n't) \\
+ e [0.2164141] \cos (3nt - 2n't - \omega) - e [2.1251665] \cos (nt - 2n't + \omega) \\
- e [0.2061947] \cos (5nt - 4n't - \omega) - e [1.6631353] \cos (3nt - 4n't + \omega) \\
+ e' [1.6772320] \cos (2nt - n't - \omega') \\
- e' [1.7024289] \cos (2nt - 3n't + \omega') \\
+ e' [9.7478501] \cos (4nt - 3n't - \omega') \\
- e' [0.7653491] \cos (4nt - 5n't + \omega') \\
+ e^2 [0.3652717] \cos (4nt - 2n't - 2\omega) \\
+ e^2 [2.0581154] \cos 2 (n't - \omega) - e^2 [0.4081872] \cos (6nt - 4n't - 2\omega) \\
- e^2 [2.7052353] \cos (2nt - 4n't + 2\omega)
\end{array}\right\} . \quad (395)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e'^2\,[2.4108064]\cos(2nt - 4n't + 2\omega') \\
&+ e'^2\,[1.1165337]\cos 2(nt - \omega') - e'^2\,[1.4079766]\cos(4nt - 6n't + 2\omega') \\
&- e'^2\,[9.0848301]\cos(4nt - 2n't - 2\omega') \\
&+ ee'\,[1.9747179]\cos(3nt - n't - \omega - \omega') \\
&- ee'\,[2.9011166]\cos(nt - 3n't + \omega + \omega') \\
&- ee'\,[2.0119104]\cos(3nt - 3n't - \omega + \omega') \\
&- ee'\,[2.8009750]\cos(nt - n't + \omega - \omega') \\
&+ ee'\,[0.1512004]\cos(5nt - 3n't - \omega - \omega') \\
&- ee'\,[1.1104975]\cos(5nt - 5n't - \omega + \omega') \\
&- ee'\,[2.4954529]\cos(3nt - 5n't + \omega + \omega') \\
&+ ee'\,[1.7723938]\cos(3nt - 3n't + \omega - \omega') \\
&- \gamma^2\,[9.5501909]\cos(4nt - 2n't - 2\Omega) - \gamma^2\,[0.7716802]\cos 2(n't - \Omega) \\
&+ \gamma^2\,[9.5565117]\cos(2nt - 2n't + 2\omega - 2\Omega) \\
&- \gamma^2\,[9.5565117]\cos(2nt - 2n't - 2\omega + 2\Omega) \\
&+ \gamma^2\,[8.2408564]\cos(6nt - 4n't - 2\Omega) \\
&- \gamma^2\,[1.7054527]\cos(2nt - 4n't + 2\Omega) \\
&- \gamma^2\,[9.4537723]\cos(4nt - 4n't + 2\omega - 2\Omega) \\
&+ \gamma^2\,[9.4537723]\cos(4nt - 4n't - 2\omega + 2\Omega) \\
&+ \frac{a}{a'}[2.8714963]\cos(nt - n't) + \frac{a}{a'}[1.3057240]\cos 3(nt - n't) \\
&\qquad\qquad - \frac{a}{a'}[9.2679081]\cos 5(nt - n't) \Big\}
\end{aligned}\right\} . \quad (395)
$$

This value of $\delta^2 r$ gives the first term of $\delta\,\dfrac{d\delta_1 v}{dt}$, as follows:

$$
\left.\begin{aligned}
&\overset{\text{1st term of}}{\delta\frac{d\delta_1 v}{dt}} = \frac{\overline{m}^4}{\mu^2}\, n \Big\{ - e\,(40.72060)\cos(nt - \omega) + e'\,(91.14685)\cos(n't - \omega') \\
&- e^2\,(287.1110)\cos 2(nt - \omega) + e'^2\,(231.2612)\cos 2(n't - \omega') \\
&- \gamma^2\,(73.69129)\cos 2(nt - \Omega) - ee'\,(5721.874)\cos(nt + n't - \omega - \omega') \\
&+ ee'\,(8819.084)\cos(nt - n't - \omega + \omega') \\
&- \{2.881340 + 696.3625e^2 + 433.4342e'^2 - 12.76408\gamma^2\}\cos 2(nt - n't) \\
&+ \{1.286366 + 340.60192e^2 - 10.98834e'^2 - 1.0611452\gamma^2\}\cos 4(nt - n't) \\
&- e\,(7.61389)\cos(3nt - 2n't - \omega) \\
&+ e\,(262.4846)\cos(nt - 2n't + \omega) + e\,(5.144871)\cos(5nt - 4n't - \omega) \\
&+ e\,(94.00955)\cos(3nt - 4n't + \omega) \\
&- e'\,(95.11784)\cos(2nt - n't - \omega')
\end{aligned}\right\} . \quad (396)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ e'(100.79962)\cos(2nt - 3n't + \omega') \\
&- e'(1.1191288)\cos(4nt - 3n't - \omega') \\
&+ e'(11.651426)\cos(4nt - 5n't + \omega') \\
&- e^2(16.05853)\cos(4nt - 2n't - 2\omega) + e^2(165.0905)\cos 2(n't - \omega) \\
&+ e^2(12.83668)\cos(6nt - 4n't - 2\omega) \\
&+ e^2(1155.5453)\cos(2nt - 4n't + 2\omega) \\
&+ e'^2(515.0346)\cos(2nt - 4n't + 2\omega') - e'^2(26.15554)\cos 2(nt - \omega') \\
&+ e'^2(51.16896)\cos(4nt - 6n't + 2\omega') \\
&+ e'^2(0.2431422)\cos(4nt - 2n't - 2\omega') \\
&- ee'(331.36632)\cos(3nt - n't - \omega - \omega') \\
&+ ee'(1743.945)\cos(nt - 3n't + \omega + \omega') \\
&+ ee'(356.7602)\cos(3nt - 3n't - \omega + \omega') \\
&+ ee'(1122.0740)\cos(nt - n't + \omega - \omega') \\
&- ee'(4.511587)\cos(5nt - 3n't - \omega - \omega') \\
&+ ee'(43.27166)\cos(5nt - 5n't - \omega + \omega') \\
&+ ee'(643.3453)\cos(3nt - 5n't + \omega + \omega') \\
&- ee'(120.09837)\cos(3nt - 3n't + \omega - \omega') \\
&+ \gamma^2(1.430274)\cos(4nt - 2n't - 2\Omega) + \gamma^2(12.542855)\cos 2(n't - \Omega) \\
&- \gamma^2(0.3564161)\cos(6nt - 4n't - 2\Omega) \\
&+ \gamma^2(101.18229)\cos(2nt - 4n't + 2\Omega) \\
&- \gamma^2(0.7203470)\cos(2nt - 2n't + 2\omega - 2\Omega) \\
&+ \gamma^2(0.7203470)\cos(2nt - 2n't - 2\omega + 2\Omega) \\
&+ \gamma^2(0.5685940)\cos(4nt - 4n't + 2\omega - 2\Omega) \\
&- \gamma^2(0.5685940)\cos(4nt - 4n't - 2\omega + 2\Omega) \\
&- \frac{a}{a'}(1487.7342)\cos(nt - n't) - \frac{a}{a'}(40.43468)\cos 3(nt - n't) \\
&\qquad\qquad + \frac{a}{a'}(0.3706280)\cos 5(nt - n't) \Big\}
\end{aligned}\right\} \quad . \quad (396)$$

The value of the first term of $\delta\dfrac{d\delta_2 v}{dt}$, which depends on $\delta^2\theta$, is

$$\left.\begin{aligned}
&\overset{\text{1st term of}}{\delta\frac{d\delta_2 v}{dt}} = \frac{\bar{m}^4}{\mu^2} n \Big\{ -\gamma^2(1.312135)\cos 2(nt - \Omega) \\
&+ \gamma^2(42.44830)\cos 2(nt - n't) - \gamma^2(4.249757)\cos 4(nt - n't) \\
&+ \gamma^2(0.319926)\cos(4nt - 2n't - 2\Omega) - \gamma^2(42.76823)\cos 2(n't - \Omega) \\
&- \gamma^2(1.099093)\cos(6nt - 4n't - 2\Omega) \\
&+ \gamma^2(5.348850)\cos(2nt - 4n't + 2\Omega) \Big\}
\end{aligned}\right\} \quad . \quad (397)$$

If we now take the sum of equations (384), (385), (387), (396), and (397), we shall obtain $\delta\dfrac{d\delta_1 v}{dt}+\delta\dfrac{d\delta_2 v}{dt}$, which includes the whole of the sun's indirect action, arising from the square of the disturbing force. Hence we find

$$
\left.
\begin{aligned}
&\delta\frac{d\delta_1 v}{dt}+\delta\frac{d\delta_2 v}{dt}=\\
&\frac{\overline{m}'^4}{\mu^2}n\Big\{+e\,(86.06323)\cos(nt-\omega)+e'\,(107.48823)\cos(n't-\omega')\\
&\quad -e^2\,(49.5472)\cos 2(nt-\omega)+e'^2\,(280.2129)\cos 2(n't-\omega')\\
&\quad -\gamma^2\,(76.39920)\cos 2(nt-\Omega)-ee'\,(5435.610)\cos(nt+n't-\omega-\omega')\\
&\quad +ee'\,(9258.668)\cos(nt-n't-\omega+\omega')\\
&\quad -\{1.593671-676.9343e^2+434.7219e'^2-48.98408\gamma^2\}\cos 2(nt-n't)\\
&\quad +\{3.7735052+815.3204e^2-32.21431e'^2-19.924194\gamma^2\}\cos 4(nt-n't)\\
&\quad -e\,(7.071636)\cos(3nt-2n't-\omega)+e\,(290.7186)\cos(nt-2n't+\omega)\\
&\quad +e\,(15.32281)\cos(5nt-4n't-\omega)\\
&\quad +e\,(211.15713)\cos(3nt-4n't+\omega)\\
&\quad -e'\,(101.50997)\cos(2nt-n't-\omega')\\
&\quad +e'\,(100.15823)\cos(2nt-3n't+\omega')\\
&\quad -e'\,(3.301772)\cos(4nt-3n't-\omega')\\
&\quad +e'\,(31.68390)\cos(4nt-5n't+\omega')\\
&\quad -e^2\,(19.582861)\cos(4nt-2n't-2\omega)+e^2\,(107.2304)\cos 2(n't-\omega)\\
&\quad +e^2\,(40.47466)\cos(6nt-4n't-2\omega)\\
&\quad +e^2\,(2516.1565)\cos(2nt-4n't+2\omega)\\
&\quad +e'^2\,(497.3605)\cos(2nt-4n't+2\omega')-e'^2\,(32.46362)\cos 2(nt-\omega')\\
&\quad +e'^2\,(148.12995)\cos(4nt-6n't+2\omega')\\
&\quad +e'^2\,(0.7219987)\cos(4nt-2n't-2\omega')\\
&\quad -ee'\,(480.66636)\cos(3nt-n't-\omega-\omega')\\
&\quad +ee'\,(1539.901)\cos(nt-3n't+\omega+\omega')\\
&\quad +ee'\,(482.6625)\cos(3nt-3n't-\omega+\omega')\\
&\quad +ee'\,(1093.9699)\cos(nt-n't+\omega-\omega')\\
&\quad -ee'\,(13.461618)\cos(5nt-3n't-\omega-\omega')\\
&\quad +ee'\,(125.08633)\cos(5nt-5n't-\omega+\omega')\\
&\quad +ee'\,(1419.3334)\cos(3nt-5n't+\omega+\omega')\\
&\quad -ee'\,(277.63019)\cos(3nt-3n't+\omega-\omega')\\
&\quad +\gamma^2\,(1.487589)\cos(4nt-2n't-2\Omega)-\gamma^2\,(26.26166)\cos 2(n't-\Omega)\\
&\quad -\gamma^2\,(3.696186)\cos(6nt-4n't-2\Omega)
\end{aligned}
\right\}. \quad (398)
$$

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$$\left.\begin{aligned}
&+\gamma^2(81.14459)\cos(2nt-4n't+2\Omega)\\
&-\gamma^2(0.3984296)\cos(2nt-2n't+2\omega-2\Omega)\\
&+\gamma^2(0.3984296)\cos(2nt-2n't-2\omega+2\Omega)\\
&+\gamma^2(1.812163)\cos(4nt-4n't+2\omega-2\Omega)\\
&-\gamma^2(1.812163)\cos(4nt-4n't-2\omega+2\Omega)\\
&-\frac{a}{a'}(1551.2401)\cos(nt-n't)-\frac{a}{a'}(91.87709)\cos3(nt-n't)\\
&\qquad\qquad\qquad\qquad+\frac{a}{a'}(2.2295813)\cos5(nt-n't)\Big\}
\end{aligned}\right\}. \quad (398)$$

If we now take the sum of equations (366) and (398), we shall obtain the complete value of $\delta\dfrac{d\delta v}{dt}$, arising from the square of the disturbing force, as follows:

$$\left.\begin{aligned}
\delta\frac{d\delta v}{dt}=\frac{\overline{m}^4}{\mu^2}n\Big\{&-e(49.7455)\cos(nt-\omega)+e'(82.94815)\cos(n't-\omega')\\
&-e'^2(323.1496)\cos2(nt-\omega)+e'^2(215.8849)\cos2(n't-\omega')\\
&-\gamma^2(105.12966)\cos2(nt-\Omega)-ee'(5760.722)\cos(nt+n't-\omega-\omega')\\
&+ee'(8826.946)\cos(nt-n't-\omega+\omega')\\
&-\{1.593671-686.4405e^2+565.7042e'^2-49.31340\gamma^2\}\cos2(nt-n't)\\
&+\{5.037100+923.0856e^2-43.01143e'^2-22.799994\gamma^2\}\cos4(nt-n't)\\
&-e(7.667764)\cos(3nt-2n't-\omega)+e(292.7167)\cos(nt-2n't+\omega)\\
&+e(19.299374)\cos(5nt-4n't-\omega)\\
&+e(249.80493)\cos(3nt-4n't+\omega)\\
&-e'(132.72201)\cos(2nt-n't-\omega')\\
&+e'(134.28164)\cos(2nt-3n't+\omega')\\
&-e'(4.459118)\cos(4nt-3n't-\omega')\\
&+e'(41.44871)\cos(4nt-5n't+\omega')\\
&-e^2(21.305466)\cos(4nt-2n't-2\omega)+e^2(137.8428)\cos2(n't-\omega)\\
&+e^2(49.31715)\cos(6nt-4n't-2\omega)\\
&+e^2(2312.322)\cos(2nt-4n't+2\omega)\\
&+e'^2(649.2737)\cos(2nt-4n't+2\omega')-e'^2(40.46554)\cos2(nt-\omega')\\
&+e'^2(193.32625)\cos(4nt-6n't+2\omega')\\
&+e'^2(0.9877656)\cos(4nt-2n't-2\omega')\\
&-ee'(577.49536)\cos(3nt-n't-\omega-\omega')\\
&+ee'(1516.527)\cos(nt-3n't+\omega+\omega')\\
&+ee'(586.2769)\cos(3nt-3n't-\omega+\omega')\\
&+ee'(1108.8655)\cos(nt-n't+\omega-\omega')
\end{aligned}\right\}. \quad (399)$$

(Continued on the next page.)

$$\left.\begin{array}{l}
- ee'(17.114027)\cos(5nt - 3n't - \omega - \omega') \\
+ ee'(155.8657)\cos(5nt - 5n't - \omega + \omega') \\
+ ee'(1657.042)\cos(3nt - 5n't + \omega + \omega') \\
- ee'(335.8442)\cos(3nt - 3n't + \omega - \omega') \\
+ \gamma^2(1.504367)\cos(4nt - 2n't - 2\Omega) - \gamma^2(22.41414)\cos 2(n't - \Omega) \\
- \gamma^2(4.531792)\cos(6nt - 4n't - 2\Omega) \\
+ \gamma^2(106.67045)\cos(2nt - 4n't + 2\Omega) \\
- \gamma^2(0.3984296)\cos(2nt - 2n't + 2\omega - 2\Omega) \\
+ \gamma^2(0.3984296)\cos(2nt - 2n't - 2\omega + 2\Omega) \\
+ \gamma^2(2.443960)\cos(4nt - 4n't + 2\omega - 2\Omega) \\
- \gamma^2(2.443960)\cos(4nt - 4n't - 2\omega + 2\Omega) \\
- \dfrac{a}{a'}(1500.4688)\cos(nt - n't) - \dfrac{a}{a'}(110.92803)\cos 3(nt - n't) \\
\qquad\qquad\qquad + \dfrac{a}{a'}(2.204445)\cos 5(nt - n't)\Big\}
\end{array}\right\} . \quad (399)$$

Equation (399) gives by integration

$$\left.\begin{array}{l}
\delta^2 v = \dfrac{\overline{m}'^2}{\mu^2}\Big\{ - e[1.6967538]\sin(nt - \omega) + e'[3.0448976]\sin(n't - \omega') \\
- e^2[2.2083737]\sin 2(nt - \omega) + e'^2[3.1592832]\sin 2(n't - \omega') \\
- \gamma^2[1.7206980]\sin 2(nt - \Omega) - ee'[3.7291488]\sin(nt + n't - \omega - \omega') \\
+ ee'[3.9795755]\sin(nt - n't - \omega + \omega') \\
-\{[9.9351337] - [2.5693379]e^2 + [2.4853244]e'^2 \\
\qquad\qquad - [1.4256999]\gamma^2\}\sin 2(nt - n't) \\
+\{[0.1338856] + [2.3969469]e^2 - [1.0652888]e'^2 \\
\qquad\qquad - [0.7896397]\gamma^2\}\sin 4(nt - n't) \\
- e[0.4297634]\sin(3nt - 2n't - \omega) + e[2.5368255]\sin(nt - 2n't + \omega) \\
+ e[0.6133718]\sin(5nt - 4n't - \omega) \\
+ e[1.9661093]\sin(3nt - 4n't + \omega) \\
- e'[1.8384673]\sin(2nt - n't - \omega') \\
+ e'[1.8786724]\sin(2nt - 3nt + \omega') \\
- e'[0.0722635]\sin(4nt - 3n't - \omega') \\
+ e'[1.0580840]\sin(4nt - 5n't + \omega') \\
- e^2[0.7429854]\sin(4nt - 2n't - 2\omega) + e^2[2.9644450]\sin 2(n't - \omega) \\
+ e^2[0.9370626]\sin(6nt - 4n't - 2\omega) \\
+ e^2[3.1333963]\sin(2nt - 4n't + 2\omega)
\end{array}\right\} . \quad (400)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+e'^2[2.5817759]\sin(2nt-4n't+2\omega')-e'^2[1.3060553]\sin 2(nt-\omega')\\
&+e'^2[1.7359167]\sin(4nt-6n't+2\omega')\\
&+e'^2[9.4091483]\sin(4nt-2n't-2\omega')\\
&-ee'[2.2953932]\sin(3nt-n't-\omega-\omega')\\
&+ee'[3.2912145]\sin(nt-3n't+\omega+\omega')\\
&+ee'[2.3247465]\sin(3nt-3n't-\omega+\omega')\\
&+ee'[3.0786439]\sin(nt-n't+\omega-\omega')\\
&-ee'[0.5543247]\sin(5nt-3n't-\omega-\omega')\\
&+ee'[1.5275455]\sin(5nt-5n't-\omega+\omega')\\
&+ee'[2.8000400]\sin(3nt-5n't+\omega+\omega')\\
&-ee'[2.0827815]\sin(3nt-3n't+\omega-\omega')\\
&+\gamma^2[9.5918454]\sin(4nt-2n't-2\Omega)-\gamma^2[2.1755830]\sin 2(n't-\Omega)\\
&-\gamma^2[9.9003345]\sin(6nt-4n't-2\Omega)\\
&+\gamma^2[1.7973516]\sin(2nt-4n't+2\Omega)\\
&-\gamma^2[9.3330866]\sin(2nt-2n't+2\omega-2\Omega)\\
&+\gamma^2[9.3330866]\sin(2nt-2n't-2\omega+2\Omega)\\
&+\gamma^2[9.8197990]\sin(4nt-4n't+2\omega-2\Omega)\\
&-\gamma^2[9.8197990]\sin(4nt-4n't-2\omega+2\Omega)\\
&-\frac{a}{a'}[3.2099920]\sin(nt-n't)-\frac{a}{a'}[1.6016849]\sin 3(nt-n't)\\
&\qquad\qquad\qquad\qquad+\frac{a}{a'}[9.6780943]\sin 5(nt-n't)\Big\}
\end{aligned}\right\}. \quad (400)$$

In this value of $\delta^2 v$ the numbers in brackets are logarithms.

Equation (395) gives

$$\left.\begin{aligned}
&\delta\frac{d\delta_1\theta}{dt}\ {\scriptstyle\text{1st term of}}=\frac{\overline{m}^4}{\mu^2}n\Big\{-\gamma\,(2.223699)\cos(nt-\Omega)\\
&-\gamma\,(1.440670)\cos(3nt-2n't-\Omega)-\gamma\,(1.440670)\cos(nt-2n't+\Omega)\\
&+\gamma\,(0.6431830)\cos(5nt-4n't-\Omega)\\
&+\gamma\,(0.6431830)\cos(3nt-4n't+\Omega)-e\gamma\,(22.57900)\cos(2nt-\omega-\Omega)\\
&-e\gamma\,(18.13660)\cos(\omega-\Omega)-e\gamma\,(5.247615)\cos(4nt-2n't-\omega-\Omega)\\
&-e\gamma\,(2.366275)\cos(2nt-2n't-\omega+\Omega)\\
&+e\gamma\,(132.6830)\cos(2nt-2n't+\omega-\Omega)\\
&+e\gamma\,(129.8016)\cos(2n't-\omega-\Omega)\\
&+e\gamma\,(3.215619)\cos(6nt-4n't-\omega-\Omega)
\end{aligned}\right\}. \quad (401)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+ e\gamma\,(1.929252)\cos(4nt-4n't-\omega+\Omega)\\
&+ e\gamma\,(46.36160)\cos(4nt-4n't+\omega-\Omega)\\
&+ e\gamma\,(47.64796)\cos(2nt-4n't+\omega+\Omega)\\
&+ e'\gamma\,(45.57342)\cos(nt+n't-\omega'-\Omega)\\
&+ e'\gamma\,(45.57342)\cos(nt-n't+\omega'-\Omega)\\
&- e'\gamma\,(47.55891)\cos(3nt-n't-\omega'-\Omega)\\
&- e'\gamma\,(47.55891)\cos(nt-n't-\omega'+\Omega)\\
&+ e'\gamma\,(50.39981)\cos(3nt-3n't+\omega'-\Omega)\\
&+ e'\gamma\,(50.39981)\cos(nt-3n't+\omega'+\Omega)\\
&- e'\gamma\,(0.5595643)\cos(5nt-3n't-\omega'-\Omega)\\
&- e'\gamma\,(0.5595643)\cos(3nt-3n't-\omega'+\Omega)\\
&+ e'\gamma\,(5.825714)\cos(5nt-5n't+\omega'-\Omega)\\
&+ e'\gamma\,(5.825714)\cos(3nt-5n't+\omega'+\Omega)\Big\}
\end{aligned}\right\}. \quad (401)
$$

Equation (400) gives

$$
\left.\begin{aligned}
\delta\frac{d\delta_2\theta}{dt} = \frac{\overline{m}'^4}{\mu^2}\,n\Big\{&-\gamma\,(0.4306294)\cos(3nt-2n't-\Omega)\\
&+\gamma\,(0.4306294)\cos(nt-2n't+\Omega)+\gamma\,(0.680543)\cos(5nt-4n't-\Omega)\\
&-\gamma\,(0.680543)\cos(3nt-4n't+\Omega)\\
&-e\gamma\,(24.87285)\cos(2nt-\omega-\Omega)+e\gamma\,(24.87285)\cos(\omega-\Omega)\\
&-e\gamma\,(1.929658)\cos(4nt-2n't-\omega-\Omega)\\
&+e\gamma\,(1.068399)\cos(2nt-2n't-\omega+\Omega)\\
&+e\gamma\,(172.1108)\cos(2nt-2n't+\omega-\Omega)\\
&-e\gamma\,(171.2495)\cos(2n't-\omega-\Omega)\\
&+e\gamma\,(3.413863)\cos(6nt-4n't-\omega-\Omega)\\
&-e\gamma\,(2.052777)\cos(4nt-4n't-\omega+\Omega)\\
&+e\gamma\,(46.24655)\cos(4nt-4n't+\omega-\Omega)\\
&-e\gamma\,(47.60764)\cos(2nt-4n't+\omega+\Omega)\\
&+e'\gamma\,(554.4565)\cos(nt+n't-\omega'-\Omega)\\
&-e'\gamma\,(554.4565)\cos(nt-n't+\omega'-\Omega)\\
&-e'\gamma\,(34.469685)\cos(3nt-n't-\omega'-\Omega)\\
&+e'\gamma\,(34.469685)\cos(nt-n't-\omega'+\Omega)\\
&+e'\gamma\,(37.80811)\cos(3nt-3n't+\omega'-\Omega)\\
&-e'\gamma\,(37.80811)\cos(nt-3n't+\omega'+\Omega)\\
&-e'\gamma\,(0.5905185)\cos(5nt-3n't-\omega'-\Omega)
\end{aligned}\right\}. \quad (402)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ e'\gamma\,(0.5905185) \cos (3nt - 3n't - \omega' + \Omega) \\
&+ e'\gamma\,(5.715500) \cos (5nt - 5n't + \omega' - \Omega) \\
&- e'\gamma\,(5.715500) \cos (3nt - 5n't + \omega' + \Omega) \Big\}
\end{aligned}\right\} . \quad (402)$$

If we now take the sum of equations (389), (390), (392), (401), and (402), we shall obtain the following equation, which contains the whole of the indirect perturbation arising from the square of the sun's disturbing force:

$$\left.\begin{aligned}
&\delta\frac{d\delta_1\theta}{dt} + \delta\frac{d\delta_2\theta}{dt} = \\
&\frac{\overline{m}'^4}{\mu^2} n \Big\{ - \gamma\,(0.490327) \cos (nt - \Omega) - \gamma\,(0.922487) \cos (3nt - 2n't - \Omega) \\
&- \gamma\,(0.671183) \cos (nt - 2n't + \Omega) \\
&+ \gamma\,(4.163976) \cos (5nt - 4n't - \Omega) \\
&+ \gamma\,(0.446630) \cos (3nt - 4n't + \Omega) - e\gamma\,(31.8475) \cos (2nt - \omega - \Omega) \\
&+ e\gamma\,(51.8896) \cos (\omega - \Omega) - e\gamma\,(6.664839) \cos (4nt - 2n't - \omega - \Omega) \\
&- e\gamma\,(1.344303) \cos (2nt - 2n't - \omega + \Omega) \\
&+ e\gamma\,(329.7053) \cos (2nt - 2n't + \omega - \Omega) \\
&- e\gamma\,(38.0502) \cos (2n't - \omega - \Omega) \\
&+ e\gamma\,(18.493041) \cos (6nt - 4n't - \omega - \Omega) \\
&- e\gamma\,(1.149627) \cos (4nt - 4n't - \omega + \Omega) \\
&+ e\gamma\,(251.50511) \cos (4nt - 4n't + \omega - \Omega) \\
&+ e\gamma\,(18.58975) \cos (2nt - 4n't + \omega + \Omega) \\
&+ e'\gamma\,(598.3264) \cos (nt + n't - \omega' - \Omega) \\
&- e'\gamma\,(496.9842) \cos (nt - n't + \omega' - \Omega) \\
&- e'\gamma\,(131.15386) \cos (3nt - n't - \omega' - \Omega) \\
&- e'\gamma\,(7.25708) \cos (nt - n't - \omega' + \Omega) \\
&+ e'\gamma\,(132.17577) \cos (3nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma\,(4.88347) \cos (nt - 3n't + \omega' + \Omega) \\
&- e'\gamma\,(3.627115) \cos (5nt - 3n't - \omega' - \Omega) \\
&- e'\gamma\,(0.396404) \cos (3nt - 3n't - \omega' + \Omega) \\
&+ e'\gamma\,(25.55414) \cos (5nt - 5n't + \omega' - \Omega) \\
&- e'\gamma\,(5.03168) \cos (3nt - 5n't + \omega' + \Omega) \Big\}
\end{aligned}\right\} . \quad (403)$$

If we now take the sum of equations (360) and (403), we shall obtain the complete value of $\delta\,\frac{d\delta\theta}{dt}$, arising from the square of the disturbing force, as follows:

$$\left.\begin{aligned}
\delta\frac{d\delta\theta}{dt} = \frac{\overline{m}^4}{\mu^2} n \Big\{ &+ \gamma\,(1.312135)\cos(nt - \Omega) \\
&- \gamma\,(0.911915)\cos(3nt - 2n't - \Omega) - \gamma\,(36.37001)\cos(nt - 2n't + \Omega) \\
&+ \gamma\,(5.166611)\cos(5nt - 4n't - \Omega) \\
&+ \gamma\,(14.44615)\cos(3nt - 4n't + \Omega) \\
&- e\gamma\,(95.1472)\cos(2nt - \omega - \Omega) + e\gamma\,(3.3890)\cos(\omega - \Omega) \\
&- e\gamma\,(8.926955)\cos(4nt - 2n't - \omega - \Omega) \\
&+ e\gamma\,(303.7977)\cos(2nt - 2n't + \omega - \Omega) \\
&- e\gamma\,(94.11332)\cos(2nt - 2n't - \omega + \Omega) \\
&- e\gamma\,(67.9762)\cos(2n't - \omega - \Omega) \\
&+ e\gamma\,(22.65678)\cos(6nt - 4n't - \omega - \Omega) \\
&+ e\gamma\,(281.49246)\cos(4nt - 4n't + \omega - \Omega) \\
&+ e\gamma\,(47.50768)\cos(4nt - 4n't - \omega + \Omega) \\
&+ e\gamma\,(345.6460)\cos(2nt - 4n't + \omega + \Omega) \\
&+ e'\gamma\,(554.0630)\cos(nt + n't - \omega' - \Omega) \\
&- e'\gamma\,(126.1295)\cos(nt - n't + \omega' - \Omega) \\
&- e'\gamma\,(116.70862)\cos(3nt - n't - \omega' - \Omega) \\
&- e'\gamma\,(333.0181)\cos(nt - n't - \omega' + \Omega) \\
&+ e'\gamma\,(121.94012)\cos(3nt - 3n't + \omega' - \Omega) \\
&+ e'\gamma\,(53.4166)\cos(nt - 3n't + \omega' + \Omega) \\
&- e'\gamma\,(4.543364)\cos(5nt - 3n't - \omega' - \Omega) \\
&- e'\gamma\,(20.10365)\cos(3nt - 3n't - \omega' + \Omega) \\
&+ e'\gamma\,(33.30601)\cos(5nt - 5n't + \omega' - \Omega) \\
&+ e'\gamma\,(84.84277)\cos(3nt - 5n't + \omega' + \Omega) \Big\}
\end{aligned}\right\} . \quad (404)$$

This equation gives by integration

$$\left.\begin{aligned}
\delta^2\theta = \frac{\overline{m}^4}{\mu^2} \Big\{ &+ \gamma\,[0.1179785]\sin(nt - \Omega) \\
&- \gamma\,[9.5050490]\sin(3nt - 2n't - \Omega) \\
&- \gamma\,[1.6311214]\sin(nt - 2n't + \Omega) \\
&+ \gamma\,[0.0410344]\sin(5nt - 4n't - \Omega)
\end{aligned}\right\} . \quad (405)$$

(Continued on the next page.)

$$
\left.\begin{array}{l}
+\gamma\,[0.7282605]\sin(3nt-4n't+\Omega)-e\gamma\,[1.6773660]\sin(2nt-\omega-\Omega) \\
-e\gamma\,[0.3651978]\sin(4nt-2n't-\omega-\Omega) \\
+e\gamma\,[2.2153195]\sin(2nt-2n't+\omega-\Omega) \\
-e\gamma\,[1.7063861]\sin(2nt-2n't-\omega+\Omega) \\
-e\gamma\,[2.6574178]\sin(2n't-\omega-\Omega) \\
+e\gamma\,[0.5992628]\sin(6nt-4n't-\omega-\Omega) \\
+e\gamma\,[1.8811719]\sin(4nt-4n't+\omega-\Omega) \\
+e\gamma\,[1.1084698]\sin(4nt-4n't-\omega+\Omega) \\
+e\gamma\,[2.3079797]\sin(2nt-4n't+\omega+\Omega) \\
+e'\gamma\,[2.7122309]\sin(nt+n't-\omega'-\Omega) \\
-e'\gamma\,[2.1345815]\sin(nt-n't+\omega'-\Omega) \\
-e'\gamma\,[1.6009475]\sin(3nt-n't-\omega'-\Omega) \\
-e'\gamma\,[2.5562328]\sin(nt-n't-\omega'+\Omega) \\
+e'\gamma\,[1.6427903]\sin(3nt-3n't+\omega'-\Omega) \\
+e'\gamma\,[1.8380407]\sin(nt-3n't+\omega'+\Omega) \\
-e'\gamma\,[9.9783500]\sin(5nt-3n't-\omega'-\Omega) \\
-e'\gamma\,[0.8599186]\sin(3nt-3n't-\omega'+\Omega) \\
+e'\gamma\,[0.8573176]\sin(5nt-5n't+\omega'-\Omega) \\
+e'\gamma\,[0.5093212]\sin(3nt-5n't+\omega'+\Omega)\Big\}
\end{array}\right\} . \quad (405)
$$

In this value of $\delta^2\theta$ the numbers in brackets are logarithms.

We have thus fully determined the perturbations of the co-ordinates arising from the square of the disturbing force.

CHAPTER V.

PERTURBATIONS ARISING FROM THE CUBE OF THE DISTURBING FORCE.

34. We shall now compute the perturbations of the radius vector and longitude arising from the cube of the disturbing force. In this investigation we shall neglect the eccentricity and inclination of the moon's orbit, since these quantities would produce inequalities in the moon's longitude which may be neglected on account of their extreme smallness. We shall therefore neglect the force $\left(\frac{dR}{d\theta}\right)$; and the second variations of the forces $\left(\frac{dR}{dr}\right)$ and $\left(\frac{dR}{dv}\right)$ may be determined by the following equations:

$$\left.\begin{aligned}
\delta^2\left(\frac{dR}{dr}\right) = \frac{\overline{m}^2}{a^3}\Big\{&-\tfrac{1}{2} - \tfrac{3}{2}\cos 2(nt-n't) - \tfrac{9}{4}\frac{a}{a'}\cos(nt-n't) \\
&\qquad - \tfrac{15}{4}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta^2 r \\
+ \frac{\overline{m}^2}{a^2}\Big\{&+3\sin 2(nt-n't) + \tfrac{9}{8}\frac{a}{a'}\sin(nt-n't) \\
&\qquad + \tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta^2 v \\
+ \frac{\overline{m}^2}{a^4}\Big\{&-\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't) - \tfrac{15}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta r^2 \\
+ \frac{\overline{m}^2}{a^2}\Big\{&+3\cos 2(nt-n't) + \tfrac{9}{16}\frac{a}{a'}\cos(nt-n't) \\
&\qquad + \tfrac{135}{16}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta v^2 \\
+ \frac{\overline{m}^2}{a^3}\Big\{&+3\sin 2(nt-n't) + \tfrac{9}{4}\frac{a}{a'}\sin(nt-n't). \\
&\qquad + \tfrac{45}{4}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta r\,\delta v
\end{aligned}\right\} \quad (406)$$

$$\left.\begin{aligned}
\delta^2\left(\frac{dR}{dv}\right) = \frac{\overline{m}^2}{a^2}\{&+3\sin 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't) \\
&\qquad\qquad+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\}\,\delta^2 r \\
+\frac{\overline{m}^2}{a}\{&+3\cos 2(nt-n't)+\tfrac{3}{8}\frac{a}{a'}\cos(nt-n't) \\
&\qquad\qquad+\tfrac{45}{8}\frac{a}{a'}\cos 3(nt-n't)\}\,\delta^2 v \\
+\frac{\overline{m}^2}{a^3}\{&+\tfrac{3}{2}\sin 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't) \\
&\qquad\qquad+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\}\,\delta r^2 \\
+\frac{\overline{m}^2}{a}\{&-3\sin 2(nt-n't)-\tfrac{3}{16}\frac{a}{a'}\sin(nt-n't) \\
&\qquad\qquad-\tfrac{135}{16}\frac{a}{a'}\sin 3(nt-n't)\}\,\delta v^2 \\
+\frac{\overline{m}^2}{a^2}\{&+6\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't) \\
&\qquad\qquad+\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\}\,\delta r\delta v
\end{aligned}\right\} \quad . \quad (407)$$

We shall also have the following equations:

$$\delta^2(c_1\cos\beta) = \mp\sin nt\,\delta^2 v - \tfrac{1}{2}\cos nt\,\delta v^2 \quad \Big\} \quad . \quad (408)$$

$$\left.\begin{aligned}
\delta^2(c_2\cos\beta) = &+a^2 ndt\sin nt\,\{\delta^2 v+\frac{1}{ndt}\delta v d\delta v+\frac{2}{a}\delta r\delta v\} \\
&\pm a^2 ndt\cos nt\,\{\tfrac{1}{2}\delta v^2-\frac{1}{ndt}d\delta^2 v-\frac{2}{a}\delta^2 r-\frac{2}{andt}\delta r d\delta v-\frac{1}{a^2}\delta r^2\}
\end{aligned}\right\} \quad . \quad (409)$$

$$\delta^2(c_3\sin\beta) =$$

$$\left.\begin{aligned}
&2andt\sin nt\,\{\frac{1}{ndt}d\delta^2 v+\frac{1}{a}\delta^2 r-\tfrac{1}{2}\delta v^2+\frac{1}{andt}\delta r d\delta v+\frac{1}{2andt}d\delta r\delta v\} \\
&\qquad\pm 2andt\cos nt\,\{\delta^2 v-\frac{1}{2andt}d\delta^2 r+\frac{1}{ndt}\delta v d\delta v+\frac{1}{a}\delta r\delta v\}
\end{aligned}\right\} \quad . \quad (410)$$

In the preceding chapters we have obtained the following values:

$$\left.\begin{aligned}\delta r = a\frac{\overline{m}^2}{\mu}\Big\{ &-\tfrac{1}{8} - (1.2876697)\cos 2(nt-n't) \\ &+\frac{a}{a'}(13.441234)\cos(nt-n't) - \frac{a}{a'}(0.48121913)\cos 3(nt-n't)\Big\}\end{aligned}\right\}. \quad (411)$$

$$\left.\begin{aligned}\delta v = \frac{\overline{m}^2}{\mu}\Big\{ &+(1.829864)\sin 2(nt-n't) - \frac{a}{a'}(28.61780)\sin(nt-n't) \\ &+\frac{a}{a'}(0.5901323)\sin 3(nt-n't)\Big\}\end{aligned}\right\}. \quad (412)$$

$$\left.\begin{aligned}d\delta r = a\frac{\overline{m}^2}{\mu}ndt\Big\{ &+(2.382700)\sin 2(nt-n't) \\ &-\frac{a}{a'}(12.43582)\sin(nt-n't) + \frac{a}{a'}(1.335670)\sin 3(nt-n't)\Big\}\end{aligned}\right\}. \quad (413)$$

$$\left.\begin{aligned}d\delta v = \frac{\overline{m}^2}{\mu}ndt\Big\{ &+(3.385976)\cos 2(nt-n't) \\ &-\frac{a}{a'}(26.47716)\cos(nt-n't) + \frac{a}{a'}(1.637969)\cos 3(nt-n't)\Big\}\end{aligned}\right\}. \quad (414)$$

$$\left.\begin{aligned}d\delta^2 r = a\frac{\overline{m}^4}{\mu^2}ndt\Big\{ &-(2.665812)\sin 2(nt-n't) \\ &+(2.380288)\sin 4(nt-n't) - \frac{a}{a'}(689.2248)\sin(nt-n't) \\ &-\frac{a}{a'}(56.11517)\sin 3(nt-n't) + \frac{a}{a'}(0.857261)\sin 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (415)$$

$$\left.\begin{aligned}d\delta^2 v = \frac{\overline{m}^4}{\mu^2}ndt\Big\{ &-(1.593671)\cos 2(nt-n't) \\ &+(5.037100)\cos 4(nt-n't) - \frac{a}{a'}(1500.4688)\cos(nt-n't) \\ &-\frac{a}{a'}(110.92803)\cos 3(nt-n't) + \frac{a}{a'}(2.204445)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (416)$$

$$\left.\begin{aligned}\delta^2 r = a\frac{\overline{m}^4}{\mu^2}\Big\{ &+ (1.417405) + (1.440670)\cos 2(nt-n't) \\ &- (0.6431830)\cos 4(nt-n't) + \frac{a}{a'}(743.8670)\cos(nt-n't) \\ &+ \frac{a}{a'}(20.21734)\cos 3(nt-n't) - \frac{a}{a'}(0.1853139)\cos 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (417)$$

$$\left.\begin{aligned}\delta^2 v = \frac{\overline{m}^4}{\mu^2}\Big\{ &- (0.8612588)\sin 2(nt-n't) + (1.361086)\sin 4(nt-n't) \\ &- \frac{a}{a'}(1621.780)\sin(nt-n't) - \frac{a}{a'}(39.96547)\sin 3(nt-n't) \\ &+ \frac{a}{a'}(0.4765345)\sin 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (418)$$

These equations will give the following:

$$\left.\begin{aligned}\delta r^2 = a^2\frac{\overline{m}^4}{\mu^2}\Big\{ &+ (0.8568242) + (0.4292231)\cos 2(nt-n't) \\ &+ (0.8290464)\cos 4(nt-n't) - \frac{a}{a'}(21.16864)\cos(nt-n't) \\ &- \frac{a}{a'}(17.14747)\cos 3(nt-n't) + \frac{a}{a'}(0.6196511)\cos 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (419)$$

$$\left.\begin{aligned}\delta v^2 = \frac{\overline{m}^4}{\mu^2}\Big\{ &+ (1.674202) - (1.674202)\cos 4(nt-n't) \\ &+ \frac{a}{a'}(51.28686)\cos(nt-n't) + \frac{a}{a'}(52.36672)\cos 3(nt-n't) \\ &- \frac{a}{a'}(1.0798625)\cos 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (420)$$

$$\left.\begin{aligned}\delta r\delta v = a\frac{\overline{m}^4}{\mu^2}\Big\{ &- (0.3049774)\sin 2(nt-n't) - (1.178130)\sin 4(nt-n't) \\ &- \frac{a}{a'}(1.29735)\sin(nt-n't) + \frac{a}{a'}(30.62461)\sin 3(nt-n't) \\ &- \frac{a}{a'}(0.8202306)\sin 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (421)$$

$$\left.\begin{aligned}\delta v d\delta r = a\frac{\overline{m}^4}{\mu^2}ndt\Big\{&+(2.180009)-(2.180009)\cos 4(nt-n't)\\&-\frac{a}{a'}(43.54667)\cos(nt-n't)+\frac{a}{a'}(45.47177)\cos 3(nt-n't)\\&-\frac{a}{a'}(1.925102)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (422)$$

$$\left.\begin{aligned}\delta r d\delta v = a\frac{\overline{m}^4}{\mu^2}ndt\Big\{&-(2.180009)-(0.5643293)\cos 2(nt-n't)\\&-(2.180009)\cos 4(nt-n't)+\frac{a}{a'}(42.34636)\cos(nt-n't)\\&+\frac{a}{a'}(39.52979)\cos 3(nt-n't)-\frac{a}{a'}(1.869279)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (423)$$

$$\left.\begin{aligned}\delta v d\delta v = \frac{\overline{m}^4}{\mu^2}ndt\Big\{&+(3.097939)\sin 4(nt-n't)\\&+\frac{a}{a'}(23.72527)\sin(nt-n't)-\frac{a}{a'}(72.67443)\sin 3(nt-n't)\\&+\frac{a}{a'}(2.497718)\sin 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (424)$$

$$\left.\begin{aligned}\delta r^3 = a^3\frac{\overline{m}^6}{\mu^3}\Big\{&-(0.4191527)-(1.708612)\cos 2(nt-n't)\\&-(0.4145231)\cos 4(nt-n't)-(0.5337690)\cos 6(nt-n't)\\&+\frac{a}{a'}(42.29603)\cos(nt-n't)+\frac{a}{a'}(24.13211)\cos 3(nt-n't)\\&+\frac{a}{a'}(16.40530)\cos 5(nt-n't)-\frac{a}{a'}(0.5984295)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (425)$$

$$\left.\begin{aligned}\delta r\delta^2 r = a^3\frac{\overline{m}^6}{\mu^3}\Big\{&-(1.163788)-(1.651156)\cos 2(nt-n't)\\&-(0.8203565)\cos 4(nt-n't)+(0.4141036)\cos 6(nt-n't)\\&-\frac{a}{a'}(587.3794)\cos(nt-n't)-\frac{a}{a'}(477.5001)\cos 3(nt-n't)\\&-\frac{a}{a'}(17.71644)\cos 5(nt-n't)+\frac{a}{a'}(0.2740676)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (426)$$

If we now substitute equations (411–424) in equations (406–410), we shall obtain the following equations:

$$\left.\begin{aligned}\delta^2\left(\frac{dR}{dr}\right)=\frac{\overline{m}^6}{a^2\mu^2}\Big\{&-(3.538560)+(0.421682)\cos 2(nt-n't)\\&+(0.990442)\cos 4(nt-n't)-(2.303350)\cos 6(nt-n't)\\&-\frac{a}{a'}(3414.576)\cos(nt-n't)+\frac{a}{a'}(1793.077)\cos 3(nt-n't)\\&+\frac{a}{a'}(78.87132)\cos 5(nt-n't)-\frac{a}{a'}(4.80064)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (427)$$

$$\left.\begin{aligned}\delta^2\left(\frac{dR}{dv}\right)=\frac{\overline{m}^6}{a\mu^2}\Big\{&-(3.146230)\sin 2(nt-n't)\\&+(0.276102)\sin 4(nt-n't)+(0.675553)\sin 6(nt-n't)\\&+\frac{a}{a'}(3702.834)\sin(nt-n't)-\frac{a}{a'}(1264.598)\sin 3(nt-n't)\\&-\frac{a}{a'}(29.04038)\sin 5(nt-n't)+\frac{a}{a'}(1.524035)\sin 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (428)$$

$$\left.\begin{aligned}\delta^3(c_1\cos\beta)=\frac{\overline{m}^4}{\mu^2}\Big\{&-(0.837101)\cos nt-(0.4306294)\cos(3nt-2n't)\\&\pm(0.4306294)\cos(nt-2n't)+(1.099093)\cos(5nt-4n't)\\&\mp(0.261993)\cos(3nt-4n't)-\frac{a}{a'}(798.068)\cos(2nt-n't)\\&+\frac{a}{a'}(823.712)\cos n't-\frac{a}{a'}(33.07441)\cos(4nt-3n't)\\&\pm\frac{a}{a'}(6.89105)\cos(2nt-3n't)+\frac{a}{a'}(0.5082328)\cos(6nt-5n't)\\&\pm\frac{a}{a'}(0.0316984)\cos(4nt-5n't)\Big\}\end{aligned}\right\}. \quad (429)$$

$$\left.\begin{aligned}\delta^3(c_2\cos\beta)=a^2\frac{\overline{m}^4}{\mu^2}ndt\Big\{&\pm(1.505485)\cos nt\\&\pm(0.441489)\cos(3nt-2n't)-(1.029724)\cos(nt-2n't)\\&\mp(1.579813)\cos(5nt-4n't)+(0.522951)\cos(3nt-4n't)\\&\pm\frac{a}{a'}(762.107)\cos(2nt-n't)\mp\frac{a}{a'}(838.541)\cos n't\end{aligned}\right\}. \quad (430)$$

(Continued on the next page.)

$$\left.\begin{array}{l} \pm \frac{a}{a'}(43.07762)\cos(4nt-3n't) - \frac{a}{a'}(8.31305)\cos(2nt-3n't) \\ \mp \frac{a}{a'}(0.294315)\cos(6nt-5n't) + \frac{a}{a'}(1.039476)\cos(4nt-5n't) \Big\} \end{array}\right\} . \quad (430)$$

$$\left.\begin{array}{l} \delta^2(c_3\sin\beta) = a\frac{\overline{m}'^4}{\mu^2}ndt\Big\{ -(1.019401)\sin nt - (0.550660)\sin(3nt-2n't) \\ \pm (0.884000)\sin(nt-2n't) + (4.051756)\sin(5nt-4n't) \\ \pm (0.129746)\sin(3nt-4n't) - \frac{a}{a'}(1965.126)\sin(2nt-n't) \\ + \frac{a}{a'}(544.354)\sin n't - \frac{a}{a'}(108.58609)\sin(4nt-3n't) \\ \pm \frac{a}{a'}(0.67067)\sin(2nt-3n't) + \frac{a}{a'}(1.452624)\sin(6nt-5n't) \\ \pm \frac{a}{a'}(1.998159)\sin(4nt-5n't) \Big\} \end{array}\right\} . \quad (431)$$

35. The value of $\delta^2 \frac{d\delta_1 r}{dt}$ is given by the equation

$$\left.\begin{array}{l} \sqrt{a\mu}\,\delta^2\frac{d\delta_1 r}{dt} = \\ c_1\cos\beta\int\Big\{ c_2\cos\beta\,\delta^2\left(\frac{dR}{dr}\right) + \delta(c_2\cos\beta)\,\delta\left(\frac{dR}{dr}\right) + \delta^2(c_2\cos\beta)\left(\frac{dR}{dr}\right) \\ \quad + c_3\sin\beta\,\delta^2\left(\frac{dR}{dv}\right) + \delta(c_3\sin\beta)\,\delta\left(\frac{dR}{dv}\right) + \delta^2(c_3\sin\beta)\left(\frac{dR}{dv}\right)\Big\} \\ - c_1\sin\beta\int\Big\{ c_2\sin\beta\,\delta^2\left(\frac{dR}{dr}\right) + \delta(c_2\sin\beta)\,\delta\left(\frac{dR}{dr}\right) + \delta^2(c_2\sin\beta)\left(\frac{dR}{dr}\right) \\ \quad + c_3\cos\beta\,\delta^2\left(\frac{dR}{dv}\right) + \delta(c_3\cos\beta)\,\delta\left(\frac{dR}{dv}\right) + \delta^2(c_3\cos\beta)\left(\frac{dR}{dv}\right)\Big\} \\ + \delta(c_1\cos\beta)\int\Big\{ c_2\cos\beta\,\delta\left(\frac{dR}{dr}\right) + \delta(c_2\cos\beta)\left(\frac{dR}{dr}\right) + c_3\sin\beta\,\delta\left(\frac{dR}{dv}\right) \\ \quad + \delta(c_3\sin\beta)\left(\frac{dR}{dv}\right)\Big\} \\ - \delta(c_1\sin\beta)\int\Big\{ c_2\sin\beta\,\delta\left(\frac{dR}{dr}\right) + \delta(c_2\sin\beta)\left(\frac{dR}{dr}\right) + c_3\cos\beta\,\delta\left(\frac{dR}{dv}\right) \\ \quad + \delta(c_3\sin\beta)\left(\frac{dR}{dv}\right)\Big\} \end{array}\right\} . \quad (432)$$

(Continued on the next page.)

$$\left.\begin{array}{l} + \delta^2 (c_1 \cos\beta) \int \left\{ c_2 \cos\beta \left(\dfrac{dR}{dr}\right) + c_3 \sin\beta \left(\dfrac{dR}{dv}\right) \right\} \\ \qquad - \delta^2 (c_1 \sin\beta) \int \left\{ c_2 \sin\beta \left(\dfrac{dR}{dr}\right) + c_3 \cos\beta \left(\dfrac{dR}{dv}\right) \right\} \end{array}\right\} . \quad (432)$$

We shall now compute the values of the different terms of this equation. The value of $c_2 \cos\beta$ is given by equation (228); and if we multiply equations (228) and (427) together, we shall obtain

$$c_2 \cos\beta\, \delta^2 \left(\frac{dR}{dr}\right) =$$

$$\left.\begin{array}{l} \dfrac{\overline{m}^6}{\mu^2} ndt \left\{ \pm (3.538560) \cos nt \mp (0.210841) \cos (3nt - 2n't) \right. \\ \quad - (0.210841) \cos (nt - 2n't) \mp (0.495221) \cos (5nt - 4n't) \\ \quad - (0.495221) \cos (3nt - 4n't) \pm (1.151675) \cos (7nt - 6n't) \\ \quad + (1.151675) \cos (5nt - 6n't) \pm \dfrac{a}{a'} (1707.288) \cos (2nt - n't) \\ \quad \pm \dfrac{a}{a'} (1707.288) \cos n't \mp \dfrac{a}{a'} (896.5385) \cos (4nt - 3n't) \\ \quad - \dfrac{a}{a'} (896.5385) \cos (2nt - 3n't) \mp \dfrac{a}{a'} (39.43566) \cos (6nt - 5n't) \\ \quad - \dfrac{a}{a'} (39.43566) \cos (4nt - 5n't) \pm \dfrac{a}{a'} (2.40032) \cos (8nt - 7n't) \\ \qquad\qquad \left. + \dfrac{a}{a'} (2.40032) \cos (6nt - 7n't) \right\} \end{array}\right\} . \quad (433)$$

Equations (327) and (331) will give

$$\delta (c_2 \cos\beta)\, \delta \left(\frac{dR}{dr}\right) =$$

$$\left.\begin{array}{l} \dfrac{\overline{m}^6}{\mu^2} ndt \left\{ \pm (0.902328) \cos nt \mp (5.313215) \cos (3nt - 2n't) \right. \\ \quad + (3.256776) \cos (nt - 2n't) \mp (0.8865503) \cos (5nt - 4n't) \\ \quad - (0.0687524) \cos (3nt - 4n't) \pm (1.174392) \cos (7nt - 6n't) \\ \quad - (0.4533118) \cos (5nt - 6n't) \pm \dfrac{a}{a'} (79.20381) \cos (2nt - n't) \\ \quad \mp \dfrac{a}{a'} (90.99561) \cos n't \pm \dfrac{a}{a'} (53.40131) \cos (4nt - 3n't) \end{array}\right\} . \quad (434)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&-\frac{a}{a'}(23.60514)\cos(2nt-3n't) \mp \frac{a}{a'}(35.90393)\cos(6nt-5n't) \\
&+\frac{a}{a'}(21.04292)\cos(4nt-5n't) \pm \frac{a}{a'}(2.712546)\cos(8nt-7n't) \\
&\qquad\qquad -\frac{a}{a'}(0.7917688)\cos(6nt-7n't)\Big\}
\end{aligned}\right\}. \quad (434)$$

In like manner equations (218) and (430) will give

$$\delta^2(c_2\cos\beta)\left(\frac{dR}{dr}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^6}{\mu^2}ndt\Big\{\mp(0.311566)\cos nt \mp (0.164998)\cos(3nt-2n't) \\
&-(1.006465)\cos(nt-2n't) \pm (0.458790)\cos(5nt-4n't) \\
&+(0.510818)\cos(3nt-4n't) + (1.184860)\cos(7nt-6n't) \\
&-(0.392213)\cos(5nt-6n't) \pm \frac{a}{a'}(213.934)\cos(2nt-n't) \\
&\mp\frac{a}{a'}(147.247)\cos n't \mp \frac{a}{a'}(595.466)\cos(4nt-3n't) \\
&+\frac{a}{a'}(631.157)\cos(2nt-3n't) \mp \frac{a}{a'}(33.46360)\cos(6nt-5n't) \\
&+\frac{a}{a'}(6.38626)\cos(4nt-5n't) \mp \frac{a}{a'}(1.260339)\cos(8nt-7n't) \\
&\qquad\qquad -\frac{a}{a'}(1.269874)\cos(6nt-7n't)\Big\}
\end{aligned}\right\}. \quad (435)$$

We also obtain from the force $\left(\dfrac{dR}{dv}\right)$ and its variations,

$$c_3\sin\beta\,\delta^2\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^6}{\mu^2}ndt\Big\{\pm(3.146230)\cos(3nt-2n't) - (3.146230)\cos(nt-2n't) \\
&\mp(0.276102)\cos(5nt-4n't) + (0.276102)\cos(3nt-4n't) \\
&\mp(0.675553)\cos(7nt-6n't) + (0.675553)\cos(5nt-6n't) \\
&\mp\frac{a}{a'}(3702.834)\cos(2nt-n't) \pm \frac{a}{a'}(3702.834)\cos n't \\
&\pm\frac{a}{a'}(1264.598)\cos(4nt-3n't) - \frac{a}{a'}(1264.598)\cos(2nt-3n't)
\end{aligned}\right\}. \quad (436)$$

(Continued on the next page.)

$$\left.\begin{aligned}&\pm\frac{a}{a'}(29.04038)\cos(6nt-5n't)-\frac{a}{a'}(29.04038)\cos(4nt-5n't)\\&\mp\frac{a}{a'}(1.524035)\cos(8nt-7n't)+\frac{a}{a'}(1.524035)\cos(6nt-7n't)\Big\}\end{aligned}\right\}. \quad (436)$$

$$\delta(c_3\sin\beta)\,\delta\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{aligned}\frac{\overline{m}^6}{\mu^2}ndt\Big\{&\mp(0.319257)\cos nt\mp(0.6769530)\cos(3nt-2n't)\\&+(1.196252)\cos(nt-2n't)\pm(0.819754)\cos(5nt-4n't)\\&-(0.500497)\cos(3nt-4n't)\mp(1.112919)\cos(7nt-6n't)\\&+(0.5936197)\cos(5nt-6n't)\pm\frac{a}{a'}(106.35163)\cos(2nt-n't)\\&\mp\frac{a}{a'}(74.28136)\cos n't\mp\frac{a}{a'}(95.64895)\cos(4nt-3n't)\\&+\frac{a}{a'}(31.03693)\cos(2nt-3n't)\pm\frac{a}{a'}(47.91723)\cos(6nt-5n't)\\&-\frac{a}{a'}(14.36068)\cos(4nt-5n't)\mp\frac{a}{a'}(2.749070)\cos(8nt-7n't)\\&\qquad\qquad-\frac{a}{a'}(1.734264)\cos(6nt-7n't)\Big\}\end{aligned}\right\}. \quad (437)$$

$$\delta^2(c_3\sin\beta)\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{aligned}\frac{\overline{m}^6}{\mu^2}ndt\Big\{&\pm(0.250005)\cos nt\pm(3.803368)\cos(3nt-2n't)\\&-(0.667241)\cos(nt-2n't)\pm(0.412995)\cos(5nt-4n't)\\&-(0.663000)\cos(3nt-4n't)\mp(3.038817)\cos(7nt-6n't)\\&-(0.097310)\cos(5nt-6n't)\mp\frac{a}{a'}(484.990)\cos(2nt-n't)\\&\mp\frac{a}{a'}(1473.761)\cos n't\pm\frac{a}{a'}(1476.753)\cos(4nt-3n't)\\&+\frac{a}{a'}(408.667)\cos(2nt-3n't)\pm\frac{a}{a'}(81.19611)\cos(6nt-5n't)\\&-\frac{a}{a'}(1.35608)\cos(4nt-5n't)\mp\frac{a}{a'}(4.887989)\cos(8nt-7n't)\\&\qquad\qquad-\frac{a}{a'}(1.620256)\cos(6nt-7n't)\Big\}\end{aligned}\right\}. \quad (438)$$

If, for brevity, we now put the sum of equations (433–438) equal to $h \cos H$, we shall have

$$\left. \begin{aligned} h \cos H = \frac{\overline{m}^6}{\mu^2} ndt \Big\{ & \pm (4.060070) \cos nt \pm (0.583591) \cos (3nt - 2n't) \\ & - (0.577749) \cos (nt - 2n't) \pm (0.033666) \cos (5nt - 4n't) \\ & - (0.940550) \cos (3nt - 4n't) \mp (1.316362) \cos (7nt - 6n't) \\ & + (1.478013) \cos (5nt - 6n't) \mp \frac{a}{a'} (2081.046) \cos (2nt - n't) \\ & \pm \frac{a}{a'} (3623.837) \cos n't \pm \frac{a}{a'} (1207.099) \cos (4nt - 3n't) \\ & - \frac{a}{a'} (1113.880) \cos (2nt - 3n't) \pm \frac{a}{a'} (49.35053) \cos (6nt - 5n't) \\ & - \frac{a}{a'} (56.76362) \cos (4nt - 5n't) \mp \frac{a}{a'} (5.30856) \cos (8nt - 7n't) \\ & + \frac{a}{a'} (1.97671) \cos (6nt - 7n't) \Big\} \end{aligned} \right\} . \quad (439)$$

This gives by integration

$$\left. \begin{aligned} \int h \cos H = \frac{\overline{m}^6}{\mu^2} \Big\{ & + (4.060070) \sin nt + (0.2047402) \sin (3nt - 2n't) \\ & \mp (0.6793870) \sin (nt - 2n't) + (0.007161766) \sin (5nt - 4n't) \\ & \mp (0.3482494) \sin (3nt - 4n't) - (0.2009348) \sin (7nt - 6n't) \\ & \pm (0.3247529) \sin (5nt - 6n't) - \frac{a}{a'} (1080.951) \sin (2nt - n't) \\ & + \frac{a}{a'} (48446.18) \sin n't + \frac{a}{a'} (319.7108) \sin (4nt - 3n't) \\ & \mp \frac{a}{a'} (627.3276) \sin (2nt - 3n't) + \frac{a}{a'} (8.771878) \sin (6nt - 5n't) \\ & \mp \frac{a}{a'} (15.65464) \sin (4nt - 5n't) - \frac{a}{a'} (0.7100430) \sin (8nt - 7n't) \\ & \pm \frac{a}{a'} (0.3609512) \sin (6nt - 7n't) \Big\} \end{aligned} \right\} . \quad (440)$$

Equation (440) will give the value of $\int h \sin H$ by using the lower signs and changing *sin* to *cos* in the second member.

The first term of the value of $\delta^2 \frac{d\delta_1 r}{dt}$ in equation (432) is evidently equal to $\frac{1}{\sqrt{a\mu}} \left\{ c_1 \cos\beta \int h \cos H - c_1 \sin\beta \int h \sin H \right\}.$

Now by using the value of $c_1 \cos\beta$ and $c_1 \sin\beta$, given by equation (240), we shall obtain

$$\left.\begin{aligned}
&\text{1st term of}\\
&\delta^2 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^6}{\mu^3} n \Big\{ -(0.4746468) \sin 2(nt-n't)\\
&\quad -(0.3410876) \sin 4(nt-n't) + (0.1238181) \sin 6(nt-n't)\\
&\quad -\frac{a}{a'}(49527.13) \sin (nt-n't) - \frac{a}{a'}(307.6168) \sin 3(nt-n't)\\
&\quad -\frac{a}{a'}(6.88276) \sin 5(nt-n't) - \frac{a}{a'}(0.3490918) \sin 7(nt-n't) \Big\}
\end{aligned}\right\} . \quad (441)$$

The second term of $\delta^2 \frac{d\delta_1 r}{dt}$ is obtained from equations (330) and (342), as follows:

$$\left.\begin{aligned}
&\text{2nd term of}\\
&\delta^2 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^6}{\mu^3} n \Big\{ +(3.371945) \sin 2(nt-n't)\\
&\quad -(2.059793) \sin 4(nt-n't) + (1.010216) \sin 6(nt-n't)\\
&\quad -\frac{a}{a'}(679.7844) \sin (nt-n't) - \frac{a}{a'}(547.3829) \sin 3(nt-n't)\\
&\quad -\frac{a}{a'}(34.08278) \sin 5(nt-n't) + \frac{a}{a'}(1.468533) \sin 7(nt-n't) \Big\}
\end{aligned}\right\} . \quad (442)$$

The third term of $\delta^2 \frac{d\delta_1 r}{dt}$ is obtained from equations (272) and (429), as follows:

$$\left.\begin{aligned}
&\text{3rd term of}\\
&\delta^2 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^6}{\mu^3} n \Big\{ -(0.581550) \sin 2(nt-n't)\\
&\quad -(1.933221) \sin 4(nt-n't) + (2.976939) \sin 6(nt-n't)\\
&\quad +\frac{a}{a'}(3190.664) \sin (nt-n't) - \frac{a}{a'}(2406.833) \sin 5(nt-n't)\\
&\quad -\frac{a}{a'}(104.15013) \sin 5(nt-n't) + \frac{a}{a'}(3.142347) \sin 7(nt-n't) \Big\}
\end{aligned}\right\} . \quad (443)$$

The sum of equations (441–443) is the complete value of $\delta^2 \frac{d\delta_1 r}{dt}$. Therefore we shall have

$$\left.\begin{aligned}\delta^2 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^6}{\mu^3} n \Big\{ &+ (2.315748)\sin 2(nt-n't) \\ &- (4.334102)\sin 4(nt-n't) + (4.110973)\sin 6(nt-n't) \\ &- \frac{a}{a'}(47016.25)\sin(nt-n't) - \frac{a}{a'}(3261.833)\sin 3(nt-n't) \\ &- \frac{a}{a'}(145.1157)\sin 5(nt-n't) + \frac{a}{a'}(4.261788)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (444)$$

We must now find the value of $\delta^2 \frac{d\delta_4 r}{dt}$. It is evident by development of equation (252‴) that we shall have

$$\left.\begin{aligned}\delta^2 \frac{d\delta_4 r}{dt} = \frac{1}{\sqrt{a\mu}}\,\delta\frac{d\delta_1 r}{dt}\int\left(\frac{dR}{dv}\right)dt \\ + \frac{1}{\sqrt{a\mu}}\frac{d\delta_1 r}{dt}\left\{\int\delta\left(\frac{dR}{dv}\right)dt + \frac{1}{\sqrt{a\mu}}\left[\int\left(\frac{dR}{dv}\right)dt\right]^2\right\}\end{aligned}\right\} . \quad (445)$$

Now we have found in the preceding chapters

$$\left.\begin{aligned}\frac{d\delta_1 r}{dt} = a\frac{\overline{m}^2}{\mu} n \Big\{ &+ (2.382700)\sin 2(nt-n't) \\ &- \frac{a}{a'}(12.43582)\sin(nt-n't) + \frac{a}{a'}(1.335670)\sin 3(nt-n't)\Big\}\end{aligned}\right\} . \quad (446)$$

$$\left.\begin{aligned}\delta\frac{d\delta_1 r}{dt} = a\frac{\overline{m}^4}{\mu^2} n \Big\{ &- (2.665813)\sin 2(nt-n't) \\ &+ (3.346039)\sin 4(nt-n't) - \frac{a}{a'}(683.9649)\sin(nt-n't) \\ &- \frac{a}{a'}(60.67276)\sin 3(nt-n't) + \frac{a}{a'}(2.203426)\sin 5(nt-n't)\Big\}\end{aligned}\right\} . \quad (447)$$

$$\left.\begin{aligned}\int\left(\frac{dR}{dv}\right)dt = a^2\frac{\overline{m}^2}{\mu} n \Big\{ &- (0.8106367)\cos 2(nt-n't) \\ &- \frac{a}{a'}(0.4053184)\cos(nt-n't) - \frac{a}{a'}(0.6755306)\cos 3(nt-n't)\Big\}\end{aligned}\right\} . \quad (448)$$

$$\left.\begin{aligned}\int\delta\left(\frac{dR}{dv}\right)dt = a^2\frac{\overline{m}^4}{\mu^2}n\Big\{ & +(0.2702122)\cos 2(nt-n't)\\ & -(0.2197617)\cos 4(nt-n't)-\frac{a}{a'}(61.40028)\cos(nt-n't)\\ & +\frac{a}{a'}(8.676904)\cos 3(nt-n't)-\frac{a}{a'}(0.364958)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (449)$$

Equation (448) gives

$$\left.\begin{aligned}\left[\int\left(\frac{dR}{dv}\right)dt\right]^2 = a^4\frac{\overline{m}^4}{\mu^2}n^2\Big\{ & +(0.3285660)+(0.3285660)\cos 4(nt-n't)\\ & +\frac{a}{a'}(0.4380880)\cos(nt-n't)+\frac{a}{a'}(0.1642830)\cos 3(nt-n't)\\ & +\frac{a}{a'}(0.2738050)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (450)$$

From equations (449) and (450) we get

$$\frac{1}{\sqrt{a\mu}}\int\delta\left(\frac{dR}{dv}\right)dt+\frac{1}{a\mu}\left[\int\left(\frac{dR}{dv}\right)dt\right]^2=$$

$$\left.\begin{aligned}a^2\frac{\overline{m}^4}{\mu^2\sqrt{a\mu}}n\Big\{ & +(0.3285660)+(0.2702122)\cos 2(nt-n't)\\ & +(0.1088043)\cos 4(nt-n't)-\frac{a}{a'}(60.96219)\cos(nt-n't)\\ & +\frac{a}{a'}(8.841187)\cos 3(nt-n't)-\frac{a}{a'}(0.091153)\cos 5(nt-n't)\Big\}\end{aligned}\right\}. \quad (451)$$

If we substitute these values in equation (445), we shall obtain the following values of the first and second terms of $\delta^2\dfrac{d\delta_4 r}{dt}$:

1st term of

$$\left.\begin{aligned}\delta^2\frac{d\delta_4 r}{dt} = a\,\frac{\overline{m}^6}{\mu^3}n\Big\{ & -(1.356211)\sin 2(nt-n't)\\ & +(1.080503)\sin 4(nt-n't)-(1.356211)\sin 6(nt-n't)\\ & -\frac{a}{a'}(254.0855)\sin(nt-n't)+\frac{a}{a'}(276.1926)\sin 3(nt-n't)\\ & +\frac{a}{a'}(24.85058)\sin 5(5nt-n't)-\frac{a}{a'}(2.023265)\sin 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (452)$$

$$\left.\begin{aligned}\text{2nd term of }\delta^2\frac{d\delta_4 r}{dt} = a\frac{\overline{m}^6}{\mu^3}n\Big\{&(0.6532502)\sin 2(nt-n't)+(0.3219173)\sin 4(nt-n't)\\&+(0.1296240)\sin 6(nt-n't)-\frac{a}{a'}(85.45829)\sin(nt-n't)\\&-\frac{a}{a'}(73.08345)\sin 3(nt-n't)+\frac{a}{a'}(10.03687)\sin 5(nt-n't)\\&-\frac{a}{a'}(0.0359318)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (453)$$

If we now take the sum of equations (452) and (453), we obtain

$$\left.\begin{aligned}\delta^2\frac{d\delta_4 r}{dt} = a\frac{\overline{m}^6}{\mu^3}n\Big\{&-(0.702961)\sin 2(nt-n't+(1.402418)\sin 4(nt-n't)\\&-(1.226587)\sin 6(nt-n't)-\frac{a}{a'}(339.5438)\sin(nt-n't)\\&+\frac{a}{a'}(203.1092)\sin 3(nt-n't)+\frac{a}{a'}(34.88745)\sin 5(nt-n't)\\&-\frac{a}{a'}(2.059197)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (454)$$

The sum of equations (444) and (454) is the value of $\delta^2\left(\frac{d\delta_1 r}{dt}\right)$; but since

$$\left.\delta^2\frac{d\delta_0 r}{dt}=0,\qquad \delta^2\frac{d\delta_2 r}{dt}=0,\quad\text{and}\quad \delta^2\frac{d\delta_3 r}{dt}=0,\right\} . \quad (454')$$

when we neglect the eccentricity and inclination of the orbit, we shall have

$$\left.\delta^2\left(\frac{d\delta_1 r}{dt}\right)=\delta^2\frac{d\delta r}{dt}\right\} . \quad (455)$$

Therefore we shall have

$$\left.\begin{aligned}\delta^2\frac{d\delta r}{dt} = a\frac{\overline{m}^6}{\mu^3}n\Big\{&+(1.612787)\sin 2(nt-n't)-(2.931684)\sin 4(nt-n't)\\&+(2.884386)\sin 6(nt-n't)-\frac{a}{a'}(47355.79)\sin(nt-n't)\\&-\frac{a}{a'}(3058.724)\sin 3(nt-n't)-\frac{a}{a'}(110.22822)\sin 5(nt-n't)\\&+\frac{a}{a'}(2.202591)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (456)$$

This equation gives by integration

$$\left.\begin{aligned}\delta^3 r = a\frac{\overline{m}^6}{\mu^3}\Big\{ &-(1.794788) - (0.8715896)\cos 2(nt-n't) \\ &+ (0.7921767)\cos 4(nt-n't) - (0.5195976)\cos 6(nt-n't) \\ &+ \frac{a}{a'}(51184.46)\cos(nt-n't) + \frac{a}{a'}(1102.006)\cos 3(nt-n't) \\ &+ \frac{a}{a'}(23.82800)\cos 5(nt-n't) - \frac{a}{a'}(0.3400953)\cos 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (457)$$

The constant added to the integral being determined so as to satisfy the condition mentioned in section 23.

36. We shall now determine the perturbations of the longitude. From the development of the first of equations (244) we easily find

$$\delta^2\frac{d\delta_1 v}{dt} = -2\frac{dv_1}{r_1 dt}\delta^3 r + 6\frac{dv_1}{r_1^2 dt}\delta r\delta^2 r - 4\frac{dv_1}{r_1^3 dt}\delta r^3 \quad\Big\} . \quad (458)$$

In this equation we may put $r_1 = a$ and $\frac{dv_1}{dt} = n$; and we shall easily obtain by means of equations (457), (426), and (425) the following values:

$$\left.\begin{aligned}\overset{\text{1st term of}}{\delta^2\frac{d\delta_1 v}{dt}} = \frac{\overline{m}^6}{\mu^3}n\Big\{ &+ (1.743179)\cos 2(nt-n't) \\ &- (1.584353)\cos 4(nt-n't) + (1.039195)\cos 6(nt-n't) \\ &- \frac{a}{a'}(102368.92)\cos(nt-n't) - \frac{a}{a'}(2204.012)\cos 3(nt-n't) \\ &- \frac{a}{a'}(47.65600)\cos 5(nt-n't) + \frac{a}{a'}(0.6801906)\cos 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (459)$$

$$\left.\begin{aligned}\overset{\text{2nd term of}}{\delta^2\frac{d\delta_1 v}{dt}} = \frac{\overline{m}^6}{\mu^3}n\Big\{ &- (9.906936)\cos 2(nt-n't) \\ &- (4.9221390)\cos 4(nt-n't) + (2.484622)\cos 6(nt-n't) \\ &- \frac{a}{a'}(3524.276)\cos(nt-n't) - \frac{a}{a'}(2865.001)\cos 3(nt-n't) \\ &- \frac{a}{a'}(106.2986)\cos 5(nt-n't) + \frac{a}{a'}(1.644406)\cos 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (460)$$

3rd term of

$$\left.\begin{aligned}\delta^2\frac{d\delta_1 v}{dt} = \frac{\overline{m}^6}{\mu^3}n\Big\{ &+ (6.834448)\cos 2(nt-n't) \\ &+ (1.6580924)\cos 4(nt-n't) + (2.135076)\cos 6(nt-n't) \\ &- \frac{a}{a'}(169.1841)\cos(nt-n't) - \frac{a}{a'}(96.52844)\cos 3(nt-n't) \\ &- \frac{a}{a'}(65.62120)\cos 5(nt-n't) + \frac{a}{a'}(2.393718)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (461)$$

If we substitute equations (454–461) in equation (458), we obtain

$$\left.\begin{aligned}\delta^2\frac{d\delta_1 v}{dt} = \frac{\overline{m}^6}{\mu^3}n\Big\{ &- (1.329309)\cos 2(nt-n't) \\ &- (4.848400)\cos 4(nt-n't) + (5.658893)\cos 6(nt-n't) \\ &- \frac{a}{a'}(106062.38)\cos(nt-n't) - \frac{a}{a'}(5165.541)\cos 3(nt-n't) \\ &- \frac{a}{a'}(219.5758)\cos 5(nt-n't) + \frac{a}{a'}(4.718314)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (462)$$

We must now find the value of $\delta^2\frac{d\delta_0 v}{dt}$. If we develop equation (254), we shall easily obtain

$$\left.\begin{aligned}\delta^2\frac{d\delta_0 v}{dt} = -\frac{1}{a^2}\int\delta^2\left(\frac{dR}{dv}\right)dt + \frac{2}{a^3}\delta r\int\delta\left(\frac{dR}{dv}\right)dt \\ -\frac{3}{a^4}\delta r^2\int\left(\frac{dR}{dv}\right)dt + \frac{2}{a^3}\delta^2 r\int\left(\frac{dR}{dv}\right)dt\end{aligned}\right\}. \quad (463)$$

If we multiply equation (428) by dt and take the integral, we shall obtain

$$\left.\begin{aligned}\int\delta^2\left(\frac{dR}{dv}\right)dt = a^2\frac{\overline{m}^6}{\mu^3}n\Big\{ &+ (1.700299)\cos 2(nt-n't) \\ &- (0.07460612)\cos 4(nt-n't) - (0.1216951)\cos 6(nt-n't) \\ &- \frac{a}{a'}(4002.205)\cos(nt-n't) + \frac{a}{a'}(455.6131)\cos 3(nt-n't) \\ &+ \frac{a}{a'}(6.277654)\cos 5(nt-n't) - \frac{a}{a'}(0.2353217)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (464)$$

This equation multiplied by $-\frac{1}{a^2}$ is the first term of $\delta^2\frac{d\delta_0 v}{dt}$. Equations (411) and (449) will give

$$\left.\begin{aligned}&\text{2nd term of}\\&\delta^2\frac{d\delta_0 v}{dt}=\frac{\overline{m}^6}{\mu^3}n\Big\{+(0.1929098)\cos 2(nt-n't)\\&\quad-(0.2746901)\cos 4(nt-n't)+(0.2829805)\cos 6(nt-n't)\\&\quad+\frac{a}{a'}(91.96476)\cos(nt-n't)+\frac{a}{a'}(77.31904)\cos 3(nt-n't)\\&\quad-\frac{a}{a'}(14.13523)\cos 5(nt-n't)+\frac{a}{a'}(0.5756988)\cos 7(nt-n't)\Big\}\end{aligned}\right\}.\quad(465)$$

Equations (419) and (448) will give

$$\left.\begin{aligned}&\text{3rd term of}\\&\delta^2\frac{d\delta_0 v}{dt}=\frac{\overline{m}^6}{\mu^3}n\Big\{+(3.091803)\cos 2(nt-n't)\\&\quad+(0.5219198)\cos 4(nt-n't)+(1.008083)\cos 6(nt-n't)\\&\quad-\frac{a}{a'}(44.01286)\cos(nt-n't)-\frac{a}{a'}(22.48523)\cos 3(nt-n't)\\&\quad-\frac{a}{a'}(19.91158)\cos 5(nt-n't)+\frac{a}{a'}(1.5935374)\cos 7(nt-n't)\Big\}\end{aligned}\right\}.\quad(466)$$

Lastly, equations (417) and (448) will give

$$\left.\begin{aligned}&\text{4th term of}\\&\delta^2\frac{d\delta_0 v}{dt}=\frac{\overline{m}^6}{\mu^3}n\Big\{-(1.776613)\cos 2(nt-n't)\\&\quad-(1.167860)\cos 4(nt-n't)+(0.5213876)\cos 6(nt-n't)\\&\quad-\frac{a}{a'}(621.6665)\cos(nt-n't)-\frac{a}{a'}(605.0880)\cos 3(nt-n't)\\&\quad-\frac{a}{a'}(17.10145)\cos 5(nt-n't)+\frac{a}{a'}(0.5847120)\cos 7(nt-n't)\Big\}\end{aligned}\right\}.\quad(467)$$

If we substitute equations (464–467) in equation (463), we obtain

$$\left.\begin{aligned}&\delta^2\frac{d\delta_0 v}{dt}=\frac{\overline{m}^6}{\mu^3}n\Big\{-(0.192199)\cos 2(nt-n't)\\&\quad-(0.846024)\cos 4(nt-n't)+(1.934147)\cos 6(nt-n't)\end{aligned}\right\}.\quad(468)$$

(Continued on the next page.)

$$
\left.
\begin{aligned}
&+\frac{\overline{m}^2}{a^4}\left\{+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't)+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta r^2\delta v \\
&+\frac{\overline{m}^2}{a^3}\left\{+3\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't)\right. \\
&\qquad\qquad\qquad\left.+\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\right\}\delta r\delta v^2
\end{aligned}
\right\}. \quad (471)
$$

$$
\left.
\begin{aligned}
\delta^3\left(\frac{dR}{dv}\right)=&\frac{\overline{m}^2}{a^2}\left\{+3\sin 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't)\right. \\
&\qquad\qquad\qquad\left.+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta^3 r \\
&+\frac{\overline{m}^2}{a}\left\{+3\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't)\right. \\
&\qquad\qquad\qquad\left.+\tfrac{45}{8}\frac{a}{a'}\cos 3(nt-n't)\right\}\delta^3 v \\
&+\frac{\overline{m}^2}{a^3}\left\{+3\sin 2(nt-n't)+\tfrac{9}{4}\frac{a}{a'}\sin(nt-n't)\right. \\
&\qquad\qquad\qquad\left.+\tfrac{45}{4}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta r\delta^2 r \\
&+\frac{\overline{m}^2}{a}\left\{-6\sin 2(nt-n't)-\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't)\right. \\
&\qquad\qquad\qquad\left.-\tfrac{135}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta v\delta^2 v \\
&+\frac{\overline{m}^2}{a^2}\left\{+6\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't)\right. \\
&\qquad\qquad\qquad\left.+\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\right\}\{\delta r\delta^2 v+\delta^2 r\delta v\} \\
&+\frac{\overline{m}^2}{a^4}\left\{+\tfrac{3}{8}\frac{a}{a'}\sin(nt-n't)+\tfrac{15}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta r^3 \\
&+\frac{\overline{m}^2}{a}\left\{-2\cos 2(nt+n't)-\tfrac{1}{16}\frac{a}{a'}\cos(nt-n't)\right. \\
&\qquad\qquad\qquad\left.-\tfrac{135}{16}\frac{a}{a'}\cos 3(nt-n't)\right\}\delta v^3
\end{aligned}
\right\}. \quad (472)
$$

(Continued on the next page.)

CHAPTER VI.

PERTURBATIONS ARISING FROM THE FOURTH POWER OF THE DISTURBING FORCE.

37. THE variations of the functions and forces arising from the fourth power of the disturbing force, which enter into the expression of the radius vector, may be determined by means of the following equations:

$$\left.\begin{aligned}
\delta^3\left(\frac{dR}{dr}\right) = \frac{\overline{m}^2}{a^3}\Big\{&-\tfrac{1}{2}-\tfrac{3}{2}\cos 2(nt-n't)-\tfrac{9}{4}\frac{a}{a'}\cos(nt-n't) \\
&\qquad -\tfrac{15}{4}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta^3 r \\
+\frac{\overline{m}^2}{a^2}\Big\{&+3\sin 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't) \\
&\qquad +\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta^3 v \\
+\frac{\overline{m}^2}{\mu^4}\Big\{&-\tfrac{9}{4}\frac{a}{a'}\cos(nt-n't)-\tfrac{15}{4}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta r\delta^2 r \\
+\frac{\overline{m}^2}{a^2}\Big\{&+6\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't) \\
&\qquad +\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta v\delta^2 v \\
+\frac{\overline{m}^2}{a^3}\Big\{&+3\sin 2(nt-n't)+\tfrac{9}{4}\frac{a}{a'}\sin(nt-n't) \\
&\qquad +\tfrac{45}{4}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\{\delta r\delta^2 v+\delta^2 r\delta v\} \\
+\frac{\overline{m}^2}{\mu^2}\Big\{&-2\sin 2(nt-n't)-\tfrac{3}{16}\frac{a}{a'}\sin(nt-n't) \\
&\qquad -\tfrac{135}{16}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta v^3
\end{aligned}\right\} \quad (471)$$

(Continued on the next page)

$$
\left.
\begin{aligned}
&+\frac{\overline{m}^2}{a^4}\left\{+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't)+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta r^2\delta v\\
&+\frac{\overline{m}^2}{a^3}\Big\{+3\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't)\\
&\qquad\qquad\qquad\qquad+\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta r\delta v^2
\end{aligned}
\right\}. \quad (471)
$$

$$
\left.
\begin{aligned}
\delta^3\left(\frac{dR}{dv}\right)=&\frac{\overline{m}^2}{a^2}\Big\{+3\sin 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\sin(nt-n't)\\
&\qquad\qquad\qquad+\tfrac{45}{8}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta^3 r\\
&+\frac{\overline{m}^2}{a}\Big\{+3\cos 2(nt-n't)+\tfrac{3}{8}\frac{a}{a'}\cos(nt-n't)\\
&\qquad\qquad\qquad+\tfrac{45}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta^3 v\\
&+\frac{\overline{m}^2}{a^3}\Big\{+3\sin 2(nt-n't)+\tfrac{9}{4}\frac{a}{a'}\sin(nt-n't)\\
&\qquad\qquad\qquad+\tfrac{45}{4}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta r\delta^2 r\\
&+\frac{\overline{m}^2}{a}\Big\{-6\sin 2(nt-n't)-\tfrac{3}{8}\frac{a}{a'}\sin(nt-n't)\\
&\qquad\qquad\qquad-\tfrac{135}{8}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta v\delta^2 v\\
&+\frac{\overline{m}^2}{a^2}\Big\{+6\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't)\\
&\qquad\qquad\qquad+\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\{\delta r\delta^2 v+\delta^2 r\delta v\}\\
&+\frac{\overline{m}^2}{a^4}\left\{+\tfrac{3}{8}\frac{a}{a'}\sin(nt-n't)+\tfrac{15}{8}\frac{a}{a'}\sin 3(nt-n't)\right\}\delta r^3\\
&+\frac{\overline{m}^2}{a}\Big\{-2\cos 2(nt+n't)-\tfrac{1}{16}\frac{a}{a'}\cos(nt-n't)\\
&\qquad\qquad\qquad-\tfrac{135}{16}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta v^3
\end{aligned}
\right\}. \quad (472)
$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\frac{\overline{m}^2}{a^3}\Big\{+3\cos 2(nt-n't)+\tfrac{9}{8}\frac{a}{a'}\cos(nt-n't) \\
&\qquad\qquad +\tfrac{135}{8}\frac{a}{a'}\cos 3(nt-n't)\Big\}\,\delta r^2\delta v \\
&+\frac{\overline{m}^2}{a^2}\Big\{-6\sin 2(nt-n't)-\tfrac{9}{16}\frac{a}{a'}\sin(nt-n't) \\
&\qquad\qquad -\tfrac{405}{16}\frac{a}{a'}\sin 3(nt-n't)\Big\}\,\delta r\delta v^2
\end{aligned}\right\} . \quad (472)
$$

$$
\delta^3(c_1\cos\beta)=\mp\sin nt\,\{(\delta^3 v-\tfrac{1}{6}\delta v^3)\}-\cos nt\,\delta v\delta^2 v \quad \Big\} . \quad (473)
$$

$$
\left.\begin{aligned}
\delta^3(c_2\cos\beta) &= a^2ndt\sin nt\,\Big\{\delta^3 v-\tfrac{1}{6}\delta v^3+\frac{1}{ndt}(\delta v d\delta^2 v+\delta^2 v d\delta v) \\
&\qquad +\frac{2}{a}(\delta r\delta^2 v+\delta^2 r\delta v)+\frac{2}{andt}\delta r\delta v d\delta v+\frac{1}{a^2}\delta r^2\delta v\Big\} \\
&\pm a^2ndt\cos nt\,\Big\{\delta v\delta^2 v-\frac{1}{ndt}d\delta^3 v+\frac{1}{2ndt}\delta v^2 d\delta v-\frac{2}{a}\delta^3 r+\frac{1}{a}\delta r\delta v^2 \\
&\qquad -\frac{2}{andt}\delta r d\delta^2 v-\frac{2}{andt}\delta^2 r d\delta v-\frac{2}{a}\delta r\delta^2 r-\frac{1}{a^2ndt}\delta r^2 d\delta v\Big\}
\end{aligned}\right\} . \quad (474)
$$

$$
\left.\begin{aligned}
\delta^3(c_3\sin\beta) &= \pm 2andt\cos nt\,\Big\{\delta^3 v-\tfrac{1}{6}\delta v^3-\frac{1}{2andt}d\delta^3 r \\
&\qquad +\frac{1}{ndt}(\delta v d\delta^2 v+\delta^2 v d\delta v)+\frac{1}{a}(\delta r\delta^2 v+\delta^2 r\delta v) \\
&\qquad +\frac{1}{andt}\delta r\delta v d\delta v+\frac{1}{4andt}d\delta r\delta v^2\Big\} \\
&+2andt\sin nt\,\Big\{\frac{1}{ndt}d\delta^3 v+\frac{1}{a}\delta^3 r-\delta v\delta^2 v+\frac{1}{andt}(\delta r d\delta^2 v+\delta^2 r d\delta v) \\
&\qquad +\frac{1}{2andt}(d\delta^2 r\delta v+d\delta r\delta^2 v)-\frac{1}{2ndt}d\delta v\delta v^2-\frac{1}{2a}\delta r\delta v^2\Big\}
\end{aligned}\right\} . \quad (475)
$$

In order to reduce the preceding equations to numbers, we shall require the following developments in addition to those already given:

$$\left.\begin{aligned}\delta v\delta^2 v = \frac{\overline{m}^6}{\mu^3}\Big\{ &-(0.7879935)+(1.245301)\cos 2(nt-n't) \\ &+(0.7879935)\cos 4(nt-n't)-(1.245301)\cos 6(nt-n't) \\ &-\frac{a}{a'}(1507.913)\cos(nt-n't)+\frac{a}{a'}(1452.455)\cos 3(nt-n't) \\ &+\frac{a}{a'}(56.29547)\cos 5(nt-n't)-\frac{a}{a'}(0.8376073)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (476)$$

$$\left.\begin{aligned}\delta r\delta^2 v = a\frac{\overline{m}^6}{\mu^3}\Big\{ &-(0.7327713)\sin 2(nt-n't) \\ &+(0.327660)\sin 4(nt-n't)-(0.8763144)\sin 6(nt-n't) \\ &-\frac{a}{a'}(754.644)\sin(nt-n't)+\frac{a}{a'}(1053.871)\sin 3(nt-n't) \\ &+\frac{a}{a'}(35.00620)\sin 5(nt-n't)-\frac{a}{a'}(0.8258466)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (477)$$

$$\left.\begin{aligned}\delta^2 r\delta v = a\frac{\overline{m}^6}{\mu^3}\Big\{ &+(3.182128)\sin 2(nt-n't) \\ &+(1.318115)\sin 4(nt-n't)-(0.5884688)\sin 6(nt-n't) \\ &+\frac{a}{a'}(642.7566)\sin(nt-n't)+\frac{a}{a'}(651.7549)\sin 3(nt-n't) \\ &+\frac{a}{a'}(28.14704)\sin 5(nt-n't)-\frac{a}{a'}(0.3593313)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (478)$$

$$\left.\begin{aligned}\delta v^3 = \frac{\overline{m}^6}{\mu^3}\Big\{ &+(4.595343)\sin 2(nt-n't)-(1.531781)\sin 6(nt-n't) \\ &-\frac{a}{a'}(142.2540)\sin(nt-n't)-\frac{a}{a'}(68.90400)\sin 3(nt-n't) \\ &+\frac{a}{a'}(71.86800)\sin 5(nt-n't)-\frac{a}{a'}(1.4820015)\sin 7(nt-n't)\Big\}\end{aligned}\right\} . \quad (479)$$

$$\left.\begin{aligned}\delta r^2\delta v = a^2\,\frac{\overline{m}^6}{\mu^3}\Big\{ &+ (0.809350)\sin 2(nt-n't) \\ &+ (0.3927127)\sin 4(nt-n't) + (0.7585212)\sin 6(nt-n't) \\ &- \frac{a}{a'}(22.17575)\sin(nt-n't) - \frac{a}{a'}(13.70818)\sin 3(nt-n't) \\ &- \frac{a}{a'}(27.42486)\sin 5(nt-n't) + \frac{a}{a'}(0.8115624)\sin 7(nt-n't)\Big\}\end{aligned}\right\} \quad (480)$$

$$\left.\begin{aligned}\delta r\delta v^2 = a\,\frac{\overline{m}^6}{\mu^3}\Big\{ &- (0.2790336) - (1.077909)\cos 2(nt-n't) \\ &+ (0.2790336)\cos 4(nt-n't) + (1.077909)\cos 6(nt-n't) \\ &+ \frac{a}{a'}(30.75872)\cos(nt-n't) + \frac{a}{a'}(12.93039)\cos 3(nt-n't) \\ &- \frac{a}{a'}(44.78720)\cos 5(nt-n't) + \frac{a}{a'}(1.0980816)\cos 7(nt-n't)\Big\}\end{aligned}\right\} \quad (481)$$

$$\left.\begin{aligned}\delta v d\delta^2 v = \frac{\overline{m}^6}{\mu^3}\,ndt\Big\{ &- (4.608605)\sin 2(nt-n't) \\ &- (1.458101)\sin 4(nt-n't) + (4.608605)\sin 6(nt-n't) \\ &- \frac{a}{a'}(1277.259)\sin(nt-n't) - \frac{a}{a'}(1261.129)\sin 3(nt-n't) \\ &- \frac{a}{a'}(174.0372)\sin 5(nt-n't) + \frac{a}{a'}(3.503195)\sin 7(nt-n't)\Big\}\end{aligned}\right\} \quad (482)$$

$$\left.\begin{aligned}\delta^2 v d\delta v = \frac{\overline{m}^6}{\mu^3}\,ndt\Big\{ &+ (2.304303)\sin 2(nt-n't) \\ &- (1.458101)\sin 4(nt-n't) + (2.304303)\sin 6(nt-n't) \\ &+ \frac{a}{a'}(2691.215)\sin(nt-n't) - \frac{a}{a'}(2751.464)\sin 3(nt-n't) \\ &- \frac{a}{a'}(86.38526)\sin 5(nt-n't) + \frac{a}{a'}(1.921476)\sin 7(nt-n't)\Big\}\end{aligned}\right\} \quad (483)$$

$$\left.\begin{aligned}\delta v^2 d\delta v = \frac{\overline{m}^6}{\mu^3}\,ndt\Big\{ &+ (2.834404)\cos 2(nt-n't) \\ &- (2.834404)\cos 6(nt-n't) - \frac{a}{a'}(55.34877)\cos(nt-n't)\end{aligned}\right\} \quad (484)$$

(Continued on the next page.)

$$\left.\begin{array}{l} -\frac{a}{a'}(64.74988)\cos 3(nt-n't)+\frac{a}{a'}(110.8203)\cos 5(nt-n't) \\ \qquad\qquad -\frac{a}{a'}(3.199340)\cos 7(nt-n't)\Big\} \end{array}\right\} . \quad (484)$$

$$\left.\begin{array}{l} \delta^2 r d\delta v = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{+(2.439038)+(3.710398)\cos 2(nt-n't) \\ +(2.439038)\cos 4(nt-n't)-(1.088901)\cos 6(nt-n't) \\ +\frac{a}{a'}(1237.638)\cos(nt-n't)+\frac{a}{a'}(1253.014)\cos 3(nt-n't) \\ +\frac{a}{a'}(46.12714)\cos 5(nt-n't)-\frac{a}{a'}(0.8404915)\cos 7(nt-n't)\Big\} \end{array}\right\} . \quad (485)$$

$$\left.\begin{array}{l} \delta r\delta v d\delta v = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{-(1.994560)\sin 2(nt-n't) \\ -(0.5163230)\sin 4(nt-n't)-(1.994560)\sin 6(nt-n't) \\ +\frac{a}{a'}(57.36588)\sin(nt-n't)+\frac{a}{a'}(16.04920)\sin 3(nt-n't) \\ +\frac{a}{a'}(67.19411)\sin 5(nt-n't)-\frac{a}{a'}(2.353511)\sin 7(nt-n't)\Big\} \end{array}\right\} . \quad (486)$$

$$\left.\begin{array}{l} \delta r^2 d\delta v = a^2\frac{\overline{m}^6}{\mu^3}ndt\Big\{+(0.7266747)+(4.311440)\cos 2(nt-n't) \\ +(0.7266747)\cos 4(nt-n't)+(1.403566)\cos 6(nt-n't) \\ -\frac{a}{a'}(93.20674)\cos(nt-n't)-\frac{a}{a'}(50.04341)\cos 3(nt-n't) \\ -\frac{a}{a'}(39.65433)\cos 5(nt-n't)+\frac{a}{a'}(1.728038)\cos 7(nt-n't)\Big\} \end{array}\right\} . \quad (487)$$

$$\left.\begin{array}{l} \delta r d\delta^2 v = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{+(1.026060)-(2.977449)\cos 2(nt-n't) \\ +(0.186543)\cos 4(nt-n't)-(3.243061)\cos 6(nt-n't) \\ +\frac{a}{a'}(1276.0125)\cos(nt-n't)+\frac{a}{a'}(1006.2647)\cos 3(nt-n't) \\ +\frac{a}{a'}(105.2878)\cos 5(nt-n't)-\frac{a}{a'}(2.631273)\cos 7(nt-n't)\Big\} \end{array}\right\} . \quad (488)$$

$$\left.\begin{aligned} d\delta^2 r\delta v = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{ & -(2.439038)+(2.177802)\cos 2(nt-n't) \\ & +(2.439038)\cos 4(nt-n't)-(2.177802)\cos 6(nt-n't) \\ & -\frac{a}{a'}(643.8750)\cos(nt-n't)+\frac{a}{a'}(559.1741)\cos 3(nt-n't) \\ & +\frac{a}{a'}(86.18748)\cos 5(nt-n't)-\frac{a}{a'}(1.4866786)\cos 7(nt-n't)\Big\} \end{aligned}\right\} . \quad (489)$$

$$\left.\begin{aligned} d\delta r\delta^2 v = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{ & -(1.026060)+(1.621530)\cos 2(nt-n't) \\ & +(1.026060)\cos 4(nt-n't)-(1.621530)\cos 6(nt-n't) \\ & -\frac{a}{a'}(1974.032)\cos(nt-n't)+\frac{a}{a'}(1918.858)\cos 3(nt-n't) \\ & +\frac{a}{a'}(56.65115)\cos 5(nt-n't)-\frac{a}{a'}(1.476700)\cos 7(nt-n't)\Big\} \end{aligned}\right\} . \quad (490)$$

$$\left.\begin{aligned} d\delta r\delta v^2 = a\frac{\overline{m}^6}{\mu^3}ndt\Big\{ & +(5.983682)\sin 2(nt-n't) \\ & -(1.994560)\sin 6(nt-n't)-\frac{a}{a'}(143.1897)\sin(nt-n't) \\ & -\frac{a}{a'}(67.98797)\sin 3(nt-n't)+\frac{a}{a'}(72.79713)\sin 5(nt-n't) \\ & \qquad -\frac{a}{a'}(2.404585)\sin 7(nt-n't)\Big\} \end{aligned}\right\} . \quad (491)$$

From these equations we get

$$\left.\begin{aligned} \delta^3 v-\tfrac{1}{6}\delta v^3 = \frac{\overline{m}^6}{\mu^3}\Big\{ & -(1.5881507)\sin 2(nt-n't) \\ & -(1.538703)\sin 4(nt-n't)+(1.623118)\sin 6(nt-n't) \\ & -\frac{a}{a'}(110908.1)\sin(nt-n't)-\frac{a}{a'}(2211.969)\sin 3(nt-n't) \\ & -\frac{a}{a'}(71.85740)\sin 5(nt-n't)+\frac{a}{a'}(1.437105)\sin 7(nt-n't)\Big\} \end{aligned}\right\} . \quad (492)$$

$$\delta r \delta^2 v + \delta^2 r \delta v =$$

$$\left.\begin{aligned}
a\frac{\overline{m}^6}{\mu^3}\Big\{ &+ (2.449357) \sin 2(nt - n't) + (1.645775) \sin 4(nt - n't) \\
&- (1.464783) \sin 6(nt - n't) - \frac{a}{a'}(111.887) \sin (nt - n't) \\
&+ \frac{a}{a'}(1705.626) \sin 3(nt - n't) + \frac{a}{a'}(63.15324) \sin 5(nt - n't) \\
&\qquad\qquad - \frac{a}{a'}(1.185178) \sin 7(nt - n't) \Big\}
\end{aligned}\right\} . \quad (493)$$

$$\begin{aligned}
\Big\{\delta^3 v - \tfrac{1}{6}\delta v^3 + \frac{1}{ndt}\delta v d\delta^2 v + \frac{1}{ndt}\delta^2 v d\delta v + \frac{2}{a}(\delta r \delta^2 v + \delta^2 r \delta v) \\
+ \frac{2}{andt}\delta r \delta v d\delta v + \frac{1}{a^2}\delta r^2 \delta v\Big\} =
\end{aligned}$$

$$\left.\begin{aligned}
\frac{\overline{m}^6}{\mu^3}\Big\{ &- (2.173510) \sin 2(nt - n't) - (1.803288) \sin 4(nt - n't) \\
&+ (2.375859) \sin 6(nt - n't) - \frac{a}{a'}(109625.4) \sin (nt - n't) \\
&- \frac{a}{a'}(2794.920) \sin 3(nt - n't) - \frac{a}{a'}(99.0101) \sin 5(nt - n't) \\
&\qquad\qquad + \frac{a}{a'}(0.595960) \sin 7(nt - n't) \Big\}
\end{aligned}\right\} . \quad (494)$$

$$\begin{aligned}
\Big\{\delta v \delta^2 v - \frac{1}{ndt} d\delta^3 v + \tfrac{1}{2}\delta v^2 d\delta v - \frac{2}{a}\delta^3 r + \frac{1}{a}\delta r \delta v^2 - \frac{2}{andt}\delta r d\delta^2 v \\
- \frac{2}{andt}\delta^2 r d\delta v - \frac{2}{a}\delta r \delta^2 r - \frac{1}{a^2 ndt}\delta r^2 d\delta v\Big\} =
\end{aligned}$$

$$\left.\begin{aligned}
\frac{\overline{m}^6}{\mu^3}\Big\{ &- (2.806746) + (2.374255) \cos 2(nt - n't) \\
&+ (0.839975) \cos 4(nt - n't) - (1.706288) \cos 6(nt - n't) \\
&- \frac{a}{a'}(4999.18) \cos (nt - n't) + \frac{a}{a'}(1886.892) \cos 3(nt - n't) \\
&+ \frac{a}{a'}(68.52151) \cos 5(nt - n't) - \frac{a}{a'}(3.699232) \cos 7(nt - n't) \Big\}
\end{aligned}\right\} . \quad (495)$$

$$\left.\begin{aligned}
&\{\delta^3 v - \tfrac{1}{6}\delta v^3 - \frac{1}{2andt}d\delta^3 v + \frac{1}{ndt}\delta v d\delta^2 v + \frac{1}{ndt}\delta^2 v d\delta v + \frac{1}{a}(\delta r\delta^2 v + \delta^2 r\delta v) \\
&\qquad\qquad + \frac{1}{andt}\delta r\delta v d\delta v + \frac{1}{4andt}d\delta r\delta v^2\} = \\
&\frac{\overline{m}^6}{\mu^3}\Big\{-(2.748130)\sin 2(nt-n't) - (1.859611)\sin 4(nt-n't) \\
&\quad + (3.135850)\sin 6(nt-n't) - \frac{a}{a'}(85906.5)\sin(nt-n't) \\
&\quad - \frac{a}{a'}(2990.522)\sin 3(nt-n't) - \frac{a}{a'}(128.6192)\sin 5(nt-n't) \\
&\qquad\qquad + \frac{a}{a'}(1.628645)\sin 7(nt-n't)\Big\}
\end{aligned}\right\} \quad (496)$$

$$\left.\begin{aligned}
&\{\frac{1}{ndt}d\delta^3 v + \frac{1}{a}\delta^3 r - \delta v\delta^2 v + \frac{1}{andt}(\delta r d\delta^2 v + \delta^2 r d\delta v) \\
&\qquad + \frac{1}{2andt}(d\delta^2 r\delta v + d\delta r\delta^2 v) - \frac{1}{2ndt}d\delta v\delta v^2 - \frac{1}{2a}\delta r\delta v^2\} = \\
&\frac{\overline{m}^6}{\mu^3}\Big\{+(0.865271) - (1.884032)\cos 2(nt-n't) \\
&\quad - (1.471628)\cos 4(nt-n't) + (2.965362)\cos 6(nt-n't) \\
&\quad - \frac{a}{a'}(48724.53)\cos(nt-n't) - \frac{a}{a'}(2997.652)\cos 3(nt-n't) \\
&\quad - \frac{a}{a'}(119.65152)\cos 5(nt-n't) + \frac{a}{a'}(4.302272)\cos 7(nt-n't)\Big\}
\end{aligned}\right\} \quad (497)$$

If we now make the necessary substitutions in equations (471–475), we shall obtain the following values:

$$\left.\begin{aligned}
&\delta^3\left(\frac{dR}{dr}\right) = \frac{\overline{m}^8}{a^2\mu^3}\Big\{+(1.515429) - (0.088079)\cos 2(nt-n't) \\
&\quad + (4.187337)\cos 4(nt-n't) + (2.287588)\cos 6(nt-n't) \\
&\quad - (3.115678)\cos 8(nt-n't) - \frac{a}{a'}(232002.2)\cos(nt-n't) \\
&\quad + \frac{a}{a'}(123031.9)\cos 3(nt-n't) + \frac{a}{a'}(4247.481)\cos 5(nt-n't) \\
&\quad + \frac{a}{a'}(164.8234)\cos 7(nt-n't) - \frac{a}{a'}(7.517723)\cos 9(nt-n't)\Big\}
\end{aligned}\right\} \quad (498)$$

$$\delta^3\left(\frac{dR}{dv}\right) = \frac{\overline{m}^8}{a\mu^3}\left\{ \begin{aligned} &+ (3.988125)\sin 2(nt-n't) \\ &- (1.569922)\sin 4(nt-n't) - (0.025011)\sin 6(nt-n't) \\ &+ (0.670879)\sin 8(nt-n't) + \frac{a}{a'}(242016.1)\sin(nt-n't) \\ &- \frac{a}{a'}(86278.3)\sin 3(nt-n't) - \frac{a}{a'}(1631.179)\sin 5(nt-n't) \\ &- \frac{a}{a'}(38.99440)\sin 7(nt-n't) + \frac{a}{a'}(1.631205)\sin 9(nt-n't) \end{aligned} \right\} \quad (499)$$

$$\delta^3(c_1\cos\beta) =$$

$$\frac{\overline{m}^6}{\mu^3}\left\{ \begin{aligned} &+ (0.7879935)\cos nt - (1.416726)\cos(3nt-2n't) \\ &\pm (0.1714248)\cos(nt-2n't) - (1.163348)\cos(5nt-4n't) \\ &\pm (0.3753548)\cos(3nt-4n't) + (1.434209)\cos(7nt-6n't) \\ &\mp (0.188909)\cos(5nt-6n't) - \frac{a}{a'}(54700.09)\cos(2nt-n't) \\ &+ \frac{a}{a'}(56208.01)\cos n't - \frac{a}{a'}(1832.212)\cos(4nt-3n't) \\ &\pm \frac{a}{a'}(379.7570)\cos(2nt-3n't) - \frac{a}{a'}(64.07643)\cos(6nt-5n't) \\ &\pm \frac{a}{a'}(7.78097)\cos(4nt-5n't) + \frac{a}{a'}(1.137356)\cos(8nt-7n't) \\ &\mp \frac{a}{a'}(0.2997489)\cos(6nt-7n't) \end{aligned} \right\} \quad (500)$$

$$\delta^3(c_2\cos\beta) =$$

$$a^2\frac{\overline{m}^6}{\mu^3}ndt\left\{ \begin{aligned} &\mp (2.806746)\cos nt \pm (2.273882)\cos(3nt-2n't) \\ &+ (0.100520)\cos(nt-2n't) \pm (1.321631)\cos(5nt-4n't) \\ &- (0.481657)\cos(3nt-4n't) \mp (2.041074)\cos(7nt-6n't) \\ &+ (0.334785)\cos(5nt-6n't) \pm \frac{a}{a'}(52313.1)\cos(2nt-n't) \\ &\mp \frac{a}{a'}(57312.3)\cos n't \pm \frac{a}{a'}(2340.906)\cos(4nt-3n't) \end{aligned} \right. \quad (501)$$

(Continued on the next page.)

$$\left.\begin{aligned}&-\frac{a}{a'}(454.014)\cos(2nt-3n't)\pm\frac{a}{a'}(83.7658)\cos(6nt-5n't)\\&-\frac{a}{a'}(15.2442)\cos(4nt-5n't)\mp\frac{a}{a'}(2.147596)\cos(8nt-7n't)\\&\qquad\qquad\qquad\qquad-\frac{a}{a'}(1.551636)\cos(6nt-7n't)\Big\}\end{aligned}\right\} \quad . \quad (501)$$

$$\left.\begin{aligned}&\delta^3(c_3\sin\beta)=\\&a\frac{\overline{m}^6}{\mu^3}ndt\Big\{+(1.730542)\sin nt-(4.632162)\sin(3nt-2n't)\\&\quad\mp(0.864098)\sin(nt-2n't)-(3.331239)\sin(5nt-4n't)\\&\quad\mp(0.387983)\sin(3nt-4n't)+(6.101212)\sin(7nt-6n't)\\&\quad\pm(0.170488)\sin(5nt-6n't)-\frac{a}{a'}(124631.0)\sin(2nt-n't)\\&\quad+\frac{a}{a'}(37182.0)\sin n't-\frac{a}{a'}(5988.174)\sin(4nt-3n't)\\&\quad\pm\frac{a}{a'}(7.130)\sin(2nt-3n't)-\frac{a}{a'}(248.2707)\sin(6nt-5n't)\\&\quad\mp\frac{a}{a'}(8.9677)\sin(4nt-5n't)+\frac{a}{a'}(5.930917)\sin(8nt-7n't)\\&\qquad\qquad\qquad\qquad\mp\frac{a}{a'}(2.673627)\sin(6nt-7n't)\Big\}\end{aligned}\right\} \quad . \quad (502)$$

We have thus developed all the functions and forces which enter into the expression of the radius vector, and it now remains to make the required substitutions in order to obtain the numerical values of the various inequalities.

38. The variation of $\frac{d\delta_1 r}{dt}$ arising from the fourth power of the disturbing force is given by means of the following equation:

$$\left.\begin{aligned}&\sqrt{a\mu}\delta^3\frac{d\delta_1 r}{dt}=\\&c_1\cos\beta\int\Big\{c_2\cos\beta\,\delta^3\left(\frac{dR}{dr}\right)+\delta(c_2\cos\beta)\,\delta^2\left(\frac{dR}{dr}\right)+\delta^2(c_2\cos\beta)\,\delta\left(\frac{dR}{dr}\right)\\&\quad+\delta^3(c_2\cos\beta)\left(\frac{dR}{dr}\right)+c_3\sin\beta\,\delta^3\left(\frac{dR}{dv}\right)+\delta(c_3\sin\beta)\,\delta^2\left(\frac{dR}{dv}\right)\\&\qquad\qquad+\delta^2(c_3\sin\beta)\,\delta\left(\frac{dR}{dv}\right)+\delta^3(c_3\sin\beta)\left(\frac{dR}{dv}\right)\Big\}\end{aligned}\right\} \quad . \quad (503)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- c_1 \sin\beta \int \Big\{ c_2 \sin\beta\, \delta^3\left(\frac{dR}{dr}\right) + \delta\,(c_2 \sin\beta)\, \delta^2\left(\frac{dR}{dr}\right) + \delta^2\,(c_2 \sin\beta)\, \delta\left(\frac{dR}{dr}\right) \\
&\qquad + \delta^3\,(c_2 \sin\beta)\left(\frac{dR}{dr}\right) + c_3 \cos\beta\, \delta^3\left(\frac{dR}{dv}\right) + \delta\,(c_3 \cos\beta)\, \delta^2\left(\frac{dR}{dv}\right) \\
&\qquad\qquad + \delta^2\,(c_3 \cos\beta)\, \delta\left(\frac{dR}{dv}\right) + \delta^3\,(c_3 \cos\beta)\left(\frac{dR}{dv}\right)\Big\} \\
&+ \delta\,(c_1 \cos\beta) \int \Big\{ c_2 \cos\beta\, \delta^2\left(\frac{dR}{dr}\right) + \delta\,(c_2 \cos\beta)\, \delta\left(\frac{dR}{dr}\right) + \delta^2\,(c_2 \cos\beta)\left(\frac{dR}{dr}\right) \\
&\qquad + c_3 \sin\beta\, \delta^2\left(\frac{dR}{dv}\right) + \delta\,(c_3 \sin\beta\, \delta)\left(\frac{dR}{dv}\right) + \delta^2\,(c_3 \sin\beta)\left(\frac{dR}{dv}\right)\Big\} \\
&- \delta\,(c_1 \sin\beta) \int \Big\{ c_2 \sin\beta\, \delta^2\left(\frac{dR}{dr}\right) + \delta\,(c_2 \sin\beta)\, \delta\left(\frac{dR}{dr}\right) + \delta^2\,(c_2 \sin\beta)\left(\frac{dR}{dr}\right) \\
&\qquad + c_3 \cos\beta\, \delta^2\left(\frac{dR}{dv}\right) + \delta\,(c_3 \cos\beta)\, \delta\left(\frac{dR}{dv}\right) + \delta^2\,(c_3 \cos\beta)\left(\frac{dR}{dv}\right)\Big\} \\
&+ \delta^2\,(c_1 \cos\beta) \int \Big\{ c_2 \cos\beta\, \delta\left(\frac{dR}{dr}\right) + \delta\,(c_2 \cos\beta)\left(\frac{dR}{dr}\right) + c_3 \sin\beta\, \delta\left(\frac{dR}{dv}\right) \\
&\qquad\qquad + \delta\,(c_3 \sin\beta)\left(\frac{dR}{dv}\right)\Big\} \\
&- \delta^2\,(c_1 \sin\beta) \int \Big\{ c_2 \sin\beta\, \delta\left(\frac{dR}{dr}\right) + \delta\,(c_2 \sin\beta)\left(\frac{dR}{dr}\right) + c_3 \cos\beta\, \delta\left(\frac{dR}{dv}\right) \\
&\qquad\qquad + \delta\,(c_3 \cos\beta)\left(\frac{dR}{dv}\right)\Big\} \\
&+ \delta^3\,(c_1 \cos\beta) \int \Big\{ c_2 \cos\beta\left(\frac{dR}{dr}\right) + c_3 \sin\beta\left(\frac{dR}{dv}\right)\Big\} \\
&\qquad - \delta^3\,(c_1 \sin\beta) \int \Big\{ c_2 \sin\beta\left(\frac{dR}{dr}\right) + c_3 \cos\beta\left(\frac{dR}{dv}\right)\Big\}
\end{aligned}\right\} . \quad (503)
$$

We shall now develop the different terms of this equation. If we multiply equations (228) and (498) together, we shall obtain

$$c_2 \cos\beta\, \delta^3 \left(\frac{dR}{dr}\right) =$$

$$\left.\begin{array}{l}
\dfrac{\overline{m}^3}{\mu^3} ndt \Big\{ \mp (1.515429) \cos nt \pm (0.0440395) \cos (3nt - 2n't) \\
\quad + (0.0440395) \cos (nt - 2n't) \mp (2.0936685) \cos (5nt - 4n't) \\
\quad - (2.0936685) \cos (3nt - 4n't) \mp (1.143794) \cos (7nt - 6n't) \\
\quad - (1.143794) \cos (5nt - 6n't) \pm (1.557839) \cos (9nt - 8n't) \\
\quad + (1.557839) \cos (7nt - 8n't) \pm \dfrac{a}{a'} (116001.1) \cos (2nt - n't) \\
\quad \pm \dfrac{a}{a'} (116001.1) \cos n't \mp \dfrac{a}{a'} (61515.95) \cos (4nt - 3n't) \\
\quad - \dfrac{a}{a'} (61515.95) \cos (2nt - 3n't) \mp \dfrac{a}{a'} (2123.790) \cos (6nt - 5n't) \\
\quad - \dfrac{a}{a'} (2123.790) \cos (4nt - 5n't) \mp \dfrac{a}{a'} (82.41168) \cos (8nt - 7n't) \\
\quad - \dfrac{a}{a'} (82.41168) \cos (6nt - 7n't) \pm \dfrac{a}{a'} (3.758861) \cos (10nt - 9n't) \\
\qquad\qquad + \dfrac{a}{a'} (3.758861) \cos (8nt - 9n't) \Big\}
\end{array}\right\} . \quad (504)$$

Equations (331) and (427) will give

$$\delta (c_2 \cos\beta)\, \delta^2 \left(\frac{dR}{dr}\right) =$$

$$\left.\begin{array}{l}
\dfrac{\overline{m}^3}{\mu^3} ndt \Big\{ - (1.350436) \cos nt \pm (4.994436) \cos (3nt - 2n't) \\
\quad - (2.386834) \cos (nt - 2n't) \mp (0.7001987) \cos (5nt - 4n't) \\
\quad + (1.793021) \cos (3nt - 4n't) \mp (1.0377075) \cos (7nt - 6n't) \\
\quad - (0.1315203) \cos (5nt - 6n't) \pm (1.520500) \cos (9nt - 8n't) \\
\quad - (0.5869093) \cos (7nt - 8n't) \pm \dfrac{a}{a'} (2088.711) \cos (2nt - n't) \\
\quad \mp \dfrac{a}{a'} (2568.806) \cos n't + \dfrac{a}{a'} (2571.070) \cos (4nt - 3n't)
\end{array}\right\} . \quad (505)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&-\frac{a}{a'}(618.4698)\cos(2nt-3n't) \mp \frac{a}{a'}(1148.168)\cos(6nt-5n't)\\
&+\frac{a}{a'}(449.7613)\cos(4nt-5n't) \mp \frac{a}{a'}(69.42426)\cos(8nt-7n't)\\
&+\frac{a}{a'}(35.98828)\cos(6nt-7n't) \pm \frac{a}{a'}(3.897840)\cos(10nt-9n't)\\
&\qquad\qquad -\frac{a}{a'}(1.174060)\cos(8nt-9n't)\Big\}
\end{aligned}\right\} . \quad (505)$$

Equations (327) and (430) will give

$$\delta^2(c_2\cos\beta)\,\delta\left(\frac{dR}{dr}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^8}{\mu^3}ndt\Big\{\pm(6.388842)\cos nt \pm (2.557701)\cos(3nt-2n't)\\
&-(3.392822)\cos(nt-2n't) \mp (7.135477)\cos(5nt-4n't)\\
&+(0.184650)\cos(3nt-4n't) \mp (1.098760)\cos(7nt-6n't)\\
&+(1.149678)\cos(5nt-6n't) \pm (1.405278)\cos(9nt-8n't)\\
&-(0.4651763)\cos(7nt-8n't) \pm \frac{a}{a'}(2451.316)\cos(2nt-n't)\\
&\mp\frac{a}{a'}(2876.089)\cos n't \pm \frac{a}{a'}(707.4526)\cos(4nt-3n't)\\
&-\frac{a}{a'}(1046.581)\cos(2nt-3n't) \mp \frac{a}{a'}(616.7230)\cos(6nt-5n't)\\
&+\frac{a}{a'}(116.4626)\cos(4nt-5n't) \mp \frac{a}{a'}(66.12894)\cos(8nt-7n't)\\
&+\frac{a}{a'}(18.460152)\cos(6nt-7n't) \pm \frac{a}{a'}(2.834049)\cos(10nt-9n't)\\
&\qquad\qquad -\frac{a}{a'}(1.776104)\cos(8nt-9n't)\Big\}
\end{aligned}\right\} . \quad (506)$$

Equations (218) and (501) will give

$$\delta^3(c_2\cos\beta)\left(\frac{dR}{dr}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^8}{\mu^3}ndt\Big\{\mp(0.37743)\cos nt \mp (0.023104)\cos(3nt-2n't)\\
&+(2.416043)\cos(nt-2n't) \mp (0.835421)\cos(5nt-4n't)
\end{aligned}\right\} . \quad (507)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&-(0.08565)\cos(3nt-4n't)\pm(0.029314)\cos(7nt-6n't)\\
&+(0.193851)\cos(5nt-6n't)\pm(1.530806)\cos(9nt-8n't)\\
&-(0.251089)\cos(7nt-8n't)\pm\frac{a}{a'}(15071.0)\cos(2nt-n't)\\
&\mp\frac{a}{a'}(10238.2)\cos n't\mp\frac{a}{a'}(40465.6)\cos(4nt-3n't)\\
&+\frac{a}{a'}(43270.2)\cos(2nt-3n't)\mp\frac{a}{a'}(1797.679)\cos(6nt-5n't)\\
&+\frac{a}{a'}(349.285)\cos(4nt-5n't)\mp\frac{a}{a'}(61.8415)\cos(8nt-7n't)\\
&+\frac{a}{a'}(12.4723)\cos(6nt-7n't)\pm\frac{a}{a'}(3.524204)\cos(10nt-9n't)\\
&\qquad\qquad\qquad\qquad+\frac{a}{a'}(0.849866)\cos(8nt-9n't)\Big\}
\end{aligned}\right\} . \quad (507)
$$

In like manner equations (234) and (499), (332) and (428), (328) and (431), (219) and (502), will give the four following equations:

$$
\delta c_3 \sin\beta\, \delta^3\left(\frac{dR}{dv}\right)=
$$

$$
\left.\begin{aligned}
&\frac{\overline{m}^3}{\mu^3}ndt\Big\{\mp(3.988125)\cos(3nt-2n't)+(3.988125)\cos(nt-2n't)\\
&\qquad\pm(1.569922)\cos(5nt-4n't)-(1.569922)\cos(3nt-4n't)\\
&\qquad\pm(0.025011)\cos(7nt-6n't)-(0.025011)\cos(5nt-6n't)\\
&\qquad\mp(0.670879)\cos(9nt-8n't)+(0.670879)\cos(7nt-8n't)\\
&\qquad\mp\frac{a}{a'}(242016.1)\cos(2nt-n't)\pm\frac{a}{a'}(242016.1)\cos n't\\
&\qquad\pm\frac{a}{a'}(86278.3)\cos(4nt-3n't)-\frac{a}{a'}(86278.3)\cos(2nt-3n't)\\
&\qquad\pm\frac{a}{a'}(1631.179)\cos(6nt-5n't)-\frac{a}{a'}(1631.179)\cos(4nt-5n't)\\
&\qquad\pm\frac{a}{a'}(38.99440)\cos(8nt-7n't)-\frac{a}{a'}(38.99440)\cos(6nt-7n't)\\
&\qquad\mp\frac{a}{a'}(1.631205)\cos(10nt-9n't)+\frac{a}{a'}(1.631205)\cos(8nt-9n't)\Big\}
\end{aligned}\right\} . \quad (508)
$$

$$\delta\,(c_3 \sin\beta)\,\delta^2\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^3}{\mu^3}\,ndt\Big\{\mp(2.008911)\cos nt\mp(0.7258974)\cos(3nt-2n't)\\
&\quad+(0.9021924)\cos(nt-2n't)\pm(3.858266)\cos(5nt-4n't)\\
&\quad-(1.418005)\cos(3nt-4n't)\mp(0.2652286)\cos(7nt-6n't)\\
&\quad+(0.0889336)\cos(5nt-6n't)\mp(0.9244336)\cos(9nt-8n't)\\
&\quad+(0.4930837)\cos(7nt-8n't)\pm\frac{a}{a'}(6779.817)\cos(2nt-n't)\\
&\quad\mp(5152.893)\cos n't\mp\frac{a}{a'}(5314.021)\cos(4nt-3n't)\\
&\quad+(2855.076)\cos(2nt-3n't)\pm\frac{a}{a'}(1727.963)\cos(6nt-5n't)\\
&\quad-\frac{a}{a'}(928.7185)\cos(4nt-5n't)\pm\frac{a}{a'}(51.81359)\cos(8nt-7n't)\\
&\quad-\frac{a}{a'}(18.11713)\cos(6nt-7n't)\mp\frac{a}{a'}(2.449982)\cos(10nt-9n't)\\
&\qquad\qquad+\frac{a}{a'}(1.529357)\cos(8nt-9n't)\Big\}
\end{aligned}\right\} \quad (509)$$

$$\delta^2\,(c_3 \sin\beta)\,\delta\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^3}{\mu^3}\,ndt\Big\{\pm(1.617058)\cos nt\mp(0.908312)\cos(3nt-2n't)\\
&\quad-(0.0015102)\cos(nt-2n't)\pm(0.2768708)\cos(5nt-4n't)\\
&\quad-(0.1935358)\cos(3nt-4n't)\pm(1.236861)\cos(7nt-6n't)\\
&\quad-(0.3270390)\cos(5nt-6n't)\mp(1.647632)\cos(9nt-8n't)\\
&\quad-(0.05276074)\cos(7nt-8n't)\pm\frac{a}{a'}(118.0866)\cos(2nt-n't)\\
&\quad\pm(512.0916)\cos n't\mp\frac{a}{a'}(593.8078)\cos(4nt-3n't)\\
&\quad-\frac{a}{a'}(945.3127)\cos(2nt-3n't)\pm\frac{a}{a'}(591.4314)\cos(6nt-5n't)
\end{aligned}\right\} \quad (510)$$

(Continued on the next page.)

$$\left.\begin{array}{l} + \frac{a}{a'}(227.6264)\cos(4nt-5n't) \pm \frac{a}{a'}(93.77454)\cos(8nt-7n't) \\ + \frac{a}{a'}(1.042963)\cos(6nt-7n't) \mp \frac{a}{a'}(4.010980)\cos(10nt-9n't) \\ \qquad\qquad - \frac{a}{a'}(0.9220684)\cos(8nt-9n't)\Big\} \end{array}\right\} . \quad (510)$$

$$\delta^3(c_3\sin\beta)\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{array}{l} \frac{\overline{m}^8}{\mu^3}ndt\Big\{ \mp(4.122196)\cos nt \mp (3.796336)\cos(3nt-2n't) \\ + (1.006920)\cos(nt-2n't) \pm (8.050031)\cos(5nt-4n't) \\ + (0.775940)\cos(3nt-4n't) \pm (2.498429)\cos(7nt-6n't) \\ + (0.290987)\cos(5nt-6n't) \mp (4.575909)\cos(9nt-8n't) \\ - (0.127866)\cos(7nt-8n't) \mp \frac{a}{a'}(32382.7)\cos(2nt-n't) \\ \mp \frac{a}{a'}(93472.5)\cos n't \pm \frac{a}{a'}(93291.4)\cos(4nt-3n't) \\ + \frac{a}{a'}(27881.7)\cos(2nt-3n't) \pm \frac{a}{a'}(4501.690)\cos(6nt-5n't) \\ - \frac{a}{a'}(6.438)\cos(4nt-5n't) \pm \frac{a}{a'}(188.1820)\cos(8nt-7n't) \\ + \frac{a}{a'}(7.0575)\cos(6nt-7n't) \mp \frac{a}{a'}(10.16805)\cos(10nt-9n't) \\ \qquad\qquad + \frac{a}{a'}(1.845388)\cos(8nt-9n't)\Big\} \end{array}\right\} . \quad (511)$$

If we now put the sum of equations (504–511) equal to $h\cos H$, we shall have

$$h\cos H=$$

$$\left.\begin{array}{l} \frac{\overline{m}^8}{\mu^3}ndt\Big\{ \mp(1.368502)\cos nt \mp (1.845798)\cos(3nt-2n't) \\ + (2.576153)\cos(nt-2n't) \pm (2.990325)\cos(5nt-4n't) \\ - (2.607170)\cos(3nt-4n't) \pm (0.244125)\cos(7nt-6n't) \\ + (0.096086)\cos(5nt-6n't) \mp (1.804431)\cos(9nt-8n't) \end{array}\right\} . \quad (512)$$

(*Continued on the next page.*)

$$\left.\begin{aligned}
&+(1.238001)\cos(7nt-8n't)\mp\frac{a}{a'}(131888.8)\cos(2nt-n't)\\
&\pm\frac{a}{a'}(244220.8)\cos n't\pm\frac{a}{a'}(74958.9)\cos(4nt-3n't)\\
&-\frac{a}{a'}(76397.6)\cos(2nt-3n't)\pm\frac{a}{a'}(2765.903)\cos(6nt-5n't)\\
&-\frac{a}{a'}(3546.990)\cos(4nt-5n't)\pm\frac{a}{a'}(92.9581)\cos(8nt-7n't)\\
&-\frac{a}{a'}(64.5019)\cos(6nt-7n't)\mp\frac{a}{a'}(4.245262)\cos(10nt-9n't)\\
&\qquad\qquad\qquad\qquad+\frac{a}{a'}(5.742445)\cos(8nt-9n't)\Big\}
\end{aligned}\right\}. \quad (512)$$

This equation gives by integration

$$\int h\cos H=$$

$$\left.\begin{aligned}
&\frac{\overline{m}^8}{\mu^3}\Big\{-(1.368502)\sin nt-(0.6475580)\sin(3nt-2n't)\\
&\pm(3.029352)\sin(nt-2n't)+(0.6361315)\sin(5nt-4n't)\\
&\mp(0.9653344)\sin(3nt-4n't)+(0.0372642)\sin(7nt-6n't)\\
&\pm(0.02106371)\sin(5nt-6n't)-(0.2147725)\sin(9nt-8n't)\\
&\pm(0.1933896)\sin(7nt-8n't)-\frac{a}{a'}(68506.59)\sin(2nt-n't)\\
&+\frac{a}{a'}(3264927.0)\sin n't+\frac{a}{a'}(19853.53)\sin(4nt-3n't)\\
&\mp\frac{a}{a'}(43026.46)\sin(2nt-3n't)+\frac{a}{a'}(491.6295)\sin(6nt-5n't)\\
&\mp\frac{a}{a'}(978.2120)\sin(4nt-5n't)+\frac{a}{a'}(12.43356)\sin(8nt-7n't)\\
&\mp\frac{a}{a'}(11.77818)\sin(6nt-7n't)-\frac{a}{a'}(0.4551684)\sin(10nt-9n't)\\
&\qquad\qquad\qquad\qquad\pm\frac{a}{a'}(0.7837602)\sin(8nt-9n't)\Big\}
\end{aligned}\right\}. \quad (513)$$

This equation will give $\int h \sin H$, by using the lower signs and changing *sin* to *cos* in the second member.

The first term of the value $\delta^3 \frac{d\delta_1 r}{dt}$ in equation (503) is equal to

$$\frac{1}{\sqrt{a\mu}} c_1 \cos\beta \int h \cos H - \frac{1}{\sqrt{a\mu}} c_1 \sin\beta \int h \sin H.$$

Therefore by means of equations (240) and (513) we shall obtain—

$$\left.\begin{aligned}
&\text{1st term of } \delta^3 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^8}{\mu^4} n \Big\{ + (2.381794) \sin 2(nt - n't) \\
&\quad - (0.3292029) \sin 4(nt - n't) + (0.0583279) \sin 6(nt - n't) \\
&\quad - (0.0213829) \sin 8(nt - n't) - \frac{a}{a'}(3333433.6) \sin (nt - n't) \\
&\quad - \frac{a}{a'}(24172.93) \sin 3(nt - n't) - \frac{a}{a'}(486.5825) \sin 5(nt - n't) \\
&\quad + \frac{a}{a'}(0.65538) \sin 7(nt - n't) + \frac{a}{a'}(0.3285918) \sin 9(nt - n't) \Big\}
\end{aligned}\right\} . \quad (514)$$

Equations (330) and (440), (342) and (429), (272) and (500), will give the three remaining terms of the value of $\delta^3 \frac{d\delta_1 r}{dt}$, as follows:

$$\left.\begin{aligned}
&\text{2nd term of } \delta^3 \frac{d\delta_1 r}{dt} = a\frac{\overline{m}^8}{\mu^4} n \Big\{ - (7.104198) \sin 2(nt - n't) \\
&\quad - (1.289885) \sin 4(nt - n't) - (0.3251771) \sin 6(nt - n't) \\
&\quad + (0.4809686) \sin 8(nt - n't) - \frac{a}{a'} - (44344.10) \sin (nt - n't) \\
&\quad - \frac{a}{a'}(43308.61) \sin 3(nt - n't) + \frac{a}{a'}(855.1089) \sin 5(nt - n't) \\
&\quad - \frac{a}{a'}(29.97550) \sin 7(nt - n't) + \frac{a}{a'}(1.135000) \sin 9(nt - n't) \Big\}
\end{aligned}\right\} . \quad (515)$$

3rd term of

$$\left.\begin{aligned}\delta^3 \frac{d\delta_1 r}{dt} = a \frac{\overline{m}^3}{\mu^4} n \Big\{ & - (0.920683) \sin 2(nt - n't) \\ & + (3.655966) \sin 4(nt - n't) - (2.740416) \sin 6(nt - n't) \\ & + (1.037936) \sin 8(nt - n't) - \frac{a}{a'}(4809.732) \sin (nt - n't) \\ & + \frac{a}{a'}(2750.120) \sin 3(nt - n't) - \frac{a}{a'}(1585.9033) \sin 5(nt - n't) \\ & - \frac{a}{a'}(49.18362) \sin 7(nt - n't) + \frac{a}{a'}(2.093701) \sin 9(nt - n't) \Big\}\end{aligned}\right\} . \quad (516)$$

4th term of

$$\left.\begin{aligned}\delta^3 \frac{d\delta_1 r}{dt} = a \frac{\overline{m}^3}{\mu^4} n \Big\{ & + (1.372404) \sin 2(nt - n't) \\ & - (2.146967) \sin 4(nt - n't) - (3.988334) \sin 6(nt - n't) \\ & + (3.844367) \sin 8(nt - n't) + \frac{a}{a'}(217068.3) \sin (nt - n't) \\ & - \frac{a}{a'}(158423.3) \sin 3(nt - n't) - \frac{a}{a'}(4900.780) \sin 5(nt - n't) \\ & - \frac{a}{a'}(192.2301) \sin 7(nt - n't) + \frac{a}{a'}(5.406769) \sin 9(nt - n't) \Big\}\end{aligned}\right\} . \quad (517)$$

If we now take the sum of equations (514–517), we shall obtain the complete value of $\delta^3 \frac{d\delta_1 r}{dt}$, as follows:

$$\left.\begin{aligned}\delta^3 \frac{d\delta_1 r}{dt} = a \frac{\overline{m}^3}{\mu^4} n \Big\{ & - (4.270583) \sin 2(nt - n't) \\ & - (0.110089) \sin 4(nt - n't) - (6.995599) \sin 6(nt - n't) \\ & + (5.341889) \sin 8(nt - n't) - \frac{a}{a'}(3165519.1) \sin (nt - n't) \\ & - \frac{a}{a'}(223154.7) \sin 3(nt - n't) - \frac{a}{a'}(7828.375) \sin 5(nt - n't) \\ & - \frac{a}{a'}(270.7338) \sin 7(nt - n't) + \frac{a}{a'}(8.964061) \sin 9(nt - n't) \Big\}\end{aligned}\right\} . \quad (518)$$

We must now find the value of $\delta^3 \frac{d\delta_4 r}{dt}$. If we develop equation (252′), we shall easily find

$$\left.\begin{aligned}\delta^3 \frac{d\delta_4 r}{dt} = \frac{1}{\sqrt{a\mu}} \frac{d\delta_1 r}{dt} \Bigg\{ \int \delta^2 \left(\frac{dR}{dv}\right) dt + \frac{2}{\sqrt{a\mu}} \int \left(\frac{dR}{dv}\right) dt \int \delta \left(\frac{dR}{dv}\right) dt \\ + \frac{1}{a\mu} \left[\int \left(\frac{dR}{dv}\right) dt\right]^3 \Bigg\} \\ + \frac{1}{\sqrt{a\mu}}\, \delta \frac{d\delta_1 r}{dt} \left\{ \int \delta \left(\frac{dR}{dv}\right) dt + \frac{1}{\sqrt{a\mu}} \left[\int \left(\frac{dR}{dv}\right) dt\right]^2 \right\} \\ + \frac{1}{\sqrt{a\mu}}\, \delta^2 \frac{d\delta_1 r}{dt} \int \left(\frac{dR}{dv}\right) dt\end{aligned}\right\} . \quad (519)$$

If we multiply equations (449) and (450) by (448), we shall obtain the two following equations:

$$\left.\begin{aligned}&\int \left(\frac{dR}{dv}\right) dt \int \delta \left(\frac{dR}{dv}\right) dt = \\ &a^4 \frac{\overline{m}^6}{\mu^3} n^2 \Big\{ -(0.1095220) + (0.08907344) \cos 2(nt - n't) \\ &\quad - (0.1095220) \cos 4(nt - n't) - (0.08907344) \cos 6(nt - n't) \\ &\quad + \frac{a}{a'} (21.29796) \cos (nt - n't) + \frac{a}{a'} (25.02037) \cos 3(nt - n't) \\ &\quad - \frac{a}{a'} (3.563640) \cos 5(nt - n't) + \frac{a}{a'} (0.2221521) \cos 7(nt - n't) \Big\}\end{aligned}\right\} . \quad (520)$$

$$\left.\begin{aligned}&\left[\int \left(\frac{dR}{dv}\right) dt\right]^3 = \\ &a^6 \frac{\overline{m}^6}{\mu^3} n^3 \Big\{ -(0.3995215) \cos 2(nt - n't) - (0.1331738) \cos 6(nt - n't) \\ &\quad - \frac{a}{a'} (0.4883040) \cos (nt - n't) - \frac{a}{a'} (0.5770867) \cos 3(nt - n't) \\ &\quad - \frac{a}{a'} (0.1331738) \cos 5(nt - n't) - \frac{a}{a'} (0.2219564) \cos 7(nt - n't) \Big\}\end{aligned}\right\} . \quad (521)$$

Equation (428) gives by integration

$$\int \delta^2\left(\frac{dR}{dv}\right)dt =$$

$$\left.\begin{aligned}\frac{\overline{m}^6}{a\mu^2 n}\Big\{ &+(1.700299)\cos 2(nt-n't)-(0.07460612)\cos 4(nt-n't)\\ &-(0.1216951)\cos 6(nt-n't)-\frac{a}{a'}(4002.205)\cos(nt-n't)\\ &+\frac{a}{a'}(455.6131)\cos 3(nt-n't)+\frac{a}{a'}(6.277654)\cos 5(nt-n't)\\ &\qquad\qquad\qquad -\frac{a}{a'}(0.2353217)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (522)$$

Whence we deduce

$$\frac{1}{\sqrt{a\mu}}\left\{\int\delta^2\left(\frac{dR}{dv}\right)dt+\frac{2}{\sqrt{a\mu}}\int\left(\frac{dR}{dv}\right)dt\int\delta\left(\frac{dR}{dv}\right)dt+\frac{1}{a\mu}\left[\int\left(\frac{dR}{dv}\right)dt\right]^3\right\}=$$

$$\left.\begin{aligned}\frac{\overline{m}^6}{\mu^3}\Big\{ &-(0.2190440)+(1.478924)\cos 2(nt-n't)\\ &-(0.2936501)\cos 4(nt-n't)-(0.4330158)\cos 6(nt-n't)\\ &-\frac{a}{a'}(3960.097)\cos(nt-n't)+\frac{a}{a'}(505.0768)\cos 3(nt-n't)\\ &-\frac{a}{a'}(0.982800)\cos 5(nt-n't)-\frac{a}{a'}(0.0129739)\cos 7(nt-n't)\Big\}\end{aligned}\right\}. \quad (523)$$

If we now multiply equations (446) and (523) together, we shall obtain the following value of

$$\left.\begin{aligned}\overset{\text{1st term of}}{\delta^3}\frac{d\delta_4 r}{dt}=a\frac{\overline{m}^8}{\mu^4}n\Big\{ &-(0.1720761)\sin 2(nt-n't)\\ &+(2.277789)\sin 4(nt-n't)-(0.3498400)\sin 6(nt-n't)\\ &+(0.5158733)\sin 8(nt-n't)-\frac{a}{a'}(5306.482)\sin(nt-n't)\\ &-\frac{a}{a'}(4727.718)\sin 3(nt-n't)+\frac{a}{a'}(607.2444)\sin 5(nt-n't)\\ &+\frac{a}{a'}(1.325484)\sin 7(nt-n't)-\frac{a}{a'}(0.3042876)\sin 9(nt-n't)\Big\}\end{aligned}\right\}. \quad (524)$$

If we multiply equations (447) and (451) together, and the product by $\frac{1}{\sqrt{a\mu}}$, we find

$$\left.\begin{aligned}
&\text{2nd term of } \delta^3\frac{d\delta_4 r}{dt} = a\frac{\overline{m}^8}{\mu^4}n\Big\{-(0.2787993)\sin 2(nt-n't)\\
&\quad+(0.739227)\sin 4(nt-n't)+(0.3070441)\sin 6(nt-n't)\\
&\quad+(0.1820317)\sin 8(nt-n't)-\frac{a}{a'}(28.3944)\sin(nt-n't)\\
&\quad-\frac{a}{a'}(95.7290)\sin 3(nt-n't)-\frac{a}{a'}(159.2020)\sin 5(nt-n't)\\
&\quad+\frac{a}{a'}(11.90516)\sin 7(nt-n't)-\frac{a}{a'}(0.0326296)\sin 9(nt-n't)\Big\}
\end{aligned}\right\}. \quad (525)$$

Lastly, equations (444) and (448) will give

$$\left.\begin{aligned}
&\text{3rd term of } \delta^3\frac{d\delta_4 r}{dt} = a\frac{\overline{m}^8}{\mu^4}n\Big\{+(1.756691)\sin 2(nt-n't)\\
&\quad-(2.604868)\sin 4(nt-n't)+(1.756691)\sin 6(nt-n't)\\
&\quad-(1.666253)\sin 8(nt-n't)-\frac{a}{a'}(17732.88)\sin(nt-n't)\\
&\quad+\frac{a}{a'}(19117.35)\sin 3(nt-n't)-\frac{a}{a'}(1.142262)\sin 5(nt-n't)\\
&\quad+\frac{a}{a'}(59.44884)\sin 7(nt-n't)-\frac{a}{a'}(3.115927)\sin 9(nt-n't)\Big\}
\end{aligned}\right\}. \quad (526)$$

If we now take the sum of equations (524–526), we shall get

$$\left.\begin{aligned}
&\delta^3\frac{d\delta_4 r}{dt} = a\frac{\overline{m}^8}{\mu^4}n\Big\{+(1.305816)\sin 2(nt-n't)\\
&\quad+(0.412148)\sin 4(nt-n't)+(1.713895)\sin 6(nt-n't)\\
&\quad-(0.968348)\sin 8(nt-n't)-\frac{a}{a'}(23067.75)\sin(nt-n't)\\
&\quad+\frac{a}{a'}(14293.90)\sin 3(nt-n't)+\frac{a}{a'}(446.9001)\sin 5(nt-n't)\\
&\quad+\frac{a}{a'}(72.67948)\sin 7(nt-n't)-\frac{a}{a'}(3.452845)\sin 9(nt-n't)\Big\}
\end{aligned}\right\}. \quad (527)$$

Adding equations (518) and (527) together, we get the complete value of $\delta^3\left(\frac{d\delta_1 r}{dt}\right)$ or $\delta^3 \frac{d\delta r}{dt}$, as follows:

$$\left.\begin{aligned}\delta^3 \frac{d\delta r}{dt} = a\frac{\overline{m}^3}{\mu^4} n \Big\{ & - (2.964767) \sin 2(nt - n't) \\ & + (0.302059) \sin 4(nt - n't) - (5.281704) \sin 6(nt - n't) \\ & + (4.373541) \sin 8(nt - n't) - \frac{a}{a'}(3188586.8) \sin (nt - n't) \\ & - \frac{a}{a'}(208860.8) \sin 3(nt - n't) - \frac{a}{a'}(7381.475) \sin 5(nt - n't) \\ & - \frac{a}{a'}(198.0543) \sin 7(nt - n't) + \frac{a}{a'}(5.511216) \sin 9(nt - n't) \Big\}\end{aligned}\right\} . \quad (528)$$

This equation will give by integration, the numbers in brackets being logarithms,

$$\left.\begin{aligned}\delta^4 r = a\frac{\overline{m}^3}{\mu^4} \Big\{ & + [0.4832425] + [0.2047256] \cos 2(nt - n't) \\ & - [8.9117968] \cos 4(nt - n't) + [9.9783877] \cos 6(nt - n't) \\ & - [9.7715082] \cos 8(nt - n't) + \frac{a}{a'}[6.5373632] \cos (nt - n't) \\ & + \frac{a}{a'}[4.8765007] \cos 3(nt - n't) + \frac{a}{a'}[3.2029381] \cos 5(nt - n't) \\ & + \frac{a}{a'}[1.4854512] \cos 7(nt - n't) - \frac{a}{a'}[9.8207700] \cos 9(nt - n't) \Big\}\end{aligned}\right\} . \quad (529)$$

39. We must now determine the values of $\delta^3 \frac{d\delta_1 v}{dt}$ and $\delta^3 \frac{d\delta_0 v}{dt}$. These quantities are given by the following equations:

$$\left.\begin{aligned}\delta^3 \frac{d\delta_1 v}{dt} = -2\frac{dv_1}{r_1 dt}\delta^4 r + 6\frac{dv_1}{r_1^2 dt}\{\delta r \delta^3 r + \tfrac{1}{2}(\delta^2 r)^2\} & \\ - 12\frac{dv_1}{r_1^3 dt}\delta r^2 \delta^2 r + 5\frac{dv_1}{r_1^4 dt}\delta r^4 &\end{aligned}\right\} . \quad (530)$$

$$\left.\begin{aligned}\delta^3 \frac{d\delta_0 v}{dt} = -\frac{1}{r_1^2}\int\delta^3\left(\frac{dR}{dv}\right)dt + \frac{2}{r_1^3}\delta r\int\delta^2\left(\frac{dR}{dv}\right)dt + \frac{2}{r_1^3}\delta^2 r\int\delta\left(\frac{dR}{dv}\right)dt & \\ + \frac{2}{r_1^3}\delta^3 r\int\left(\frac{dR}{dv}\right)dt - \frac{3}{r_1^4}\delta r^2\int\delta\left(\frac{dR}{dv}\right)dt - \frac{6}{r_1^4}\delta r\delta^2 r\int\left(\frac{dR}{dv}\right)dt & \\ + \frac{4}{r_1^5}\delta r^3\int\left(\frac{dR}{dv}\right)dt &\end{aligned}\right\} . \quad (531)$$

In these equations we may put $r = a$ and $\frac{dv_1}{dt} = n$. The first term of $\delta^3 \frac{d\delta_1 v}{dt}$ will therefore become—

$$\left.\begin{aligned}
\text{1st term of } \delta^3 \frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4} n \Big\{ &-(3.204464)\cos 2(nt-n't) + (0.1632401)\cos 4(nt-n't) \\
&-(1.902907)\cos 6(nt-n't) + (1.181784)\cos 8(nt-n't) \\
&-\frac{a}{a'}(6892762.0)\cos(nt-n't) - \frac{a}{a'}(150498.0)\cos 3(nt-n't) \\
&-\frac{a}{a'}(3191.304)\cos 5(nt-n't) - \frac{a}{a'}(61.16192)\cos 7(nt-n't) \\
&+\frac{a}{a'}(1.323732)\cos 9(nt-n't) \Big\}
\end{aligned}\right\} . \quad (532)$$

We also obtain, from what precedes,

$$\left.\begin{aligned}
\delta r \delta^3 r = a^2 \frac{\overline{m}^8}{\mu^4} \Big\{ &+(0.8602908) + (1.946328)\cos 2(nt-n't) \\
&+(0.7636650)\cos 4(nt-n't) - (0.4234314)\cos 6(nt-n't) \\
&+(0.3345350)\cos 8(nt-n't) - \frac{a}{a'}(42222.59)\cos(nt-n't) \\
&-\frac{a}{a'}(33152.90)\cos 3(nt-n't) - \frac{a}{a'}(711.6585)\cos 5(nt-n't) \\
&-\frac{a}{a'}(18.96724)\cos 7(nt-n't) + \frac{a}{a'}(0.3439853)\cos 9(nt-n't) \Big\}
\end{aligned}\right\} . \quad (533)$$

$$\left.\begin{aligned}
(\delta^2 r)^2 = a^2 \frac{\overline{m}^8}{\mu^4} \Big\{ &+(3.256109) + (3.157412)\cos 2(nt-n't) \\
&-(0.783071)\cos 4(nt-n't) - (0.9266144)\cos 6(nt-n't) \\
&+(0.2068421)\cos 8(nt-n't) + \frac{a}{a'}(3196.570)\cos(nt-n't) \\
&+\frac{a}{a'}(650.6704)\cos 3(nt-n't) - \frac{a}{a'}(449.5788)\cos 5(nt-n't) \\
&-\frac{a}{a'}(13.13694)\cos 7(nt-n't) + \frac{a}{a'}(0.0595954)\cos 9(nt-n't) \Big\}
\end{aligned}\right\} . \quad (534)$$

$$\left.\begin{aligned}\delta r^2\delta^2 r = a^3\frac{\overline{m}^8}{\mu^4}\Big\{ &+ (1.257040) + (2.301944)\cos 2(nt-n't)\\ &+ (0.933186)\cos 4(nt-n't) + (0.4591558)\cos 6(nt-n't)\\ &- (0.2666142)\cos 8(nt-n't) + \frac{a}{a'}(757.3580)\cos(nt-n't)\\ &+ \frac{a}{a'}(452.9772)\cos 3(nt-n't) + \frac{a}{a'}(307.8641)\cos 5(nt-n't)\\ &+ \frac{a}{a'}(14.30162)\cos 7(nt-n't) - \frac{a}{a'}(0.2778807)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (535)$$

$$\left.\begin{aligned}\delta r^4 = a^4\frac{\overline{m}^8}{\mu^4}\Big\{ &+ (1.169924) + (1.091390)\cos 2(nt-n't)\\ &+ (1.512811)\cos 4(nt-n't) + (0.3558483)\cos 6(nt-n't)\\ &+ (0.3436589)\cos 8(nt-n't) - \frac{a}{a'}(66.42423)\cos(nt-n't)\\ &- \frac{a}{a'}(55.75468)\cos 3(nt-n't) - \frac{a}{a'}(14.29128)\cos 5(nt-n't)\\ &- \frac{a}{a'}(13.95008)\cos 7(nt-n't) + \frac{a}{a'}(0.5137195)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (536)$$

These four equations give the following terms of $\delta^3\dfrac{d\delta_1 v}{dt}$:

2nd term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4}n\Big\{ &+ (11.677968)\cos 2(nt-n't)\\ &+ (4.581990)\cos 4(nt-n't) - (2.540588)\cos 6(nt-n't)\\ &+ (2.007210)\cos 8(nt-n't) - \frac{a}{a'}(253335.54)\cos(nt-n't)\\ &- \frac{a}{a'}(198917.4)\cos 3(nt-n't) - \frac{a}{a'}(4269.951)\cos 5(nt-n't)\\ &- \frac{a}{a'}(113.80344)\cos 7(nt-n't) + \frac{a}{a'}(2.063912)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (537)$$

3rd term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4}n\Big\{ &+ (9.472236)\cos 2(nt-n't) - (2.349213)\cos 4(nt-n't) \\ &- (2.779843)\cos 6(nt-n't) + (0.6205263)\cos 8(nt-n't) \\ &+ \frac{a}{a'}(9589.71)\cos(nt-n't) + \frac{a}{a'}(1952.01)\cos 3(nt-n't) \\ &- \frac{a}{a'}(1348.7364)\cos 5(nt-n't) - \frac{a}{a'}(39.41081)\cos 7(nt-n't) \\ &\qquad\qquad + \frac{a}{a'}(0.1787861)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (538)$$

4th term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4}n\Big\{ &- (27.623328)\cos 2(nt-n't) \\ &- (11.198232)\cos 4(nt-n't) - (5.5098696)\cos 6(nt-n't) \\ &+ (3.1993704)\cos 8(nt-n't) - \frac{a}{a'}(9088.30)\cos(nt-n't) \\ &- \frac{a}{a'}(5435.62)\cos 3(nt-n't) - \frac{a}{a'}(3694.3692)\cos 5(nt-n't) \\ &- \frac{a}{a'}(171.61944)\cos 7(nt-n't) + \frac{a}{a'}(3.3345684)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (539)$$

5th term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4}n\Big\{ &+ (5.456954)\cos 2(nt-n't) + (7.564055)\cos 4(nt-n't) \\ &+ (1.779242)\cos 6(nt-n't) + (1.7182945)\cos 8(nt-n't) \\ &- \frac{a}{a'}(332.12)\cos(nt-n't) - \frac{a}{a'}(278.77)\cos 3(nt-n't) \\ &- \frac{a}{a'}(71.4564)\cos 5(nt-n't) - \frac{a}{a'}(69.75040)\cos 7(nt-n't) \\ &\qquad\qquad + \frac{a}{a'}(2.5685975)\cos 9(nt-n't)\Big\}\end{aligned}\right\} . \quad (540)$$

If we now take the sum of the different terms of $\delta^3\frac{d\delta_1 v}{dt}$, which we have computed, we shall obtain the following expression for its complete value:

$$\left.\begin{aligned}\delta^3\frac{d\delta_1 v}{dt} = \frac{\overline{m}^8}{\mu^4}n\Big\{ &-(4.220634)\cos 2(nt-n't) - (1.239160)\cos 4(nt-n't) \\ &- (10.953967)\cos 6(nt-n't) + (8.7271856)\cos 8(nt-n't)\end{aligned}\right\} . \quad (541)$$

(Continued on the next page.)

$$\left.\begin{aligned}&-\frac{a}{a'}(7145928.0)\cos(nt-n't)-\frac{a}{a'}(353177.8)\cos 3(nt-n't)\\&-\frac{a}{a'}(12575.82)\cos 5(nt-n't)-\frac{a}{a'}(455.8460)\cos 7(nt-n't)\\&\qquad\qquad+\frac{a}{a'}(9.469596)\cos 9(nt-n't)\Big\}\end{aligned}\right\}. \quad (541)$$

We shall now find the value of $\delta^3\dfrac{d\delta_0 v}{dt}$. If we multiply equation (499) by dt, we shall find by integration—

$$\int\delta^3\left(\frac{dR}{dv}\right)dt=$$

$$\left.\begin{aligned}&a^2\frac{\overline{m}^8}{\mu^4}n\Big\{-(2.155280)\cos 2(nt-n't)+(0.4242121)\cos 4(nt-n't)\\&+(0.0045055)\cos 6(nt-n't)-(0.0906398)\cos 8(nt-n't)\\&-\frac{a}{a'}(261582.8)\cos(nt-n't)+\frac{a}{a'}(31084.60)\cos 3(nt-n't)\\&+\frac{a}{a'}(352.6116)\cos 5(nt-n't)+\frac{a}{a'}(6.021007)\cos 7(nt-n't)\\&\qquad\qquad-\frac{a}{a'}(0.1958985)\cos 9(nt-n't)\Big\}\end{aligned}\right\}. \quad (542)$$

If we divide this equation by $-a^2$, we shall obtain the first term of $\delta^3\dfrac{d\delta_0 v}{dt}$.

Equations (411) and (464), (417) and (449), (457) and (448), (419) and (449), (426) and (448), (425) and (448), will give the following terms of $\delta^3\dfrac{d\delta_0 v}{dt}$:

2nd term of

$$\left.\begin{aligned}&\delta^3\frac{d\delta_0 v}{dt}=\frac{\overline{m}^8}{\mu^4}n\Big\{-(0.4706983)\cos 2(nt-n't)\\&-(2.007852)\cos 4(nt-n't)+(0.1366330)\cos 6(nt-n't)\\&+(0.1567031)\cos 8(nt-n't)+\frac{a}{a'}(5922.977)\cos(nt-n't)\\&+\frac{a}{a'}(5015.471)\cos 3(nt-n't)-\frac{a}{a'}(591.9252)\cos 5(nt-n't)\\&-\frac{a}{a'}(9.604934)\cos 7(nt-n't)+\frac{a}{a'}(0.3615785)\cos 9(nt-n't)\Big\}\end{aligned}\right\}. \quad (543)$$

3rd term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_0 v}{dt}=\frac{\overline{m}^8}{\mu^4}n\Big\{&+(0.2756000)\cos 2(nt-n't)\\&-(0.1328998)\cos 4(nt-n't)-(0.4904000)\cos 6(nt-n't)\\&+(1.066832)\cos 8(nt-n't)-\frac{a}{a'}(53.2946)\cos(nt-n't)\\&-\frac{a}{a'}(12.9599)\cos 3(nt-n't)-\frac{a}{a'}(106.6769)\cos 5(nt-n't)\\&-\frac{a}{a'}(10.599691)\cos 7(nt-n't)+\frac{a}{a'}(0.2754597)\cos 9(nt-n't)\Big\}\end{aligned}\right\} \quad (544)$$

4th term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_0 v}{dt}=\frac{\overline{m}^8}{\mu^4}n\Big\{&+(2.267675)\cos 2(nt-n't)\\&+(1.1277473)\cos 4(nt-n't)-(0.6421678)\cos 6(nt-n't)\\&+(0.4212049)\cos 8(nt-n't)-\frac{a}{a'}(42383.48)\cos(nt-n't)\\&-\frac{a}{a'}(41508.52)\cos 3(nt-n't)-\frac{a}{a'}(892.5727)\cos 5(nt-n't)\\&-\frac{a}{a'}(19.64040)\cos 7(nt-n't)+\frac{a}{a'}(0.6266978)\cos 9(nt-n't)\Big\}\end{aligned}\right\} \quad (545)$$

5th term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_0 v}{dt}=\frac{\overline{m}^8}{\mu^4}n\Big\{&-(0.8891094)\cos 2(nt-n't)\\&+(0.3909182)\cos 4(nt-n't)-(0.1945364)\cos 6(nt-n't)\\&+(0.2732890)\cos 8(nt-n't)+\frac{a}{a'}(191.0694)\cos(nt-n't)\\&+\frac{a}{a'}(98.16946)\cos 3(nt-n't)+\frac{a}{a'}(71.67921)\cos 5(nt-n't)\\&-\frac{a}{a'}(16.45906)\cos 7(nt-n't)+\frac{a}{a'}(0.2088019)\cos 9(nt-n't)\Big\}\end{aligned}\right\} \quad (546)$$

6th term of

$$\left.\begin{aligned}\delta^3\frac{d\delta_0 v}{dt}=\frac{\overline{m}^8}{\mu^4}n\Big\{&-(7.654187)\cos 2(nt-n't)\\&-(3.008402)\cos 4(nt-n't)-(1.995034)\cos 6(nt-n't)\\&+(1.007063)\cos 8(nt-n't)-\frac{a}{a'}(2599.540)\cos(nt-n't)\end{aligned}\right\} \quad (547)$$

(Continued on the next page.)

$$\left.\begin{array}{l} -\dfrac{a}{a'}(1478.424)\cos 3(nt-n't)-\dfrac{a}{a'}(1164.411)\cos 5(nt-n't) \\ -\dfrac{a}{a'}(44.24381)\cos 7(nt-n't)+\dfrac{a}{a'}(1.505727)\cos 9(nt-n't)\Big\} \end{array}\right\} . \quad (547)$$

7th term of

$$\left.\begin{array}{l} \delta^3\dfrac{d\delta_0 v}{dt}=\dfrac{\overline{m}^8}{\mu^4}n\Big\{+(2.031177)\cos 2(nt-n't) \\ \quad +(3.635514)\cos 4(nt-n't)+(0.6720553)\cos 6(nt-n't) \\ \quad +(0.8653856)\cos 8(nt-n't)-\dfrac{a}{a'}(102.76509)\cos(nt-n't) \\ \quad -\dfrac{a}{a'}(91.53491)\cos 3(nt-n't)-\dfrac{a}{a'}(35.07737)\cos 5(nt-n't) \\ \quad -\dfrac{a}{a'}(25.54356)\cos 7(nt-n't)+\dfrac{a}{a'}(1.6913726)\cos 9(nt-n't)\Big\} \end{array}\right\} . \quad (548)$$

If we now divide equation (542) by $-a^2$, and add the quotient to the sum of equations (543–548), we shall obtain the complete value of $\delta^3\dfrac{d\delta_0 v}{dt}$, as follows:

$$\left.\begin{array}{l} \delta^3\dfrac{d\delta_0 v}{dt}=\dfrac{\overline{m}^8}{\mu^4}n\Big\{-(2.284262)\cos 2(nt-n't) \\ \quad -(0.419187)\cos 4(nt-n't)-(2.557956)\cos 6(nt-n't) \\ \quad +(3.881118)\cos 8(nt-n't)+\dfrac{a}{a'}(222557.1)\cos(nt-n't) \\ \quad -\dfrac{a}{a'}(69062.39)\cos 3(nt-n't)-\dfrac{a}{a'}(3071.595)\cos 5(nt-n't) \\ \quad -\dfrac{a}{a'}(132.11409)\cos 7(nt-n't)-\dfrac{a}{a'}(4.8655360)\cos 9(nt-n't)\Big\} \end{array}\right\} . \quad (549)$$

If we now take the sum of equations (541) and (549), we shall obtain the value of $\delta^3\dfrac{d\delta v}{dt}$, as follows:

$$\left.\begin{array}{l} \delta^3\dfrac{d\delta v}{dt}=\dfrac{\overline{m}^8}{\mu^4}n\Big\{-(6.504896)\cos 2(nt-n't) \\ \quad -(1.658341)\cos 4(nt-n't)-(13.511923)\cos 6(nt-n't) \\ \quad +(12.608304)\cos 8(nt-n't)-\dfrac{a}{a'}(6923370.0)\cos(nt-n't) \end{array}\right\} . \quad (550)$$

(Continued on the next page.)

$$\left.\begin{aligned} &-\frac{a}{a'}(422240.2)\cos 3(nt-n't)-\frac{a}{a'}(15647.412)\cos 5(nt-n't) \\ &-\frac{a}{a'}(587.96010)\cos 7(nt-n't)+\frac{a}{a'}(14.33513)\cos 9(nt-n't)\Big\} \end{aligned}\right\} . \quad (550)$$

This equation gives by integration

$$\left.\begin{aligned} \delta^4 v = \frac{\overline{m}^8}{\mu^4}\Big\{ &-[0.5459753]\sin 2(nt-n't)-[9.6513789]\sin 4(nt-n't) \\ &-[0.3863308]\sin 6(nt-n't)+[0.2313316]\sin 8(nt-n't) \\ &-\frac{a}{a'}[6.8740825]\sin (nt-n't)-\frac{a}{a'}[5.1822033]\sin 3(nt-n't) \\ &-\frac{a}{a'}[3.5292295]\sin 5(nt-n't)-\frac{a}{a'}[1.9580148]\sin 7(nt-n't) \\ &\qquad\qquad +\frac{a}{a'}[0.2359242]\sin 9(nt-n't)\Big\} \end{aligned}\right\} . \quad (551)$$

The numbers in brackets are logarithms.

We have thus obtained the perturbations in the longitude and in the radius vector, arising from the fourth power of the sun's disturbing force. The only term of sensible magnitude in the value of $\delta^4 v$ is the one depending on sin $(nt-n't)$, or the parallactic inequality as it is called. This term amounts to about 3.″87, and if we estimate by induction, the sum of all the succeeding terms of the parallactic inequality would amount to about 2.″10. It therefore seems hardly worth while to compute the terms arising from the higher powers of the disturbing force.

CHAPTER VII.

PERTURBATIONS OF THE ELEMENTS ARISING FROM THE FIRST AND SECOND POWERS OF THE DISTURBING FORCE.

40. WE have given the differential equations of the elements in equations (E–I), page 40, 41. The values of $\frac{d\Omega}{dt}$, $\frac{d\gamma}{dt}$, and $d\frac{\mu}{a}$ are there given in a very convenient form for computation; but the values of $\frac{d\omega}{dt}$ and $\frac{de}{dt}$ have coefficients of complicated form, and we shall simplify the expressions by the following substitutions.

If we put

$$\boxed{0} = r^2\left\{\frac{\cos\theta}{\cos\theta_0}\cos(v-\omega)\frac{dv}{dt} - \frac{\sin\theta}{\cos\theta_0}\sin(v-\omega)\frac{d\theta}{dt}\right\}; \tag{552}$$

$$\boxed{1} = \left\{\frac{\cos(v-\omega)}{\cos\theta_0\cos\theta}\frac{dr}{dt} - 2r\frac{\cos\theta}{\cos\theta_0}\sin(v-\omega)\frac{dv}{dt}\right\}; \tag{553}$$

$$\boxed{2} = -\left\{\frac{\sin\theta}{\cos\theta_0}\frac{dr}{dt} + 2r\frac{\cos\theta}{\cos\theta_0}\frac{d\theta}{dt}\right\}\sin(v-\omega); \tag{554}$$

$$\left.\begin{aligned}\boxed{3} = r^2\Big\{ &\{\sin\theta_0\cos\theta - \cos\theta_0\sin\theta\cos(v-\omega)\}\frac{d\theta}{dt}\\ &\qquad - \cos\theta_0\cos\theta\sin(v-\omega)\frac{dv}{dt}\Big\};\end{aligned}\right\}. \tag{555}$$

$$\left.\begin{aligned}\boxed{4} = -\Big\{&\frac{\cos\theta_0}{\cos\theta}\sin(v-\omega)\frac{dr}{dt} + 2r\,\{\cos\theta_0\cos\theta\cos(v-\omega)\\ &\qquad + \sin\theta_0\sin\theta\}\frac{dv}{dt}\Big\};\end{aligned}\right\}. \tag{556}$$

$$\left.\begin{aligned}\boxed{5} = &\{\sin\theta_0\cos\theta - \cos\theta_0\sin\theta\cos(v-\omega)\}\frac{dr}{dt}\\ &- 2r\,\{\cos\theta_0\cos\theta\cos(v-\omega) + \sin\theta_0\sin\theta\}\frac{d\theta}{dt};\end{aligned}\right\}. \tag{557}$$

Equations (G) and (H) will become

$$ue\frac{d\omega}{dt} = \boxed{0}\left(\frac{dR}{dr}\right) + \boxed{1}\left(\frac{dR}{dv}\right) + \boxed{2}\left(\frac{dR}{d\theta}\right); \tag{558}$$

$$\mu\frac{de}{dt} = \boxed{3}\left(\frac{dR}{dr}\right) + \boxed{4}\left(\frac{dR}{dv}\right) + \boxed{5}\left(\frac{dR}{d\theta}\right); \tag{559}$$

We shall now give the development of these equations. For this purpose we must first substitute the elliptical values of r, v, and θ in equations (552–557), by which means they will become

$$\left.\begin{aligned}\boxed{0} = a^2n\Big\{ &-e\{1-\tfrac{1}{2}e^2\} + \{1-\tfrac{13}{8}e^2+\tfrac{109}{192}e^4+\tfrac{1}{8}\gamma^4\}\cos(nt-\omega) \\ &+e\{1-\tfrac{11}{6}e^2\}\cos 2(nt-\omega) + \tfrac{9}{8}e^2\{1-\tfrac{33}{16}e^2\}\cos 3(nt-\omega) \\ &+\tfrac{4}{3}e^3\cos 4(nt-\omega) + \tfrac{625}{384}e^4\cos 5(nt-\omega) \\ &-\tfrac{1}{4}\gamma^2\{1-\tfrac{13}{8}e^2-\tfrac{1}{2}\gamma^2\}\cos(nt+\omega-2\Omega) \\ &-\tfrac{1}{4}\gamma^2\{1-\tfrac{13}{8}e^2-\tfrac{1}{2}\gamma^2\}\cos(nt-3\omega+2\Omega) \\ &-\tfrac{1}{4}e\gamma^2\cos 2(nt-\Omega) + \tfrac{1}{2}e\gamma^2\cos 2(\omega-\Omega) - \tfrac{1}{4}e\gamma^2\cos(2nt-4\omega+2\Omega) \\ &-\tfrac{9}{32}e^2\gamma^2\cos(3nt-\omega-2\Omega) - \tfrac{9}{32}e^2\gamma^2\cos(3nt-5\omega+2\Omega)\Big\}\end{aligned}\right\} \tag{560}$$

$$\left.\begin{aligned}\boxed{1} = 2an\Big\{ &-\{1-\tfrac{7}{8}e^2+\tfrac{37}{192}e^4+\tfrac{1}{4}e^2\gamma^2\}\sin(nt-\omega) \\ &-\tfrac{5}{4}e\{1-\tfrac{37}{30}e^2-\tfrac{1}{10}\gamma^2\}\sin 2(nt-\omega) \\ &-\{\tfrac{13}{8}e^2-\tfrac{319}{128}e^4-\tfrac{1}{4}e^2\gamma^2+\tfrac{1}{8}\gamma^4\}\sin 3(nt-\omega) - \tfrac{103}{48}e^3\sin 4(nt-\omega) \\ &-\tfrac{1097}{384}e^4\sin 5(nt-\omega) + \tfrac{1}{4}\gamma^2\{1-e^2-\tfrac{1}{2}\gamma^2\}\sin(nt-3\omega+2\Omega) \\ &+\tfrac{1}{4}\gamma^2\{1-8e^2-\tfrac{1}{2}\gamma^2\}\sin(3nt-\omega-2\Omega) + \tfrac{5}{16}e^2\gamma^2\sin(nt+\omega-2\Omega) \\ &-\tfrac{9}{16}e\gamma^2\sin 2(nt-\Omega) + \tfrac{5}{16}e\gamma^2\sin(2nt-4\omega+2\Omega) \\ &+\tfrac{13}{16}e\gamma^2\sin(4nt-2\omega-2\Omega) + \tfrac{1}{16}e\gamma^2\sin 2(\omega-\Omega) \\ &+\tfrac{13}{32}e^2\gamma^2\sin(3nt-5\omega+2\Omega) + \tfrac{59}{32}e^2\gamma^2\sin(5nt-3\omega-2\Omega) \\ &+\tfrac{1}{16}\gamma^4\sin(3nt+\omega-4\Omega) - \tfrac{1}{16}\gamma^4\sin(5nt-\omega-4\Omega)\Big\}\end{aligned}\right\} \tag{561}$$

$$\left.\begin{aligned}\boxed{2} = 2an\Big\{ &-\tfrac{1}{2}\gamma\{1-\tfrac{15}{4}e^2-\tfrac{3}{8}\gamma^2\}\sin(2nt-\omega-\Omega) \\ &+\tfrac{1}{2}\gamma\{1-\tfrac{1}{2}e^2\}\sin(\omega-\Omega) - \tfrac{9}{8}e\gamma\sin(3nt-2\omega-\Omega) \\ &+\tfrac{3}{4}e\gamma\sin(nt-\Omega) - \tfrac{3}{8}e\gamma\sin(nt-2\omega+\Omega) \\ &-2e^2\gamma\sin(4nt-3\omega-\Omega) - \tfrac{3}{8}\gamma\{e^2-\tfrac{1}{2}\gamma^2\}\sin(2nt-3\omega+\Omega) \\ &+\tfrac{1}{16}\gamma^3\sin(4nt-\omega-3\Omega) - \tfrac{1}{16}\gamma^3\sin 3(\omega-\Omega)\Big\}\end{aligned}\right\} \tag{562}$$

$$\left.\begin{aligned}\boxed{3} = a^2n\Big\{ & - \{1 - \tfrac{11}{8}e^2 + \tfrac{77}{192}e^4\} \sin(nt - \omega) \\ & - e\{1 - \tfrac{5}{3}e^2\} \sin 2(nt - \omega) - \tfrac{9}{8}e^2\{1 - \tfrac{279}{144}e^2\} \sin 3(nt - \omega) \\ & - \tfrac{4}{3}e^3 \sin 4(nt - \omega) - \tfrac{625}{384} \sin 2(nt - \omega)\Big\}\end{aligned}\right\} . \quad (563)$$

$$\left.\begin{aligned}\boxed{4} = 2an\Big\{ & \tfrac{1}{4}e\{1 - \tfrac{1}{4}e^2\} - \{1 - \tfrac{9}{8}e^2 + \tfrac{29}{192}e^4 + \tfrac{1}{16}\gamma^4\} \cos(nt - \omega) \\ & - e\{\tfrac{5}{4} - \tfrac{41}{24}e^2\} \cos 2(nt - \omega) - \{\tfrac{13}{8}e^2 - \tfrac{337}{128}e^4 + \tfrac{1}{16}\gamma^4\} \cos 3(nt - \omega) \\ & - \tfrac{103}{48}e^3 \cos 4(nt - \omega) - \tfrac{1097}{384}e^4 \cos 5(nt - \omega) \\ & + \tfrac{1}{4}\gamma^2\{1 - \tfrac{7}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(nt + \omega - 2\Omega) \\ & + \tfrac{1}{4}\gamma^2\{1 - \tfrac{43}{8}e^2 - \tfrac{1}{2}\gamma^2\} \cos(3nt - \omega - 2\Omega) - \tfrac{5}{16}e\gamma^2 \cos 2(\omega - \Omega) \\ & + \tfrac{13}{16}e\gamma^2 \cos(4nt - 2\omega - 2\Omega) + \tfrac{59}{32}e^2\gamma^2 \cos(5nt - 3\omega - 2\Omega) \\ & - \tfrac{1}{32}e^2\gamma^2 \cos(nt - 3\omega + 2\Omega) - \tfrac{1}{16}\gamma^4 \cos(5nt - \omega + 4\Omega) \\ & + \tfrac{1}{16}\gamma^4 \cos(nt + 3\omega - 4\Omega)\Big\}\end{aligned}\right\} . \quad (564)$$

$$\left.\begin{aligned}\boxed{5} = 2an\Big\{ & - \tfrac{1}{2}\gamma\{1 - \tfrac{3}{8}\gamma^2 - \tfrac{11}{4}e^2\} \cos(2nt - \omega - \Omega) \\ & - \tfrac{1}{2}\gamma\{1 - \tfrac{1}{2}e^2 - \tfrac{1}{2}\gamma^2\} \cos(\omega - \Omega) - \tfrac{9}{8}e\gamma \cos(3nt - 2\omega - \Omega) \\ & - \tfrac{1}{8}e\gamma \cos(nt - 2\omega + \Omega) + \tfrac{1}{4}e\gamma \cos(nt - \Omega) \\ & - 2e^2\gamma \cos(4nt - 3\omega - \Omega) - \tfrac{1}{16}\gamma\{2e^2 - \gamma^2\} \cos(2nt - 3\omega + \Omega) \\ & + \tfrac{1}{16}\gamma^3 \cos(4nt - \omega - 3\Omega) - \tfrac{1}{16}\gamma^3 \cos 3(\omega - \Omega)\Big\}\end{aligned}\right\} . \quad (565)$$

41. In the development of equations (558) and (559) we shall omit the terms depending on the sun's parallax, and shall only retain terms of the second order in e and γ. This being premised, equations (218) and (560) will give

$$\left.\begin{aligned}\boxed{0}\left(\frac{dR}{dr}\right) = \overline{m}^2n\Big\{ & \tfrac{3}{4}e\{1 + \tfrac{3}{2}e'^2\} - \tfrac{1}{2}\{1 - \tfrac{7}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2\} \cos(nt - \omega) \\ & + \tfrac{9}{4}ee' \cos(n't - \omega') - \tfrac{1}{4}e \cos 2(nt - \omega) - \tfrac{3}{4}e' \cos(nt + n't - \omega - \omega') \\ & - \tfrac{3}{4}e' \cos(nt - n't - \omega + \omega') - \tfrac{9}{8}e'^2 \cos(nt + 2n't - \omega - 2\omega') \\ & - \tfrac{9}{8}e'^2 \cos(nt - 2n't - \omega + 2\omega') - \tfrac{3}{8}ee' \cos(2nt + n't - 2\omega - \omega') \\ & - \tfrac{3}{8}ee' \cos(2nt - n't - 2\omega + \omega') - \tfrac{3}{8}\gamma^2 \cos(3nt - \omega - 2\Omega) \\ & - \tfrac{1}{4}\gamma^2 \cos(nt + \omega - 2\Omega) + \tfrac{1}{8}\gamma^2 \cos(nt - 3\omega + 2\Omega) - \tfrac{3}{16}e^2 \cos 3(nt - \omega)\end{aligned}\right\} . \quad (566)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&+\tfrac{9}{4}e\cos 2(nt-n't)-\tfrac{3}{4}\{1-\tfrac{69}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(3nt-2n't-\omega)\\
&-\tfrac{3}{4}\{1+\tfrac{23}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\cos(nt-2n't+\omega)\\
&-\tfrac{9}{8}ee'\cos(2nt-n't-\omega')+\tfrac{63}{8}ee'\cos(2nt-3n't+\omega')\\
&-\tfrac{15}{8}e\cos(4nt-2n't-2\omega)+\tfrac{9}{8}e\cos 2(n't-\omega)\\
&+\tfrac{3}{8}e'\cos(3nt-n't-\omega-\omega')+\tfrac{3}{8}e'\cos(nt-n't+\omega-\omega')\\
&-\tfrac{21}{8}e'\cos(3nt-3n't-\omega+\omega')-\tfrac{21}{8}e'\cos(nt-3n't+\omega+\omega')\\
&-\tfrac{111}{32}e^2\cos(5nt-2n't-3\omega)-\tfrac{3}{32}e^2\cos(nt+2n't-3\omega)\\
&-\tfrac{51}{8}e'^2\cos(3nt-4n't-\omega+2\omega')-\tfrac{51}{8}e'^2\cos(nt-4n't+\omega+2\omega')\\
&+\tfrac{15}{16}ee'\cos(4nt-n't-2\omega-\omega')-\tfrac{105}{16}ee'\cos(4nt-3n't-2\omega+\omega')\\
&-\tfrac{63}{16}ee'\cos(3n't-2\omega-\omega')-\tfrac{9}{16}ee'\cos(n't-2\omega+\omega')\\
&-\tfrac{3}{8}\gamma^2\cos(nt+2n't-\omega-2\Omega)+\tfrac{3}{8}\gamma^2\cos(3nt-2n't-3\omega+2\Omega)\Big\}
\end{aligned}\right\} \quad (566)
$$

Equations (219) and (561) will give

$$
\left.\begin{aligned}
\boxed{1}\left(\frac{dR}{dv}\right)=\overline{m}^2n\Big\{&-6e\cos 2(nt-n't)\\
&+\tfrac{3}{2}\{1-\tfrac{65}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{4}\gamma^2\}\cos(3nt-2n't-\omega)\\
&-\tfrac{3}{2}\{1-\tfrac{37}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{4}\gamma^2\}\cos(nt-2n't+\omega)\\
&+\tfrac{27}{8}e\cos(4nt-2n't-2\omega)+\tfrac{21}{8}e\cos 2(n't-\omega)\\
&-\tfrac{3}{4}e'\cos(3nt-n't-\omega-\omega')+\tfrac{3}{4}e'\cos(nt-n't+\omega-\omega')\\
&+\tfrac{21}{4}e'\cos(3nt-3n't-\omega+\omega')-\tfrac{21}{4}e'\cos(nt-3n't+\omega+\omega')\\
&+\tfrac{93}{16}e^2\cos(5nt-2n't-3\omega)-\tfrac{9}{16}e^2\cos(nt+2n't-3\omega)\\
&+\tfrac{51}{4}e'^2\cos(3nt-4n't-\omega+2\omega')-\tfrac{51}{4}e'^2\cos(nt-4n't+\omega+2\omega')\\
&-\tfrac{9}{16}\gamma^2\cos(5nt-2n't-\omega-2\Omega)+\tfrac{9}{16}\gamma^2\cos(3nt-2n't+\omega-2\Omega)\\
&-\tfrac{3}{16}\gamma^2\cos(nt+2n't-\omega-2\Omega)+\tfrac{15}{16}\gamma^2\cos(nt-2n't-\omega+2\Omega)\\
&-\tfrac{3}{4}\gamma^2\cos(3nt-2n't-3\omega+2\Omega)+3ee'\cos(2nt-n't-\omega')\\
&-21ee'\cos(2nt-3n't+\omega')-\tfrac{27}{16}ee'\cos(4nt-n't-2\omega-\omega')\\
&+\tfrac{189}{16}ee'\cos(4nt-3n't-2\omega+\omega')+\tfrac{147}{16}ee'\cos(3n't-2\omega-\omega')\\
&-\tfrac{21}{16}ee'\cos(n't-2\omega+\omega')\Big\}
\end{aligned}\right\} \quad (567)
$$

Equations (220) and (562) will also give

$$
\left.\begin{aligned}
\boxed{2}\left(\frac{dR}{d\theta}\right)=\overline{m}^2n\Big\{&\tfrac{3}{4}\gamma^2\cos(3nt-\omega-2\Omega)-\tfrac{3}{4}\gamma^2\cos(nt+\omega-2\Omega)\\
&+\tfrac{3}{8}\gamma^2\cos(5nt-2n't-\omega-2\Omega)+\tfrac{3}{8}\gamma^2\cos(nt+2n't-\omega-2\Omega)\\
&-\tfrac{3}{8}\gamma^2\cos(3nt-2n't+\omega-2\Omega)-\tfrac{3}{8}\gamma^2\cos(nt-2n't-\omega+2\Omega)\Big\}
\end{aligned}\right\} \quad (568)
$$

Equations (218) and (563) will give

$$\boxed{3}\left(\frac{dR}{dr}\right)=\overline{m}^2 n\left\{\begin{aligned}
&+\tfrac{1}{2}\{1-\tfrac{9}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{2}\gamma^2\}\sin(nt-\omega)\\
&+\tfrac{1}{4}e\sin 2(nt-\omega)+\tfrac{3}{4}e'\sin(nt+n't-\omega-\omega')\\
&+\tfrac{3}{4}e'\sin(nt-n't-\omega+\omega')+\tfrac{3}{16}e^2\sin 3(nt-\omega)\\
&+\tfrac{9}{8}e'^2\sin(nt+2n't-\omega-2\omega')+\tfrac{9}{8}e'^2\sin(nt-2n't-\omega+2\omega')\\
&+\tfrac{3}{8}ee'\sin(2nt+n't-2\omega-\omega')+\tfrac{3}{8}ee'\sin(2nt-n't-2\omega+\omega')\\
&+\tfrac{3}{8}\gamma^2\sin(3nt-\omega-2\Omega)-\tfrac{3}{8}\gamma^2\sin(nt+\omega-2\Omega)\\
&-3e\sin 2(nt-n't)+\tfrac{3}{4}\{1-\tfrac{67}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\sin(3nt-2n't-\omega)\\
&-\tfrac{3}{4}\{1-\tfrac{31}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{2}\gamma^2\}\sin(nt-2n't+\omega)\\
&+\tfrac{15}{8}e\sin(4nt-2n't-2\omega)-\tfrac{9}{8}e\sin 2(n't-\omega)\\
&+\tfrac{3}{8}e'\sin(3nt-n't-\omega-\omega')+\tfrac{3}{8}e'\sin(nt-n't+\omega-\omega')\\
&+\tfrac{21}{8}e'\sin(3nt-3n't-\omega+\omega')-\tfrac{21}{8}e'\sin(nt-3n't+\omega+\omega')\\
&+\tfrac{111}{32}e^2\sin(5nt-2n't-3\omega)+\tfrac{3}{32}e^2\sin(nt+2n't-3\omega)\\
&+\tfrac{51}{8}e'^2\sin(3nt-4n't-\omega+2\omega')-\tfrac{51}{8}e'^2\sin(nt-4n't+\omega+2\omega')\\
&+\tfrac{3}{2}ee'\sin(2nt-n't-\omega')-\tfrac{21}{2}ee'\sin(2nt-3n't+\omega')\\
&-\tfrac{15}{16}ee'\sin(4nt-n't-2\omega-\omega')+\tfrac{105}{16}ee'\sin(4nt-3n't-2\omega+\omega')\\
&-\tfrac{63}{16}ee'\sin(3n't-2\omega-\omega')+\tfrac{9}{16}ee'\sin(nt-2\omega+\omega')\\
&+\tfrac{3}{8}\gamma^2\sin(nt+2n't-\omega-2\Omega)+\tfrac{9}{16}\gamma^2\sin(nt-2n't-\omega+2\Omega)\\
&+\tfrac{3}{16}\gamma^2\sin(3nt-2n't+\omega-2\Omega)-\tfrac{3}{16}\gamma^2\sin(nt-2n't+3\omega-2\Omega)\\
&-\tfrac{3}{16}\gamma^2\sin(3nt-2n't-3\omega+2\Omega)\end{aligned}\right\}. \quad (569)$$

Equations (219) and (564) will give

$$\boxed{4}\left(\frac{dR}{dv}\right)=\overline{m}^2 n\left\{\begin{aligned}
&+\tfrac{15}{4}e\sin 2(nt-n't)\\
&-\tfrac{3}{2}\{1-\tfrac{55}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{4}\gamma^2\}\sin(3nt-2n't-\omega)\\
&-\tfrac{3}{2}\{1+\tfrac{13}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{4}\gamma^2\}\sin(nt-2n't+\omega)\\
&-\tfrac{27}{8}e\sin(4nt-2n't-2\omega)-\tfrac{21}{8}e\sin 2(n't-\omega)\\
&+\tfrac{3}{4}e'\sin(3nt-n't-\omega-\omega')+\tfrac{3}{4}e'\sin(nt-n't+\omega-\omega')\\
&-\tfrac{21}{4}e'\sin(3nt-3n't-\omega+\omega')-\tfrac{21}{4}e'\sin(nt-3n't+\omega+\omega')\\
&-\tfrac{15}{8}ee'\sin(2nt-n't-\omega')+\tfrac{105}{8}ee'\sin(2nt-3n't+\omega')\\
&-\tfrac{93}{16}e^2\sin(5nt-2n't-3\omega)+\tfrac{9}{16}e^2\sin(nt+2n't-3\omega)\end{aligned}\right. \quad (570)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&-\tfrac{51}{4}e'^2\sin(3nt-4n't-\omega+2\omega')-\tfrac{51}{4}e'^2\sin(nt-4n't+\omega+2\omega')\\
&+\tfrac{27}{16}ee'\sin(4nt-n't-2\omega-\omega')-\tfrac{189}{16}ee'\sin(4nt-3n't-2\omega+\omega')\\
&-\tfrac{147}{16}ee'\sin(3n't-2\omega-\omega')+\tfrac{21}{16}ee'\sin(nt-2\omega+\omega')\\
&+\tfrac{9}{16}\gamma^2\sin(5nt-2n't-\omega-2\Omega)+\tfrac{3}{16}\gamma^2\sin(3nt-2n't+\omega-2\Omega)\\
&+\tfrac{3}{16}\gamma^2\sin(nt+2n't-\omega-2\Omega)+\tfrac{3}{16}\gamma^2\sin(nt-2n't-\omega+2\Omega)\\
&-\tfrac{3}{8}\gamma^2\sin(nt-2n't+3\omega-2\Omega)+\tfrac{3}{8}\gamma^2\sin(3nt-2n't-3\omega+2\Omega)\Big\}
\end{aligned}\right\}. \quad (570)$$

Lastly, equations (220) and (562) will give

$$\left.\begin{aligned}
\boxed{5}\left(\frac{dR}{d\theta}\right)=\overline{m}^2n\Big\{&-\tfrac{3}{4}\gamma^2\sin(3nt-\omega-2\Omega)-\tfrac{3}{4}\gamma^2\sin(nt+\omega-2\Omega)\\
&-\tfrac{3}{8}\gamma^2\sin(5nt-2n't-\omega-2\Omega)-\tfrac{3}{8}\gamma^2\sin(nt+2n't-\omega-2\Omega)\\
&-\tfrac{3}{8}\gamma^2\sin(3nt-2n't+\omega-2\Omega)+\tfrac{3}{8}\gamma^2\sin(nt-2n't-\omega+2\Omega)\Big\}
\end{aligned}\right\}. \quad (571)$$

If we now substitute equations (566–571) in equations (558–559), we shall obtain the following:

$$\left.\begin{aligned}
e\frac{d\omega}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{&\tfrac{3}{4}e\{1+\tfrac{3}{2}e'^2\}-\tfrac{1}{2}\{1-\tfrac{7}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{2}\gamma^2\}\cos(nt-\omega)\\
&+\tfrac{9}{4}ee'\cos(n't-\omega')-\tfrac{1}{4}e\cos2(nt-\omega)-\tfrac{3}{4}e'\cos(nt+n't-\omega-\omega')\\
&-\tfrac{3}{4}e'\cos(nt-n't-\omega+\omega')-\tfrac{9}{8}e'^2\cos(nt+2n't-\omega-2\omega')\\
&-\tfrac{9}{8}e'^2\cos(nt-2n't-\omega+2\omega')-\tfrac{3}{8}ee'\cos(2nt+n't-2\omega-\omega')\\
&-\tfrac{3}{8}ee'\cos(2nt-n't-2\omega+\omega')+\tfrac{3}{8}\gamma^2\cos(3nt-\omega-2\Omega)\\
&-\gamma^2\cos(nt+\omega-2\Omega)+\tfrac{1}{8}\gamma^2\cos(nt-3\omega+2\Omega)\\
&-\tfrac{3}{16}e^2\cos3(nt-\omega)-\tfrac{15}{4}e\cos2(nt-n't)\\
&+\tfrac{3}{4}\{1-\tfrac{61}{8}e^2-\tfrac{5}{2}e'^2\}\cos(3nt-2n't-\omega)\\
&-\tfrac{9}{4}\{1-\tfrac{17}{8}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{3}\gamma^2\}\cos(nt-2n't+\omega)\\
&+\tfrac{3}{2}e\cos(4nt-2n't-2\omega)+\tfrac{15}{4}e\cos2(n't-\omega)\\
&-\tfrac{3}{8}e'\cos(3nt-n't-\omega-\omega')+\tfrac{9}{8}e'\cos(nt-n't+\omega-\omega')\\
&+\tfrac{21}{8}e'\cos(3nt-3n't-\omega+\omega')-\tfrac{63}{8}e'\cos(nt-3n't+\omega+\omega')\\
&+\tfrac{15}{8}ee'\cos(2nt-n't-\omega')-\tfrac{105}{8}ee'\cos(2nt-3n't+\omega')\\
&+\tfrac{75}{32}e^2\cos(5nt-2n't-3\omega)-\tfrac{21}{32}e^2\cos(nt+2n't-3\omega)\\
&+\tfrac{51}{8}e'^2\cos(3nt-4n't-\omega+2\omega')-\tfrac{153}{8}e'^2\cos(nt-4n't+\omega+2\omega')\\
&-\tfrac{3}{4}ee'\cos(4nt-n't-2\omega-\omega')+\tfrac{21}{4}ee'\cos(4nt-3n't-2\omega+\omega')
\end{aligned}\right\}. \quad (572)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+\tfrac{105}{8}ee'\cos(3n't-2\omega-\omega')-\tfrac{15}{8}ee'\cos(n't-2\omega+\omega')\\
&-\tfrac{3}{16}\gamma^2\cos(nt+2n't-\omega-2\Omega)-\tfrac{3}{8}\gamma^2\cos(3nt-2n't-3\omega+2\Omega)\\
&+\tfrac{9}{16}\gamma^2\cos(nt-2n't-\omega+2\Omega)+\tfrac{3}{16}\gamma^2\cos(3nt-2n't+\omega-2\Omega)\\
&-\tfrac{3}{16}\gamma^2\cos(5nt-2n't-\omega-2\Omega)\Big\}
\end{aligned}\right\}. \quad (572)$$

$$\left.\begin{aligned}
\frac{de}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{&\tfrac{1}{2}\{1-\tfrac{9}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{2}\gamma^2\}\sin(nt-\omega)+\tfrac{1}{4}e\sin 2(nt-\omega)\\
&+\tfrac{3}{4}e'\sin(nt+n't-\omega-\omega')+\tfrac{3}{4}e'\sin(nt-n't-\omega+\omega')\\
&+\tfrac{3}{16}e^2\sin 3(nt-\omega)+\tfrac{9}{8}e'^2\sin(nt+2n't-\omega-2\omega')\\
&+\tfrac{9}{8}e'^2\sin(nt-2n't-\omega+2\omega')+\tfrac{3}{8}ee'\sin(2nt+n't-2\omega-\omega')\\
&+\tfrac{3}{8}ee'\sin(2nt-n't-2\omega+\omega')-\tfrac{3}{8}\gamma^2\sin(3nt-\omega-2\Omega)\\
&-\tfrac{9}{8}\gamma^2\sin(nt+\omega-2\Omega)+\tfrac{3}{4}e\sin 2(nt-n't)\\
&-\tfrac{3}{4}\{1-\tfrac{43}{8}e^2-\tfrac{5}{2}e'^2\}\sin(3nt-2n't-\omega)\\
&-\tfrac{9}{4}\{1-\tfrac{5}{24}e^2-\tfrac{5}{2}e'^2-\tfrac{1}{3}\gamma^2\}\sin(nt-2n't+\omega)\\
&-\tfrac{3}{2}e\sin(4nt-2n't-2\omega)-\tfrac{15}{4}e\sin 2(n't-\omega)\\
&+\tfrac{3}{8}e'\sin(3nt-n't-\omega-\omega')+\tfrac{9}{8}e'\sin(nt-n't+\omega-\omega')\\
&-\tfrac{21}{8}e'\sin(3nt-3n't-\omega+\omega')-\tfrac{63}{8}e'\sin(nt-3n't+\omega+\omega')\\
&-\tfrac{75}{32}e^2\sin(5nt-2n't-3\omega)+\tfrac{21}{32}e^2\sin(nt+2n't-3\omega)\\
&-\tfrac{51}{8}e'^2\sin(3nt-4n't-\omega+2\omega')-\tfrac{153}{8}e'^2\sin(nt-4n't+\omega+2\omega')\\
&-\tfrac{3}{8}ee'\sin(2nt-n't-\omega')+\tfrac{21}{8}ee'\sin(2nt-3n't+\omega')\\
&+\tfrac{3}{4}ee'\sin(4nt-n't-2\omega-\omega')-\tfrac{21}{4}ee'\sin(4nt-3n't-2\omega+\omega')\\
&-\tfrac{105}{8}ee'\sin(3n't-2\omega-\omega')+\tfrac{15}{8}ee'\sin(n't-2\omega+\omega')\\
&+\tfrac{3}{16}\gamma^2\sin(nt+2n't-\omega-2\Omega)+\tfrac{9}{8}\gamma^2\sin(nt-2n't-\omega+2\Omega)\\
&-\tfrac{9}{16}\gamma^2\sin(nt-2n't+3\omega-2\Omega)+\tfrac{3}{16}\gamma^2\sin(3nt-2n't-3\omega+2\Omega)\\
&+\tfrac{3}{16}\gamma^2\sin(5nt-2n't-\omega-2\Omega)\Big\}
\end{aligned}\right\}. \quad (573)$$

If we now put the sum of the periodic terms in equations (572) and (573) equal to $\frac{d\delta\omega}{dt}$ and $\frac{d\delta e}{dt}$, respectively, they will give by integration,

$$\left.\begin{aligned}
e\delta\omega=\frac{\overline{m}^2}{\mu}\Big\{&-\tfrac{1}{2}\{1-\tfrac{7}{8}e^2+\tfrac{3}{2}e'^2-\tfrac{3}{2}\gamma^2\}\sin(nt-\omega)\\
&+ee'(30.07969)\sin(n't-\omega')-\tfrac{1}{8}e\sin 2(nt-\omega)
\end{aligned}\right\}. \quad (574)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- e' (0.6978034) \sin (nt + n't - \omega - \omega') \\
&- e' (0.8106367) \sin (nt - n't - \omega + \omega') \\
&- e'^2 (0.9785986) \sin (nt + 2n't - \omega - 2\omega') \\
&- e'^2 (1.322911) \sin (nt - 2n't - \omega + 2\omega') \\
&- ee' (0.1807402) \sin (2nt + n't - 2\omega - \omega') \\
&- ee' (0.1947851) \sin (2nt - n't - 2\omega + \omega') \\
&+ \tfrac{1}{8}\gamma^2 \sin (3nt - \omega - 2\Omega) - \gamma^2 \sin (nt + \omega - 2\Omega) \\
&+ \tfrac{1}{8}\gamma^2 \sin (nt - 3\omega + 2\Omega) - \tfrac{1}{16}e^2 \sin 3(nt - \omega) \\
&- e (2.026592) \sin 2(nt - n't) \\
&+ \{1 - \tfrac{61}{8}e^2 - \tfrac{5}{2}e'^2\} (0.2631212) \sin (3nt - 2n't - \omega) \\
&- \{1 - \tfrac{17}{8} e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{3}\gamma^2\} (2.645822) \sin (nt - 2n't + \omega) \\
&+ e (0.3895707) \sin (4nt - 2n't - 2\omega) + e (25.06642) \sin 2(n't - \omega) \\
&- e' (0.1281964) \sin (3nt - n't - \omega - \omega') \\
&+ e' (1.215955) \sin (nt - n't + \omega - \omega') \\
&+ e' (0.9457427) \sin (3nt - 3n't - \omega + \omega') \\
&- e' (10.15348) \sin (nt - 3n't + \omega + \omega') \\
&+ ee' (0.9739255) \sin (2nt - n't - \omega') \\
&- ee' (7.391886) \sin (2nt - 3n't + \omega') \\
&+ e^2 (0.4832083) \sin (5nt - 2n't - 3\omega) \\
&- e^2 (0.5708494) \sin (nt + 2n't - 3\omega) \\
&+ e'^2 (2.360416) \sin (3nt - 4n't - \omega + 2\omega') \\
&- e'^2 (27.29673) \sin (nt - 4n't + \omega + 2\omega') \\
&- ee' (0.1910731) \sin (4nt - n't - 2\omega - \omega') \\
&+ ee' (1.390509) \sin (4nt - 3n't - 2\omega + \omega') \\
&+ ee' (58.18828) \sin (3n't - 2\omega - \omega') \\
&- ee' (25.06642) \sin (nt - 2\omega + \omega') \\
&- \gamma^2 (0.163100) \sin (nt + 2n't - \omega - 2\Omega) \\
&- \gamma^2 (0.1315606) \sin (3nt - 2n't - 3\omega + 2\Omega) \\
&+ \gamma^2 (0.6614553) \sin (nt - 2n't - \omega + 2\Omega) \\
&+ \gamma^2 (0.06578030) \sin (3nt - 2n't + \omega - 2\Omega) \\
&- \gamma^2 (0.0386566) \sin (5nt - 2n't - \omega - 2\Omega) \}
\end{aligned}\right\} . \quad (574)
$$

$$
\left.\begin{aligned}
\delta e = \frac{\overline{m}^2}{\mu}\Big\{ & -\tfrac{1}{2}\{1 - \tfrac{9}{8}e^2 + \tfrac{3}{2}e'^2 - \tfrac{3}{2}\gamma^2\} \cos(nt - \omega) - \tfrac{1}{8}e \cos 2(nt - \omega) \\
& - e' (0.6978034) \cos(nt + n't - \omega - \omega') \\
& - e' (0.8106367) \cos(nt - n't - \omega + \omega') \\
& - \tfrac{1}{16}e^2 \cos 3(nt - \omega) - e'^2 (0.9785986) \cos(nt + 2n't - \omega - 2\omega') \\
& - e'^2 (1.322911) \cos(nt - 2n't - \omega + 2\omega') \\
& - ee' (0.1807402) \cos(2nt + n't - 2\omega - \omega') \\
& - ee' (0.1947851) \cos(2nt - n't - 2\omega + \omega') \\
& + \tfrac{1}{8}\gamma^2 \cos(3nt - \omega - 2\Omega) + \tfrac{9}{8}\gamma^2 \cos(nt + \omega - 2\Omega) \\
& - e (0.4053184) \cos 2(nt - n't) \\
& + \{1 - \tfrac{43}{8}e' - \tfrac{5}{2}e'^2\}(0.2631212) \cos(3nt - 2n't - \omega) \\
& + \{1 - \tfrac{5}{24}e^2 - \tfrac{5}{2}e'^2 - \tfrac{1}{3}\gamma^2\}(2.645822) \cos(nt - 2n't + \omega) \\
& + e (0.3895707) \cos(4nt - 2n't - 2\omega) + e (25.06642) \cos 2(n't - \omega) \\
& - e' (0.1281964) \cos(3nt - n't - \omega - \omega') \\
& - e' (1.215955) \cos(nt - n't + \omega - \omega') \\
& + e' (0.9457427) \cos(3nt - 3n't - \omega + \omega') \\
& + e' (10.15348) \cos(nt - 3n't + \omega + \omega') \\
& + e^2 (0.4832083) \cos(5nt - 2n't - 3\omega) \\
& - e^2 (0.5708494) \cos(nt + 2n't - 3\omega) \\
& + e'^2 (2.360416) \cos(3nt - 4n't - \omega + 2\omega') \\
& + e'^2 (27.29673) \cos(nt - 4n't + \omega + 2\omega') \\
& + ee' (0.1947851) \cos(2nt - n't - \omega') \\
& - ee' (1.478377) \cos(2nt - 3n't + \omega') \\
& - ee' (0.1910731) \cos(4nt - n't - 2\omega - \omega') \\
& + ee' (1.390509) \cos(4nt - 3n't - 2\omega + \omega') \\
& + ee' (58.18828) \cos(3n't - 2\omega - \omega') \\
& - ee' (25.06642) \cos(n't - 2\omega + \omega') \\
& - \gamma^2 (0.163100) \cos(nt + 2n't - \omega - 2\Omega) \\
& - \gamma^2 (1.322911) \cos(nt - 2n't - \omega + 2\Omega) \\
& + \gamma^2 (0.6614553) \cos(nt - 2n't + 3\omega - 2\Omega) \\
& - \gamma^2 (0.0657803) \cos(3nt - 2n't - 3\omega + 2\Omega) \\
& - \gamma^2 (0.0386566) \cos(5nt - 2n't - \omega' - 2\Omega) \Big\}
\end{aligned}\right\} \quad (575)
$$

42. We shall now develop equations (E) and (F). The elliptical values of the coefficients of the forces are as follows:

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\sin(v-\Omega) = \left.\begin{aligned}&\{1-\tfrac{1}{2}e^2+\tfrac{3}{8}\gamma^2\}\sin(nt-\Omega)+e\sin(2nt-\omega-\Omega)\\ &-e\sin(\omega-\Omega)-\tfrac{1}{8}\gamma^2\sin 3(nt-\Omega)+\tfrac{9}{8}e^2\sin(3nt-2\omega-\Omega)\\ &+\tfrac{1}{8}\{e^2-\gamma^2\}\sin(nt-2\omega+\Omega)+\tfrac{1}{8}\gamma^2\sin(nt+2\omega-3\Omega)\end{aligned}\right\}. \quad (576)$$

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\sin(v-\Omega)\cos(v-\Omega) = \left.\begin{aligned}&\tfrac{1}{2}\{1-\tfrac{1}{2}e^2+\tfrac{1}{2}\gamma^2\}\sin 2(nt-\Omega)+e\sin(3nt-\omega-2\Omega)\\ &-e\sin(nt+\omega-2\Omega)+\tfrac{3}{8}e^2\sin 2(\omega-\Omega)+\tfrac{13}{8}e^2\sin(4nt-2\omega-2\Omega)\\ &-\tfrac{1}{8}\gamma^2\sin 4(nt-\Omega)+\tfrac{1}{8}\gamma^2\sin(2nt+2\omega-4\Omega)-\tfrac{1}{8}\gamma^2\sin 2(nt-\omega)\end{aligned}\right\}. \quad (577)$$

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\gamma\cos^2(v-\Omega) = \left.\begin{aligned}&\tfrac{1}{2}\gamma+\tfrac{1}{2}\gamma\cos 2(nt-\Omega)+e\gamma\cos(3nt-\omega-2\Omega)\\ &\qquad -e\gamma\cos(nt+\omega-2\Omega)\end{aligned}\right\}. \quad (578)$$

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{1-e^2}}\cos(v-\Omega) = \cos(nt-\Omega)-e\cos(\omega-\Omega)+e\cos(2nt-\omega-\Omega)\Big\}. \quad (579)$$

By means of equations (219) and (220); together with (576–579), we obtain the following values:

$$-\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu}(1-e^2)}\,\frac{\sin(v-\Omega)}{\gamma}\left(\frac{dR}{d\theta}\right) =$$

$$\left.\begin{aligned}&\frac{\overline{m}^2}{\mu}n\Big\{-\tfrac{3}{4}\{1+2e^2+\tfrac{3}{2}e'^2-\tfrac{1}{2}\gamma^2\}-\tfrac{3}{4}\cos 2(nt-\Omega)\\ &\quad+\tfrac{3}{2}e\cos(nt-\omega)-\tfrac{9}{4}e'\cos(n't-\omega')+\tfrac{3}{4}e\cos(3nt-\omega-2\Omega)\\ &\quad-\tfrac{9}{4}e\cos(nt+\omega-2\Omega)+\tfrac{9}{8}e'\cos(2nt+n't-\omega'-2\Omega)\\ &\quad+\tfrac{9}{8}e'\cos(2nt-n't+\omega'-2\Omega)-\tfrac{3}{4}\cos 2(nt-n't)\\ &\quad+\tfrac{3}{8}\cos(4nt-2n't-2\Omega)+\tfrac{3}{8}\cos 2(n't-\Omega)\\ &\quad-\tfrac{3}{4}e\cos(3nt-2n't-\omega)+\tfrac{9}{4}e\cos(nt-2n't+\omega)\\ &\quad+\tfrac{9}{8}e\cos(5nt-2n't-\omega-2\Omega)-\tfrac{15}{8}e\cos(3nt-2n't+\omega-2\Omega)\end{aligned}\right\}. \quad (580)$$

(Continued on the next page.)

$$\left.\begin{array}{l} -\frac{3}{8}e\cos(nt+2n't-\omega-2\Omega)-\frac{3}{8}e\cos(nt-2n't-\omega+2\Omega) \\ +\frac{3}{8}e'\cos(2nt-n't-\omega')-\frac{21}{8}e'\cos(2nt-3n't+\omega') \\ -\frac{3}{16}e'\cos(4nt-n't-\omega'-2\Omega)+\frac{21}{16}e'\cos(4nt-3n't+\omega'-2\Omega) \\ -\frac{3}{16}e'\cos(n't+\omega'-2\Omega)+\frac{21}{16}e'\cos(3n't-\omega'-2\Omega)\Big\} \end{array}\right\} . \quad (580)$$

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\sin(v-\Omega)\cos(v-\Omega)\left(\frac{dR}{dv}\right)=$$

$$\left.\begin{array}{l} \frac{\overline{m}^2}{\mu}n\Big\{-\frac{3}{8}\cos(4nt-2n't-2\Omega)+\frac{3}{8}\cos 2(nt-\Omega) \\ -\frac{3}{8}e\cos(5nt-2n't-\omega-2\Omega)-\frac{3}{8}e\cos(nt-2n't-\omega+2\Omega) \\ +\frac{15}{8}e\cos(3nt-2n't+\omega-2\Omega)-\frac{3}{8}e\cos(nt+2n't-\omega-2\Omega) \\ +\frac{3}{16}e'\cos(4nt-n't-\omega'-2\Omega)-\frac{21}{16}e'\cos(4nt-3n't+\omega'-2\Omega) \\ -\frac{3}{16}e'\cos(n't+\omega'-2\Omega)+\frac{21}{16}e'\cos(3n't-\omega'-2\Omega)\Big\} \end{array}\right\} . \quad (581)$$

$$\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\gamma\cos^2(v-\Omega)\left(\frac{dR}{dv}\right)=$$

$$\left.\frac{m^2}{\mu}n\Big\{\frac{3}{4}\gamma\sin 2(nt-n't)+\frac{3}{8}\gamma\sin(4nt-2n't-2\Omega)-\frac{3}{8}\gamma\sin 2(n't-\Omega)\Big\}\right\} . \quad (582)$$

$$-\frac{\sqrt{1+\gamma^2}}{\sqrt{a\mu(1-e^2)}}\cos(v-\Omega)\left(\frac{dR}{d\theta}\right)=$$

$$\left.\frac{\overline{m}^2}{\mu}n\Big\{-\frac{3}{4}\gamma\sin 2(nt-\Omega)-\frac{3}{8}\gamma\sin(4nt-2n't-2\Omega)-\frac{3}{8}\gamma\sin 2(n't-\Omega)\Big\}\right\} . \quad (583)$$

If we substitute equations (580–583) in equations (E) and (F), we shall obtain the following values of the differential equations of the node and inclination:

$$\left.\begin{array}{l} \frac{d\Omega}{dt}=\frac{\overline{m}^2}{\mu}n\Big\{-\frac{3}{4}\{1+2e^2+\frac{3}{2}e'^2-\frac{1}{4}\gamma^2\}+\frac{3}{4}\cos 2(nt-\Omega) \\ +\frac{3}{4}e\cos(nt-\omega)-\frac{9}{4}e'\cos(n't-\omega')-\frac{3}{4}e\cos(nt+\omega-2\Omega) \\ +\frac{3}{4}e\cos(3nt-\omega-2\Omega)+\frac{3}{8}e'\cos(2nt+n't-\omega'-2\Omega) \\ +\frac{3}{8}e'\cos(2nt-n't+\omega'-2\Omega)-\frac{3}{4}\cos 2(nt-n't) \end{array}\right\} . \quad (584)$$

(Continued on the next page.)

$$\left.\begin{array}{l} +\frac{3}{4}\cos 2(n't-\Omega)-\frac{3}{4}e\cos(3nt-2n't-\omega) \\ +\frac{9}{4}e\cos(nt-2n't+\omega)-\frac{3}{4}e\cos(nt+2n't-\omega-2\Omega) \\ -\frac{3}{4}e\cos(nt-2n't-\omega+2\Omega)+\frac{3}{8}e'\cos(2nt-n't-\omega') \\ -\frac{21}{8}e'\cos(2nt-3n't+\omega')-\frac{3}{8}e'\cos(n't+\omega'-2\Omega) \\ +\frac{21}{8}e'\cos(3n't-\omega'-2\Omega)\Big\} \end{array}\right\} \quad (584)$$

$$\frac{d\gamma}{dt}=\frac{\overline{m}^2}{\mu}n\left\{-\frac{3}{4}\gamma\sin 2(nt-\Omega)+\frac{3}{4}\gamma\sin 2(nt-n't)-\frac{3}{4}\gamma\sin 2(n't-\Omega)\right\} \quad (585)$$

Putting the periodic parts of these equations equal to $\frac{d\delta\Omega}{dt}$ and $\frac{d\delta\gamma}{dt}$, we shall obtain by integration,

$$\left.\begin{array}{l} \delta\Omega=\frac{\overline{m}^2}{\mu}\Big\{\frac{3}{8}\sin 2(nt-\Omega)+\frac{3}{2}e\,(nt-\omega)+\frac{3}{4}e\sin(nt+\omega-2\Omega) \\ \quad +\frac{1}{4}e\sin(3nt-\omega-2\Omega)-e'\,(30.07970)\sin(n't-\omega') \\ \quad +e'\,(0.542221)\sin(2nt+n't-\omega'-2\Omega) \\ \quad +e'\,(0.584355)\sin(2nt-n't+\omega'-2\Omega) \\ \quad -(0.4053184)\sin 2(nt-n't)+(5.013280)\sin 2(n't-\Omega) \\ \quad -e\,(0.2631212)\sin(3nt-2n't-\omega) \\ \quad +e\,(2.645831)\cos(nt-2n't+\omega) \\ \quad -e\,(0.6523992)\sin(nt+2n't-\omega-2\Omega) \\ \quad -e\,(0.8819403)\sin(nt-2n't-\omega+2\Omega) \\ \quad +e'\,(0.1947851)\sin(2nt-n't-\omega') \\ \quad -e'\,(1.478377)\sin(2nt-3n't+\omega') \\ \quad -e'\,(5.01328)\sin(n't+\omega'-2\Omega)+e'\,(11.69766)\sin(3n't-\omega'-2\Omega)\Big\} \end{array}\right\} \quad (586)$$

$$\left.\begin{array}{l} \delta\gamma=\frac{\overline{m}^2}{\mu}\Big\{\frac{3}{8}\gamma\cos 2(nt-\Omega)-\gamma\,(0.4053184)\cos 2(nt-n't) \\ \qquad\qquad +\gamma\,(5.013283)\cos 2(n't-\Omega)\Big\} \end{array}\right\} \quad (587)$$

43. Lastly, we shall develop equation (I), which gives the variation of the mean distance. If we multiply the values of $\left(\frac{dR}{dr}\right)$, $\left(\frac{dR}{dv}\right)$, and $\left(\frac{dR}{d\theta}\right)$ by the elliptical values of dr, dv, and $d\theta$ respectively, we shall find by retaining only the terms depending on the first power of e, e', and γ,

$$\left.\begin{aligned}2\left(\frac{dR}{dr}\right)dr = 2\,\frac{\overline{m}^2}{a}\,ndt\Big\{ & -\tfrac{1}{2}e\sin(nt-\omega) - \tfrac{3}{4}e\sin(3nt-2n't-\omega) \\ & \qquad + \tfrac{9}{4}e\sin(nt-2n't+\omega)\Big\}\end{aligned}\right\}. \tag{588}$$

$$\left.\begin{aligned}2\left(\frac{dR}{dv}\right)dv = 2\,\frac{\overline{m}^2}{a}\,ndt\Big\{ & \tfrac{3}{2}\sin 2(nt-n't) + 3e(3nt-2n't-\omega) \\ & - 3e\sin(nt-2n't+\omega) - \tfrac{3}{4}e'\sin(2nt-n't-\omega') \\ & \qquad - \tfrac{21}{4}e'\sin(2nt-3n't+\omega')\Big\}\end{aligned}\right\}. \tag{589}$$

$$2\left(\frac{dR}{d\theta}\right)d\theta = 0. \tag{590}$$

Substituting these quantities in equation (I), it will give by development of the first member,

$$\left.\begin{aligned}da = a\,\frac{\overline{m}^2}{\mu}ndt\Big\{ & e\sin(nt-\omega) - 3\sin 2(nt-n't) \\ & - \tfrac{9}{2}e\sin(3nt-2n't-\omega) + \tfrac{3}{2}e\sin(nt-2n't+\omega) \\ & + \tfrac{3}{2}e'\sin(2nt-n't-\omega') + \tfrac{21}{2}e'\sin(2nt-3n't+\omega')\Big\}\end{aligned}\right\}. \tag{591}$$

If we put the integral of this equation equal to δa, we shall find

$$\left.\begin{aligned}\delta a = a\,\frac{\overline{m}^2}{\mu}\Big\{ & -e\cos(nt-\omega) + (1.6212734)\cos 2(nt-n't) \\ & + (1.5787272)\cos(3nt-2n't-\omega) \\ & - e\,(5.291644)\cos(nt-2n't+\omega) \\ & - e'\,(0.7791404)\cos(2nt-n't-\omega') \\ & - e'\,(5.913508)\cos(2nt-3n't+\omega')\Big\}\end{aligned}\right\}. \tag{592}$$

Now, since
$$\delta n = -\tfrac{3}{2}\,\frac{n}{a}\,\delta a, \tag{593}$$

we shall have

$$\left.\begin{aligned}\delta n = \frac{\overline{m}^2}{\mu} n \Big\{ & \tfrac{3}{2} e \cos(nt-\omega) - (2.431910)\cos 2(nt - n't) \\ & - e(2.368091)\cos(3nt - 2n't - \omega) + e(7.937466)\cos(nt - 2n't + \omega) \\ & - e'\,(1.1687106)\cos(2nt - n't - \omega') \\ & + e'\,(8.870262)\cos(2nt - 3n't + \omega')\Big\}\end{aligned}\right\}. \tag{594}$$

If we multiply this equation by dt and take the integral, putting the $\int \delta n dt = \delta(nt)$ we shall have the variation of the mean longitude by means of the equation,

$$\left.\begin{aligned}\delta(nt) = \frac{\overline{m}^2}{\mu} \Big\{ & \tfrac{3}{2} e \sin(nt-\omega) - (1.314264)\sin 2(nt - n't) \\ & - e(0.8307934)\sin(3nt - 2n't - \omega) + e(9.333828)\sin(nt - 2n't + \omega) \\ & + e'\,(0.607060)\sin(2nt - n't - \omega') \\ & + e'\,(4.99566)\sin(2nt - 3n't + \omega')\Big\}\end{aligned}\right\}. \tag{595}$$

We have thus determined the variation of the elements of the orbit arising from the first power of the disturbing force; and we shall now determine the non-periodic part of the variation of the perigee and node arising from the square of the disturbing force.

44. For this purpose we must compute the variation of equation (558), and we shall find

$$\mu e \delta \frac{d\omega}{dt} = \boxed{0}\,\delta\left(\frac{dR}{dr}\right) + \delta\boxed{0}\left(\frac{dR}{dr}\right) + \boxed{1}\,\delta\left(\frac{dR}{dv}\right) + \delta\boxed{1}\left(\frac{dR}{dv}\right), \tag{596}$$

when we neglect the inclination of the orbit.

The values of $\delta\boxed{0}$ and $\delta\boxed{1}$ are given by the following equations:

$$\left.\begin{aligned}\delta\boxed{0} = an\,\{ & -e + 2\cos(nt-\omega) + 3e\cos 2nt - \omega\}\,\delta r \\ & + a^2 n\,\{-\sin(nt-\omega) - e\sin 2(nt-\omega)\}\,\delta v \\ & \qquad + a^2\,\{-2e + \cos(nt-\omega)\}\,\frac{d\delta v}{dt}\end{aligned}\right\}. \tag{597}$$

$$\left.\begin{aligned}\delta\,\boxed{1} &= 2n\{-\sin(nt-\omega) - 2e\sin 2(nt-\omega)\}\,\delta r \\ &+ an\{\tfrac{1}{2}e - 2\cos(nt-\omega) - \tfrac{5}{2}e\cos 2(nt-\omega)\}\,\delta v \\ &+ \{-e + \cos(nt-\omega) + e\cos 2(nt-\omega)\}\,\frac{d\delta r}{dt} \\ &\qquad - a\{2\sin(nt-\omega) + e\sin 2(nt-\omega)\}\,\frac{d\delta v}{dt}\end{aligned}\right\}. \tag{598}$$

If we reduce these equations to numbers, we shall find

$$\left.\begin{aligned}\delta\,\boxed{0} = a^2\frac{\overline{m}^2}{\mu}n\Big\{&\tfrac{35}{24}e - \tfrac{1}{8}\cos(nt-\omega) + e\,(29.20474)\cos 2(nt-n't) \\ &+ (1.320251)\cos(3nt-2n't-\omega) - (0.509613)\cos(nt-2n't+\omega)\Big\}\end{aligned}\right\}. \tag{599}$$

$$\left.\begin{aligned}\delta\,\boxed{1} = a\frac{\overline{m}^2}{\mu}n\Big\{&\tfrac{1}{8}\sin(nt-\omega) - e\,(79.40903)\sin 2(nt-n't) \\ &-(2.736821)\sin(3nt-2n't-\omega) + (1.459793)\sin(nt-2n't+\omega)\Big\}\end{aligned}\right\}. \tag{600}$$

Now by means of equations (218), (219), (327), (328), (552), (553), (599), and (600) we shall obtain the following values:

$$\left.\begin{aligned}\boxed{0}\,\delta\left(\frac{dR}{dr}\right) &= \frac{\overline{m}^4}{\mu}n\{+e\,(58.76057)\}, & \delta\,\boxed{0}\left(\frac{dR}{dr}\right) &= \frac{\overline{m}^4}{\mu}n\{-e\,(24.94854)\} \\ \boxed{1}\,\delta\left(\frac{dR}{dv}\right) &= \frac{\overline{m}^4}{\mu}n\{+e\,(157.4356)\}, & \delta\,\boxed{1}\left(\frac{dR}{dv}\right) &= \frac{\overline{m}^4}{\mu}n\{-e\,(64.89392)\}\end{aligned}\right\}. \tag{601}$$

If we substitute these values in equation (596), we shall find

$$\delta\frac{d\omega}{dt} = \frac{\overline{m}^4}{\mu^2}n\,\{+\,126.3572\}. \tag{602}$$

Adding this to the non-periodic part of equation(572), we find

$$\frac{d\omega}{dt} = \tfrac{3}{4}\frac{\overline{m}^2}{\mu}n + 126.3572\,\frac{\overline{m}^4}{\mu^2}n. \tag{603}$$

This equation gives by integration

$$\omega = \omega_0 + \tfrac{3}{4}\frac{\overline{m}^2}{\mu}nt + 126.3572\,\frac{\overline{m}^4}{\mu^2}nt, \tag{604}$$

ω_0 being an arbitrary constant quantity.

The only non-periodic term in the value of $\delta \frac{d\Omega}{dt}$ comes from the term

$$\frac{\sin(v-\Omega)}{\gamma}\,\delta\left(\frac{dR}{d\theta}\right) \text{ of the value of } \frac{d\Omega}{dt}.$$

It is therefore easy to deduce from equations (120) and (329),

$$\delta\frac{d\Omega}{dt} = +\frac{\overline{m}^4}{\mu^2}\,n\,\{4.088874\}. \tag{605}$$

If we add this to the non-periodic term of equation (584), we shall obtain

$$\frac{d\Omega}{dt} = -\tfrac{3}{4}\frac{\overline{m}^2}{\mu}n + 4.088874\frac{\overline{m}^4}{\mu^2}n. \tag{606}$$

This gives by integration

$$\Omega = \Omega_0 - \tfrac{3}{4}\frac{\overline{m}^2}{\mu}nt + 4.088874\frac{\overline{m}^4}{\mu}nt, \tag{607}$$

Ω_0 being an arbitrary constant quantity.

If we now put $\frac{\overline{m}^2}{\mu}$ equal to the square of the ratio of the sun's mean motion to that of the moon, we shall have $\log\frac{\overline{m}^2}{\mu} = 7.7478186$, and equation (604) will give the mean motion of the perigee equal to

$$+\{0.00419643 + 0.00395572 = 0.00815215\}\,nt. \tag{607'}$$

The first three terms of PLANA's series give the mean motion of the perigee equal to $0.00813492\,nt$. The two terms of the series which I have given approach more nearly to the true value of the motion than the three terms of PLANA's series.

In like manner equation (607) gives the motion of the node equal to

$$-\{0.00419643 - 0.00012801 = 0.00406842\}\,nt; \tag{607''}$$

which is also more converging than PLANA's series for the motion of the node.

CHAPTER VIII.

ON THE LUNAR INEQUALITIES ARISING FROM THE OBLATENESS OF THE EARTH.

45. We shall now determine the inequalities in the moon's motion arising from the oblateness of the earth. The effect of the earth's oblateness is to add some small terms to the expressions of the moon's co-ordinates, which we have already determined. In the value of Q, given by equation (1), the earth was supposed to be spherical; but we may include the effect of the earth's oblateness in that equation by adding the term

$$\{\tfrac{1}{2}\psi - \rho\} \frac{D}{r^3} \mu \{\sin^2 \mu' - \tfrac{1}{3}\},$$

so that we shall have

$$Q = \frac{\mu}{r} + \{\tfrac{1}{2}\psi - \rho\} \frac{D}{r^3} \mu \{\sin^2 \mu' - \tfrac{1}{3}\}. \tag{608}$$

This value of Q includes the effect of the earth's spheroidal form; and if we compare it with the general value of Q as given by equation (1), we shall obtain

$$R = \{\rho - \tfrac{1}{2}\psi\} \frac{D}{r^3} \mu \{\sin^2 \mu' - \tfrac{1}{3}\}. \tag{609}$$

In these equations

ρ = the ellipticity of the earth;

ψ = the ratio of the centrifugal force to the gravity at the equator;

D = the mean radius of the earth; and

μ' = the moon's declination.

The demonstration of these equations requires too much detail for the limits of the present work; and we would refer the student to the third book of the *Mécanique Céleste*, a work in which the general theory of the attraction of spheroids is given with all desirable completeness.

We shall now develop equation (609). For this purpose we shall denote the obliquity of the ecliptic to the equator by ϵ, and then we shall find

$$\sin \mu' = \sin \epsilon \cos \theta \sin v + \cos \epsilon \sin \theta; \tag{610}$$

and this gives

$$\sin^2 \mu' = \sin^2 \epsilon \cos^2 \theta \sin^2 v + 2 \sin \epsilon \cos \epsilon \sin \theta \cos \theta \sin v + \cos^2 \epsilon \sin^2 \theta \tag{611}$$

If we now put

$$m = \{\rho - \tfrac{1}{2}\psi\} \frac{D^2}{a^2}, \tag{612}$$

and substitute the value of $\sin^2 \mu'$, we shall obtain

$$R = \frac{a^2 m \mu}{r^3} \left\{ \begin{array}{l} \tfrac{1}{2} \sin^2 \epsilon \cos^2 \theta + \cos^2 \epsilon \sin^2 \theta - \tfrac{1}{2} \sin^2 \epsilon \cos^2 \theta \cos 2v \\ \qquad + 2 \sin \epsilon \cos \epsilon \sin \theta \cos \theta \sin v - \tfrac{1}{3} \end{array} \right\}. \tag{613}$$

If we take the partial differentials of this equation with respect to r, v, and θ, we shall obtain the following values:

$$\left(\frac{dR}{dr}\right) = \frac{a^2 m \mu}{r^4} \left\{ \begin{array}{l} 1 - \tfrac{3}{2} \sin^2 \epsilon \cos^2 \theta - 3 \cos^2 \epsilon \sin^2 \theta + \tfrac{3}{2} \sin^2 \epsilon \cos^2 \theta \cos 2v \\ \qquad - 6 \sin \epsilon \cos \epsilon \sin \theta \cos \theta \sin v \end{array} \right\}; \tag{614}$$

$$\left(\frac{dR}{dv}\right) = \frac{a^2 m \mu}{r^3} \left\{ \sin^2 \theta \cos \theta \sin 2v + 2 \sin \epsilon \cos \epsilon \sin \theta \cos \theta \cos v \right\}; \tag{615}$$

$$\left(\frac{dR}{d\theta}\right) = \frac{a^2 m \mu}{r^3} \left\{ \begin{array}{l} - \sin^2 \epsilon \sin \theta \cos \theta + 2 \cos^2 \epsilon \sin \theta \cos \theta \\ \qquad + \sin^2 \epsilon \sin \theta \cos \theta \cos 2v + 2 \sin \epsilon \cos \epsilon \{\cos^2 \theta - \sin^2 \theta\} \sin v \end{array} \right\}. \tag{616}$$

If we substitute the elliptical values of r, v, and θ in these equations, they will become by neglecting the eccentricity and retaining only the first power of the inclination,

$$\left(\frac{dR}{dr}\right) = \frac{m}{a^2} \mu \left\{ \begin{array}{l} 1 - \tfrac{3}{2} \sin^2 \epsilon_0 + \tfrac{3}{2} \{\sin^2 \epsilon_0 - \sin^2 \epsilon\} + \tfrac{3}{2} \sin^2 \epsilon \cos 2nt \\ \qquad + 3\gamma \sin \epsilon \cos \epsilon \cos (2nt - \Omega) - 3\gamma \sin \epsilon \cos \epsilon \cos \Omega \end{array} \right\}; \tag{617}$$

$$\left(\frac{dR}{dv}\right)=\frac{m}{a}\mu\left\{\begin{aligned}&\sin^2\epsilon\sin 2nt+\gamma\sin\epsilon\cos\epsilon\sin(2nt-\Omega)\\&\qquad-\gamma\sin\epsilon\cos\epsilon\sin\Omega\end{aligned}\right\};\tag{618}$$

$$\left(\frac{dR}{d\theta}\right)=\frac{m}{a}\mu\left\{\begin{aligned}&2\sin\epsilon\cos\epsilon\sin nt+2\gamma\left\{1-\tfrac{3}{2}\sin^2\epsilon\right\}\sin(nt-\Omega)\\&\qquad+\tfrac{1}{2}\gamma\sin^2\epsilon\sin(3nt-\Omega)-\tfrac{1}{2}\gamma\sin^2\epsilon\sin(nt+\Omega)\end{aligned}\right\}.\tag{619}$$

ϵ_0 being the value of ϵ when $t=0$.

46. Having obtained the expressions of the forces in terms of the time, we shall first consider the variation of the elements of the orbit. The factors by which it is necessary to multiply the forces in order to obtain the variations of the node and inclination are given by equations (576–579). Therefore we shall find

$$\sin(v-\Omega)\cos(v-\Omega)\left(\frac{dR}{dv}\right)=$$

$$\frac{m}{a}\mu\left\{\begin{aligned}&-\tfrac{1}{4}\sin^2\epsilon\cos(4nt-2\Omega)+\tfrac{1}{4}\sin^2\epsilon\cos 2\Omega\\&-\tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos(4nt-3\Omega)+\tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos\Omega\\&+\tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos(2nt-\Omega)-\tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos(2nt-3\Omega)\end{aligned}\right\};\tag{620}$$

$$\frac{\sin(v-\Omega)}{\gamma}\left(\frac{dR}{d\theta}\right)=$$

$$\frac{m}{a}\mu\left\{\begin{aligned}&\left\{1-\tfrac{3}{2}\sin^2\epsilon\right\}-\left\{1-\tfrac{7}{4}\sin^2\epsilon\right\}\cos 2(nt-\Omega)\\&-\tfrac{1}{4}\sin^2\epsilon\cos(4nt-2\Omega)+\tfrac{1}{4}\sin^2\epsilon\cos 2nt-\tfrac{1}{4}\sin^2\epsilon\cos 2\Omega\\&\qquad-\frac{1}{\gamma}\sin\epsilon\cos\epsilon\cos(2nt-\Omega)+\frac{1}{\gamma}\sin\epsilon\cos\epsilon\cos\Omega\end{aligned}\right\};\tag{621}$$

$$\gamma\cos^2(v-\Omega)\left(\frac{dR}{dv}\right)=$$

$$\frac{m}{a}\mu\left\{\tfrac{1}{2}\gamma\sin^2\epsilon\sin 2nt+\tfrac{1}{4}\gamma\sin^2\epsilon\sin(4nt-2\Omega)+\tfrac{1}{4}\gamma\sin^2\epsilon\sin 2\Omega\right\};\tag{622}$$

$$\cos(v-\Omega)\left(\frac{dR}{d\theta}\right)=$$

$$\frac{m}{a}\mu\left\{\begin{aligned}&\gamma\left\{1-\tfrac{3}{2}\sin^2\epsilon\right\}\sin 2(nt-\Omega)+\tfrac{1}{4}\gamma\sin^2\epsilon\sin(4nt-2\Omega)\\&+\sin\epsilon\cos\epsilon\sin\Omega-\tfrac{1}{4}\gamma\sin^2\epsilon\sin 2\Omega+\sin\epsilon\cos\epsilon\sin(2nt-\Omega)\end{aligned}\right\}.\tag{623}$$

If we substitute these values in equations (E) and (F), we shall obtain

$$\left.\begin{aligned}\frac{d\Omega}{dt} = mn\Big\{ & -\{1 - \tfrac{3}{2}\sin^2\epsilon\} + \tfrac{3}{2}\{\sin^2\epsilon - \sin^2\epsilon_0\} + \tfrac{1}{2}\sin^2\epsilon\cos 2\Omega \\ & - \frac{1}{\gamma}\sin\epsilon\cos\epsilon\cos\Omega + \frac{1}{\gamma}\sin\epsilon\cos\epsilon\cos(2nt - \Omega) \\ & + \{1 - \tfrac{7}{4}\sin^2\epsilon\}\cos 2(nt - \Omega) - \tfrac{1}{4}\sin^2\epsilon\cos 2nt \\ & - \tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos(4nt - 3\Omega) - \tfrac{1}{4}\gamma\sin\epsilon\cos\epsilon\cos(2nt - 3\Omega)\Big\}\end{aligned}\right\}; \qquad (624)$$

$$\left.\begin{aligned}\frac{d\gamma}{dt} = mn\Big\{ & -\sin\epsilon\cos\epsilon\sin\Omega + \tfrac{1}{2}\gamma\sin^2\epsilon\sin 2\Omega + \tfrac{1}{2}\gamma\sin^2\epsilon\sin 2nt \\ & - \sin\epsilon\cos\epsilon\sin(2nt - \Omega) - \gamma\{1 - \tfrac{3}{2}\sin^2\epsilon\}\sin 2(nt - \Omega)\Big\}\end{aligned}\right\}. \qquad (625)$$

In the integration of these equations we shall suppose that Ω is variable, and that we have

$$\Omega = \Omega_0 + \alpha' t, \qquad (626)$$

but shall neglect α' in the integrals of those terms in which the moon's mean motion enters into the argument. Therefore we shall find

$$\left.\begin{aligned}\Omega = \Omega_0 - mn\{1 - \tfrac{3}{2}\sin^2\epsilon\}t + \tfrac{3}{2}mn\int\{\sin^2\epsilon - \sin^2\epsilon_0\}dt & \\ + m\Big\{\tfrac{1}{4}\frac{n}{\alpha'}\sin^2\epsilon\sin 2\Omega - \frac{n}{\alpha'\gamma}\sin\epsilon\cos\epsilon\sin\Omega & \\ + \frac{1}{2\gamma}\sin\epsilon\cos\epsilon\sin(2nt - \Omega) - \tfrac{1}{8}\sin^2\epsilon\sin 2nt & \\ + \tfrac{1}{2}\{1 - \tfrac{7}{4}\sin^2\epsilon\}\sin 2(nt - \Omega) - \tfrac{1}{16}\gamma\sin\epsilon\cos\epsilon\sin(4nt - 3\Omega) & \\ - \tfrac{1}{8}\gamma\sin\epsilon\cos\epsilon\sin(2nt - 3\Omega)\Big\} &\end{aligned}\right\}; \qquad (627)$$

$$\left.\begin{aligned}\gamma = \gamma_0 + m\Big\{ & -\tfrac{1}{4}\frac{n}{\alpha'}\gamma\sin^2\epsilon\cos 2\Omega + \frac{n}{\alpha'}\sin\epsilon\cos\epsilon\cos\Omega - \tfrac{1}{4}\gamma\sin^2\epsilon\cos 2nt \\ & + \tfrac{1}{2}\gamma\{1 - \tfrac{3}{2}\sin^2\epsilon\}\cos 2(nt - \Omega) + \tfrac{1}{2}\sin\epsilon\cos\epsilon\cos(2nt - \Omega)\Big\}\end{aligned}\right\}. \qquad (628)$$

The principal term in the value of θ in equation (124) is

$$\theta = \gamma\sin(nt - \Omega). \qquad (629)$$

The only terms of ☊ and γ which sensibly affect the latitude are the following:

$$\delta\Omega = m\left\{\frac{1}{2\gamma}\sin\varepsilon\cos\varepsilon\sin(2nt-\Omega) - \frac{n}{\alpha'\gamma}\sin\varepsilon\cos\varepsilon\sin\Omega\right\}. \tag{630}$$

$$\delta\gamma = m\left\{\tfrac{1}{2}\sin\varepsilon\cos\varepsilon\cos(2nt-\Omega) + \frac{n}{\alpha'}\sin\varepsilon\cos\varepsilon\cos\Omega\right\}. \tag{631}$$

If we now compute the equation

$$\delta\theta = \left(\frac{d\theta}{d\gamma}\right)\delta\gamma + \left(\frac{d\theta}{d\Omega}\right)\delta\Omega, \tag{632}$$

we shall obtain

$$\delta\theta = m\left\{\frac{n}{\alpha'} - \tfrac{1}{2}\right\}\sin\varepsilon\cos\varepsilon\sin nt. \tag{633}$$

47. If we substitute the values of $\left(\frac{dR}{dv}\right)$ and $\left(\frac{dR}{d\theta}\right)$ in equation (263), we shall find

$$\delta_0\theta = \tfrac{1}{2}m\sin\varepsilon\cos\varepsilon\sin nt. \tag{634}$$

There are no terms of this form in the values of $\delta_1\theta$ and $\delta_2\theta$; therefore the complete value of $\delta\theta$ will be the sum of equations (633) and (634). Whence we get

$$\delta\theta = m\frac{n}{\alpha'}\sin\varepsilon\cos\varepsilon\sin nt. \tag{635}$$

48. We shall now determine the variations of the radius vector and the longitude. For this purpose we shall observe that the factors by which the forces are to be multiplied for the determination of $\frac{d\delta_1 r}{dt}$ are given by means of equations (228), (234), and (239). We shall therefore find

$$\left.\begin{aligned} &c_2\cos\beta\left(\frac{dR}{dr}\right) = \\ &\mu mndt\Big\{-\{1-\tfrac{3}{4}\sin^2\varepsilon\}\cos nt - \tfrac{3}{4}\sin^2\varepsilon\cos 3nt \\ &\qquad -\tfrac{3}{2}\gamma\sin\varepsilon\cos\varepsilon\cos(3nt-\Omega) + \tfrac{3}{2}\gamma\sin\varepsilon\cos\varepsilon\cos(nt+\Omega)\Big\}\end{aligned}\right\}. \tag{636}$$

$$c_2 \sin\beta\left(\frac{dR}{dr}\right) =$$

$$\left.\begin{aligned} \mu mndt \Big\{ & \{1 - \tfrac{3}{4}\sin^2\varepsilon\} \sin nt + \tfrac{3}{4}\sin^2\varepsilon \sin 3nt \\ & + \tfrac{3}{2}\gamma \sin\varepsilon \cos\varepsilon \sin(3nt - \Omega) - \tfrac{3}{2}\gamma \sin\varepsilon \cos\varepsilon \sin(nt + \Omega) \\ & \qquad - 3\gamma \sin\varepsilon \cos\varepsilon \sin(nt - \Omega) \Big\} \end{aligned}\right\} . \tag{637}$$

$$c_3 \sin\beta\left(\frac{dR}{dv}\right) =$$

$$\left.\begin{aligned} \mu mndt \Big\{ & \sin^2\varepsilon \cos nt - \sin^2\varepsilon \cos 3nt - \gamma \sin\varepsilon \cos\varepsilon \cos(3nt - \Omega) \\ & \qquad + \gamma \sin\varepsilon \cos\varepsilon \cos(nt + \Omega) \Big\} \end{aligned}\right\} . \tag{638}$$

$$c_3 \cos\beta\left(\frac{dR}{dv}\right) =$$

$$\left.\begin{aligned} \mu mndt \Big\{ & \sin^2\varepsilon \sin nt + \sin^2\varepsilon \sin 3nt + \gamma \sin\varepsilon \cos\varepsilon \sin(3nt - \Omega) \\ & - \gamma \sin\varepsilon \cos\varepsilon \sin(nt + \Omega) + 2\gamma \sin\varepsilon \cos\varepsilon \sin(nt - \Omega) \Big\} \end{aligned}\right\} . \tag{639}$$

$$c_4 \sin\beta\left(\frac{dR}{d\theta}\right) =$$

$$\left.\begin{aligned} \mu mndt \Big\{ & -\gamma \sin\varepsilon \cos\varepsilon \cos(3nt - \Omega) - \gamma \sin\varepsilon \cos\varepsilon \cos(nt + \Omega) \\ & \qquad + 2\gamma \sin\varepsilon \cos\varepsilon \cos(nt - \Omega) \Big\} \end{aligned}\right\} . \tag{640}$$

$$c_4 \cos\beta\left(\frac{dR}{d\theta}\right) =$$

$$\mu mndt \Big\{ \gamma \sin\varepsilon \cos\varepsilon \sin(3nt - \Omega) + \gamma \sin\varepsilon \cos\varepsilon \sin(nt + \Omega) \Big\} . \tag{641}$$

These equations give

$$c_2 \cos\beta\left(\frac{dR}{dr}\right) + c_3 \sin\beta\left(\frac{dR}{dv}\right) + c_4 \sin\beta\left(\frac{dR}{d\theta}\right) =$$

$$\left.\begin{aligned} \mu mndt \Big\{ & -\{1 - \tfrac{7}{4}\sin^2\varepsilon\} \cos nt - \tfrac{7}{4}\sin^2\varepsilon \cos 3nt \\ & - \tfrac{7}{2}\gamma \sin\varepsilon \cos\varepsilon \cos(3nt - \Omega) + \tfrac{3}{2}\gamma \sin\varepsilon \cos\varepsilon \cos(nt + \Omega) \\ & \qquad + 2\gamma \sin\varepsilon \cos\varepsilon \cos(nt - \Omega) \Big\} \end{aligned}\right\} . \tag{642}$$

$$c_2 \sin\beta\left(\frac{dR}{dr}\right) + c_3\cos\beta\left(\frac{dR}{dv}\right) + c_4\cos\beta\left(\frac{dR}{d\theta}\right) =$$

$$\left.\begin{array}{l} \mu mndt\Big\{+\{1-\frac{5}{4}\sin^2\epsilon\}\sin nt + \frac{7}{4}\sin^2\epsilon\sin 3nt \\ \qquad + \frac{7}{2}\gamma\sin\epsilon\cos\epsilon\sin(3nt-\Omega) - \frac{5}{2}\gamma\sin\epsilon\cos\epsilon\sin(nt+\Omega) \\ \qquad\qquad - \gamma\sin\epsilon\cos\epsilon\sin(nt-\Omega)\Big\} \end{array}\right\} . \quad (643)$$

Equations (642) and (643) give by integration,

$$\int\left\{c_2\cos\beta\left(\frac{dR}{dr}\right) + c_3\sin\beta\left(\frac{dR}{dv}\right) + c_4\sin\beta\left(\frac{dR}{d\theta}\right)\right\} =$$

$$\left.\begin{array}{l} \mu m\Big\{-\{1-\frac{3}{4}\sin^2\epsilon\}\sin nt - \frac{7}{12}\sin^2\epsilon\sin 3nt \\ \qquad - \frac{7}{6}\gamma\sin\epsilon\cos\epsilon\sin(3nt-\Omega) + \frac{3}{2}\frac{n}{n+\alpha'}\gamma\sin\epsilon\cos\epsilon\sin(nt+\Omega) \\ \qquad\qquad + 2\frac{n}{n-\alpha'}\gamma\sin\epsilon\cos\epsilon\sin(nt-\Omega)\Big\} \end{array}\right\} . \quad (644)$$

$$\int\left\{c_2\sin\beta\left(\frac{dR}{dr}\right) + c_3\cos\beta\left(\frac{dR}{dv}\right) + c_4\cos\beta\left(\frac{dR}{d\theta}\right)\right. =$$

$$\left.\begin{array}{l} \mu m\Big\{-\{1-\frac{5}{4}\sin^2\epsilon\}\cos nt - \frac{7}{12}\sin^2\epsilon\cos 3nt \\ \qquad - \frac{7}{6}\gamma\sin\epsilon\cos\epsilon\cos(3nt-\Omega) + \frac{5}{2}\frac{n}{n+\alpha'}\gamma\sin\epsilon\cos\epsilon\cos(nt+\Omega) \\ \qquad\qquad + \frac{n}{n-\alpha'}\gamma\sin\epsilon\cos\epsilon\cos(nt-\Omega)\Big\} \end{array}\right\} . \quad (645)$$

In finding these integrals we have neglected α' in those terms in which the coefficient of nt is greater than unity.

With these integrals and the values of $c_1\cos\beta$ and $c_1\sin\beta$, which are given by equation (240), we obtain the value of $\frac{d\delta_1 r}{dt}$, in which we have neglected α'^2 in comparison with n^2 in the last term.

$$\frac{d\delta_1 r}{dt} = amn\left\{ -\tfrac{1}{3}\sin^2\varepsilon\sin 2nt - \tfrac{2}{3}\gamma\sin\varepsilon\cos\varepsilon\sin(2nt - \Omega) \right.$$
$$\left. - 3\frac{\alpha'}{n}\gamma\sin\varepsilon\cos\varepsilon\sin\Omega \right\}. \quad (646)$$

Since $\delta_0 r$, $\delta_2 r$, and $\delta_3 r$ are each equal to nothing when we neglect e, we shall have $\frac{d\delta_1 r}{dt} = \frac{d\delta r}{dt}$, and equation (646) will give by integration,

$$\delta r = am\left\{ \tfrac{1}{3}\{1 - \tfrac{3}{2}\sin^2\varepsilon\} + \tfrac{1}{6}\sin^2\varepsilon\cos 2nt + \tfrac{1}{3}\gamma\sin\varepsilon\cos\varepsilon\cos(2nt - \Omega) \right.$$
$$\left. + 3\gamma\sin\varepsilon\cos\varepsilon\cos\Omega \right\}, \quad (647)$$

the term $\tfrac{1}{3}\{1 - \tfrac{3}{2}\sin^2\varepsilon\}$ being added to complete the integral.

Equation (618) will give

$$\int\left(\frac{dR}{dv}\right)dt =$$
$$\frac{m}{a}\mu\left\{ -\frac{1}{2n}\sin^2\varepsilon\cos 2nt - \frac{1}{2n}\gamma\sin\varepsilon\cos\varepsilon\cos(2nt - \Omega) \right.$$
$$\left. + \frac{1}{\alpha'}\gamma\sin\varepsilon\cos\varepsilon\cos\Omega \right\}. \quad (648)$$

From this we get

$$\frac{d\delta_0 v}{dt} = mn\left\{ \tfrac{1}{2}\sin^2\varepsilon\cos 2nt + \tfrac{1}{2}\gamma\sin\varepsilon\cos\varepsilon\cos(2nt - \Omega) \right.$$
$$\left. - \frac{n}{\alpha'}\gamma\sin\varepsilon\cos\varepsilon\cos\Omega \right\}. \quad (649)$$

Equation (647) will give

$$\frac{d\delta_1 v}{dt} = mn\left\{ -\tfrac{1}{3}\sin^2\varepsilon\cos 2nt - \tfrac{2}{3}\gamma\sin\varepsilon\cos\varepsilon\cos(2nt - \Omega) \right.$$
$$\left. - 6\gamma\sin\varepsilon\cos\varepsilon\cos\Omega \right\}. \quad (650)$$

The value of $\delta\theta$ (635), being substituted in (262), will give

$$\frac{d\delta_2 v}{dt} = mn\left\{ \frac{n}{\alpha'}\gamma\sin\varepsilon\cos\varepsilon\cos\Omega - \frac{n}{\alpha'}\gamma\sin\varepsilon\cos\varepsilon\cos(2nt - \Omega) \right\}. \quad (651)$$

The sum of equations (649–651) is the value $\frac{d\delta v}{dt}$. Whence we get

$$\left.\begin{aligned}\frac{d\delta v}{dt} = mn\Big\{ \tfrac{1}{6}\sin^2\epsilon\cos 2nt - \{\tfrac{1}{6} + \frac{n}{\alpha'}\}\gamma\sin\epsilon\cos\epsilon\cos(2nt - \Omega) \\ - 6\gamma\sin\epsilon\cos\epsilon\cos\Omega \Big\}\end{aligned}\right\} . \tag{652}$$

Whence we get by integration

$$\left.\begin{aligned}\delta v = m\Big\{ \tfrac{1}{12}\sin^2\epsilon\sin 2nt - \{\tfrac{1}{12} + \frac{n}{2\alpha'}\}\gamma\sin\epsilon\cos\epsilon\sin(2nt - \Omega) \\ + 6\frac{n}{\alpha'}\gamma\sin\epsilon\cos\epsilon\sin\Omega \Big\}\end{aligned}\right\} . \tag{653}$$

In this investigation we have omitted the term $\frac{3}{2}\{\sin^2\epsilon_0 - \sin^2\epsilon\}$ of the value of $\left(\frac{dR}{dr}\right)$, since it is of the form $h\cos(\alpha t - \beta)$ arising from the sun's action; and we can determine its effect on the moon's longitude by substituting its value in equation (307).

The effect of the earth's oblateness on the motion of the perigee is also easily found to be the same as on the motion of the node, except that it has a contrary sign.

49. In order to reduce these formulas to numbers we shall assume, as given by observation,

$$\left.\begin{aligned}\rho = \tfrac{1}{300}, \quad \phi = \tfrac{1}{289}, \quad D = 0.016551\,a, \quad \alpha' = -0.00402177n, \\ \gamma = 0.0900456, \quad \epsilon = 23^\circ\ 27'\ 30''\end{aligned}\right\} . \tag{654}$$

Equation (612) will give

$$m = 0''.090588. \tag{655}$$

Then we get

$$\delta v = 0''.0012\sin 2nt + 0''.371\sin(2nt - \Omega) + 4''.444\sin\Omega; \tag{656}$$

$$\delta\theta = -8''.226\sin nt; \tag{657}$$

$$\left.\begin{aligned}\omega &= \omega_0 + 0.0000003348\,nt = \omega_0 + 5''.801\,t,\\ \Omega &= \Omega_0 - 0.0000003348\,nt = \Omega_0 - 5.801\,t.\end{aligned}\right\}. \quad (658)$$

In the values of ω and Ω, t denotes the number of years.

If we neglect the terms of equation (653) which contain κ', we shall have

$$\left.\begin{aligned}\delta v &= m\{\tfrac{1}{12}\sin^2\varepsilon\sin 2nt - \tfrac{1}{12}\gamma\sin\varepsilon\cos\varepsilon\sin(2nt-\Omega),\\ &= 0''.0012\sin 2nt - 0''.0002\sin(2nt-\Omega)\end{aligned}\right\}. \quad (659)$$

This is the only perturbation of the moon's longitude produced by the earth's oblateness which is independent of the sun's action.

49′. If we neglect the eccentricity of the moon's orbit we shall have $dr = 0$, $dv = ndt$, and $d\theta = ndt\,\gamma\cos(nt-\Omega)$; and if we multiply equations (617), (618), and (619) by these values of dr, dv, and $d\theta$, respectively, and substitute the products in equation (I), page 41, we shall obtain

$$2dR = -\mu\frac{da}{a^2} = 2m\frac{\mu}{a}ndt\,\{\sin^2\varepsilon\sin 2nt + 2\gamma\sin\varepsilon\cos\varepsilon\sin(2nt-\Omega)\}. \quad (659')$$

This gives by integration,

$$\delta a = am\,\{\sin^2\varepsilon\cos 2nt + 2\gamma\sin\varepsilon\cos\varepsilon\cos(2nt-\Omega)\}. \quad (659'')$$

It follows from this equation that the moon's mean distance is not affected by any inequalities of long period depending on the oblateness of the earth.

CHAPTER IX.

DETERMINATION OF THE LONG PERIOD AND SECULAR INEQUALITIES IN THE MOON'S MOTION.

50. THE inequalities of long period in the moon's motion arising from the variation of the central force which is produced by the changes of the elements of the moon's orbit may be computed by means of the equation

$$\delta v = -2\frac{\overline{m}^2}{\mu}\frac{n^3 h'}{(n^2 - \alpha'^2)\alpha'}\sin(\alpha' t - \beta'), \tag{660}$$

which is one of the terms of equation (307).

If we put

$$h' = -\tfrac{9}{8}e^2\gamma^2, \qquad \alpha' t - \beta' = 2(\omega - \Omega), \tag{661}$$

we shall obtain the inequality in the longitude which depends on twice the difference of longitude of the perigee and node of the lunar orbit. In this case we have

$$\alpha' = 0.0249474 n, \tag{661'}$$

and if we neglect α'^2 in comparison with n^2 in the denominator of equation (660), it will become

$$\left.\begin{aligned}\delta v &= -2\frac{\overline{m}^2}{\mu}\frac{nh'}{\alpha'}\sin(\alpha' t - \beta')\\ &= +\tfrac{9}{4}\frac{\overline{m}^2}{\mu}\frac{n}{\alpha'}e^2\gamma^2\sin(\alpha' t - \beta').\end{aligned}\right\} \tag{662}$$

Now we have, very nearly, as we shall see hereafter,

$$\left.\begin{aligned}\frac{\overline{m}^2}{\mu} = \frac{n'^2}{n^2} &= 0.00559524, \qquad & e' &= 0.01677120,\\ e &= 0.05489930, \qquad & \gamma &= 0.09004560.\end{aligned}\right\} \tag{662'}$$

If we substitute these quantities in equation (662), we shall find

$$\delta v = +2''.544 \sin 2(\omega - \Omega). \tag{663}$$

Equation (307) also contains the term

$$\delta v = \frac{\overline{m}^2}{\mu}\frac{a}{a'}\left\{\tfrac{45}{16}\frac{n^2}{\alpha''^2} + \tfrac{3}{2}\frac{n}{\alpha''}\right\}h''\sin(\alpha''t - \beta''). \tag{664}$$

If in this we put

$$h'' = ee', \qquad \alpha''t - \beta'' = \omega - \omega', \tag{665}$$

we shall obtain the inequality of the longitude depending on the angular distance between the perigee of the sun and moon. The term of equation (664), which has α''^2 for a denominator, is produced by the force $\left(\frac{dR}{dv}\right)$, while the term $\frac{3}{2}\frac{n}{\alpha''}$ arises from the force $\left(\frac{dR}{dr}\right)$. The term $\frac{3}{2}\frac{n}{\alpha''}h''$, of equation (664) is derived from the term $+\frac{3}{4}ee'\cos(\omega-\omega')$ in the expression of the force $\left(\frac{dR}{dr}\right)$ in equation (218); while the term $+\frac{45}{16}\frac{n^2}{\alpha''^2}h''$ comes from the term $-\frac{15}{16}ee'\sin(\omega-\omega')$ in the expression of the force $\left(\frac{dR}{dv}\right)$ in equation (219). If we divide these terms by $\frac{3}{4}$ and $-\frac{15}{16}$ respectively, they will become

$$2\frac{n}{\alpha''}h'' \qquad \text{and} \qquad -3\frac{n^2}{\alpha''^2}h''.$$

These coefficients correspond to equal forces in the direction of, and perpendicular to, the radius vector. Whence it follows that the inequality produced by a central force is to the inequality produced by an equal tangential force as 1 to $-\frac{3}{2}\frac{n}{\alpha''}$.

If we neglect α'^2 in comparison with n^2, equation (660) shows that the inequalities produced by the central force are proportional to the product of the force into the period of the argument, while equation (664) shows that the inequalities produced by the tangential force are proportional to the product of the force into the square of the period of the argument.

If we now substitute the preceding values of e, e' and also $\alpha'' = 0.0084513n$, in equation (664), it will become

$$\delta v = \{107''.08 + 1''.45 = 108''.53\}\sin(\omega - \omega'). \tag{666}$$

In this calculation we have supposed that the sun's parallax is $8''.75$. It is evident that this inequality may itself be used to find the sun's parallax; since according to the preceding calculations its coefficient is nearly as large as the

coefficient of the parallactic equation, and its theoretical and practical determination is free from the difficulties which seem to be inseparably connected with this latter equation.

51. We shall now determine the value of the secular equation of the moon's longitude. For this purpose we have in equation (218), the term

$$\left(\frac{dR}{dr}\right)=\frac{\overline{m}^2}{a^2}\{-\tfrac{3}{4}e'^2\};\tag{667}$$

and in equation (327) we have the term

$$\delta\left(\frac{dR}{dr}\right)=\frac{\overline{m}^4}{a^2\mu}\{+34.39014e'^2\}.\tag{668}$$

Also by noticing quantities of the order e'^2 in the computation of $\delta^2\left(\frac{dR}{dr}\right)$, we shall obtain the term

$$\delta^2\left(\frac{dR}{dr}\right)=\frac{\overline{m}^6}{a^2\mu^2}\{-405.3504e'^2\}.\tag{669}$$

Moreover the term $h'\cos(\alpha't-\beta')$ of the value of δr, in equation (303), produces in the value of $\delta\left(\frac{dR}{dr}\right)$, the term

$$\frac{\overline{m}^4}{a^2\mu}\{-\tfrac{1}{2}h'\cos(\alpha't-\beta')\};\tag{670}$$

and there is a similar term in $\delta^2\left(\frac{dR}{dr}\right)$, which is equal to

$$\frac{\overline{m}^6}{a^2\mu^2}\{-\tfrac{1}{4}h'\cos(\alpha't-\beta')\}.\tag{671}$$

We have in the preceding chapters considered e' as constant, and in order to notice the variableness of e' without affecting the accuracy of the preceding calculations, we must write $e'^2-e'^2_0$ for e'^2 in the above formulas, e'_0 being the value of e' at the epoch. But we have

$$e'^2-e'^2_0=h'\cos(\alpha't-\beta');$$

therefore we shall have by putting the sum of the preceding terms equal to $\left(\frac{dR}{dr}\right)$:

$$\left(\frac{dR}{dr}\right)=\frac{\overline{m}^2}{a^2}\left\{-\tfrac{3}{4}+33.89014\frac{\overline{m}^2}{\mu}-405.6004\frac{\overline{m}^4}{\mu^2}\right\}(e'^2-e'^2_0)=\frac{\mu}{a^2}H\{e'^2-e'^2_0\}. \quad (672)$$

Or by substituting the value of $e'^2-e'^2_0$, we shall have

$$\left(\frac{dR}{dr}\right)=Hh'\cos(\alpha't-\beta'). \quad (673)$$

Now if we neglect α'^2 in comparison with n^2, we shall find the term in equation (307),

$$\delta v=\frac{\overline{m}^2}{\mu}\{-2n\frac{h'}{\alpha'}\sin(\alpha't-\beta')\}. \quad (674)$$

But
$$\frac{h'}{\alpha'}\sin(\alpha't-\beta')=\int\{e'^2-e'^2_0\}\,dt; \quad (675)$$

and in order to notice the higher powers of the disturbing force we must put $-H$ for $\frac{\overline{m}^2}{\mu}$ in equation (674). Then we shall have

$$\delta v=+2Hn\int\{e'^2-e'^2_0\}\,dt. \quad (676)$$

All that now remains in order to obtain the secular equation is to find the value of the integral $\int(e'^2-e'^2_0)\,dt$. For this purpose we shall observe that, according to the developments of physical astronomy, the values e' and ω', at any time t, may be determined by means of equations of the following form:

$$\left.\begin{aligned}e'\sin\omega'&=N'\sin(gt+\beta)+N_1'\sin(g_1t+\beta_1)+\text{etc.}\\ e'\cos\omega'&=N'\cos(gt+\beta)+N_1'\cos(g_1t+\beta_1)+\text{etc.}\end{aligned}\right\}; \quad (677)$$

in which the coefficients N', N_1', g, g_1, β, β_1, etc. are constant.

If we take the sum of the squares of equations (677), we shall find

$$\left.\begin{aligned}e'^2=&\;N'^2+N_1'^2+N_2'^2+\text{etc.},\\ &+2N'N_1'\cos\{(g_1-g)t+\beta'-\beta\}+2N'N_2'\cos\{(g_2-g)t+\beta_2-\beta\},\\ &+2N_1'N_2'\cos\{(g_2-g_1)t+\beta_2-\beta_1\}+\text{etc.},\end{aligned}\right\}. \quad (678)$$

In number (232) of the *Smithsonian Contributions to Knowledge* I have determined the following values of the quantities which enter into the above formulas:

$$\left.\begin{array}{lll}
N' = +0.0053866, & g = 5''.509545, & \beta = 87^\circ\ 43'\ 23''.3, \\
N_1' = -0.0152679, & g_1 = 7.315380, & \beta_1 = 19\ \ 43\ \ 4\ .3, \\
N_2' = +0.0122912, & g_2 = 17.217532, & \beta_2 = 331\ \ 51\ \ 47\ .6, \\
N_3' = +0.0171912, & g_3 = 17.931057, & \beta_3 = 136\ \ 5\ \ 29\ .0, \\
N_4' = +0.0000136, & g_4 = 0.616686, & \beta_4 = 67\ \ 56\ \ 38\ .9, \\
N_5' = +0.0005812, & g_5 = 2.727684, & \beta_5 = 105\ \ 5\ \ 26\ .0, \\
N_6' = +0.0162413, & g_6 = 3.716923, & \beta_6 = 28\ \ 8\ \ 50\ .4, \\
N_7' = -0.0024158, & g_7 = 22.460985, & \beta_7 = 307\ \ 56\ \ 52\ .2.
\end{array}\right\} \quad (679)$$

The quantities N', N_1', etc., g, g_1, etc., are independent of the time, and β, β_1, etc., correspond to the epoch of 1850.0.

If we now substitute the preceding values in equation (678), we shall obtain the following value of e'^2, in which the coefficients enclosed in brackets are logarithms:

$$\left.\begin{array}{l}
e'^2 = 0.000978688 \\
\quad -[96.2161253]\cos\{\ 1''.805835.t + 291^\circ\ 59'\ 41''.0\} \\
\quad +[96.2676518]\cos\{12''.421512.t + 48^\circ\ 22'\ 5''.7\} \\
\quad +[94.7967198]\cos\{\ 2''.781861.t + 342^\circ\ 37'\ 57''.3\} \\
\quad -[95.4154103]\cos\{16''.951440.t + 220^\circ\ 13'\ 28''.9\} \\
\quad -[96.7201139]\cos\{10''.615677.t + 116^\circ\ 22'\ 24''.7\} \\
\quad -[95.2491819]\cos\{\ 4''.587696.t + 274^\circ.37'\ 38''.3\} \\
\quad +[95.8678724]\cos\{15''.145605.t + 288^\circ\ 13'\ 47''.9\} \\
\quad +[93.5231839]\cos\{16''.600846.t + 263^\circ\ 55'\ 8''.7\} \\
\quad +[96.6012456]\cos\{13''.500609.t + 303^\circ\ 42'\ 57''.2\} \\
\quad +[93.6688941]\cos\{17''.314371.t + 68^\circ\ 8'\ 50''.2\} \\
\quad +[96.7469558]\cos\{14''.214134.t + 107^\circ\ 56'\ 38''.6\} \\
\quad +[92.1979621\}\cos\{\ 2''.110998.t + 37^\circ\ 8'\ 47''.2\} \\
\quad -[92.8166526]\cos\{21''.844299.t + 240^\circ\ 0'\ 13''.4\}
\end{array}\right\} \quad (680)$$

(Continued on the next page.)

$$
\left.
\begin{array}{l}
-[94.4484669]\cos\{19''.733301.t+202^\circ\ 51'\ 26''.2\} \\
+[96.1219416]\cos\{11''.707987.t+244^\circ\ 8'\ 24''.3\} \\
+[93.1649055]\cos\{\ 4''.892859.t+\ 19^\circ\ 46'\ 44''.4\} \\
+[96.2429672]\cos\{\ 1''.792622.t+\ 59^\circ\ 34'\ 32''.9\} \\
-[96.5744037]\cos\{\ 9''.902152.t+312^\circ\ 8'\ 43''.3\} \\
-[93.6173676]\cos\{\ 6''.698694.t+311^\circ\ 46'\ 25''.4\} \\
-[96.6954293]\cos\{\ 3''.598457.t+351^\circ\ 34'\ 13''.9\} \\
+[96.6259302]\cos\{\ 0''.713525.t+164^\circ\ 13'\ 41''.4\} \\
+[95.1549982]\cos\{14''.489848.t+226^\circ\ 46'\ 21''.6\} \\
-[95.7736887]\cos\{\ 5''.243453.t+336^\circ\ 5'\ 4''.6\} \\
+[95.3007084]\cos\{15''.203373.t+\ 31^\circ\ 0'\ 3''.0\} \\
-[95.9193989]\cos\{\ 4''.529928.t+171^\circ\ 51'\ 23''.2\} \\
+[93.6442095]\cos\{\ 3''.100237.t+320^\circ\ 12'\ 11''.5\} \\
+[95.2760238]\cos\{\ 0''.989239.t+283^\circ\ 3'\ 24''.4\} \\
-[95.8947143]\cos\{18''.744062.t+279^\circ\ 48'\ 1''.8\}
\end{array}
\right\} \quad . \quad (680)
$$

At the epoch we have $e_0'^2 = 0.000281273$; therefore we shall get by integration,

$$
\left.
\begin{array}{l}
\int\{(e'^2 - e_0'^2)\}\,dt = +\,0.000697415.t \\
\qquad -[1.2738723]\sin\{\ 1''.805835.t+291^\circ\ 59'\ 41''.0\} \\
\qquad +[0.4879024]\sin\{12''.421512.t+\ 48^\circ\ 22'\ 5''.7\} \\
\qquad +[9.6668095]\sin\{\ 2''.781861.t+342^\circ\ 37'\ 57''.3\} \\
\qquad -[9.5006288]\sin\{16''.951440.t+220^\circ\ 13'\ 28''.9\} \\
\qquad -[1.0085914]\sin\{10''.615677.t+116^\circ\ 22'\ 24''.7\} \\
\qquad -[9.9020124]\sin\{\ 4''.587696.t+274^\circ\ 37'\ 38''.3\} \\
\qquad +[0.0020109]\sin\{15''.145605.t+288^\circ\ 13'\ 47''.9\} \\
\qquad +[7.6174787]\sin\{16''.600846.t+263^\circ\ 55'\ 8''.7\} \\
\qquad +[0.7853173]\sin\{13''.500609.t+303^\circ\ 42'\ 57''.2\} \\
\qquad +[7.7449125]\sin\{17''.314371.t+\ 68^\circ\ 8'\ 50''.2\}
\end{array}
\right\} \quad . \quad (681)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+[0.9086606]\sin\{14''.214134.t+107^\circ\ 56'\ 38''.6\}\\
&+[7.1878994]\sin\{\ 2''.110998.t+\ 37^\circ\ \ 8'\ 47''.2\}\\
&-[6.7917396]\sin\{21''.844299.t+240^\circ\ \ 0'\ 13''.4\}\\
&-[8.4676922]\sin\{19''.733301.t+202^\circ\ 51'\ 26''.2\}\\
&+[0.3678845]\sin\{11''.707987.t+244^\circ\ \ 8'\ 24''.3\}\\
&+[7.7897679]\sin\{\ 4''.892859.t+\ 19^\circ\ 46'\ 44''.4\}\\
&+[1.3039036]\sin\{\ 1''.792622.t+\ 59^\circ\ 34'\ 32''.9\}\\
&-[0.8930992]\sin\{\ 9''.902152.t+312^\circ\ \ 8'\ 43''.3\}\\
&-[8.1058026]\sin\{\ 6''.698694.t+311^\circ\ 46'\ 25''.4\}\\
&-[1.4537382]\sin\{\ 3''.598457.t+351^\circ\ 34'\ 13''.9\}\\
&+[2.0869461]\sin\{\ 0''.713525.t+164^\circ\ 13'\ 41''.4\}\\
&+[9.3083594]\sin\{14''.489848.t+226^\circ\ 46'\ 21''.6\}\\
&-[0.3685048]\sin\{\ 5''.243453.t+336^\circ\ \ 5'\ \ 4''.6\}\\
&+[9.4331936]\sin\{15''.203373.t+\ 31^\circ\ \ 0'\ \ 3''.0\}\\
&-[0.5777327]\sin\{\ 4''.529928.t+171^\circ\ 51'\ 23''.2\}\\
&+[8.4672397]\sin\{\ 3''.100237.t+320^\circ\ 12'\ 11''.5\}\\
&+[0.5951476]\sin\{\ 0''.989239.t+283^\circ\ \ 3'\ 24''.4\}\\
&-[9.9362757]\sin\{18''.744062.t+279^\circ\ 48'\ \ 1''.8\}
\end{aligned}\right\}. \qquad (681)$$

If we reduce this integral to numbers, we shall have the table on page 302, in which i denotes the number of centuries from the epoch of 1850.0.

Now equation (672) gives

$$H=\frac{\bar{m}^2}{\mu}\left\{-\tfrac{3}{4}+33.89014\frac{\bar{m}^3}{\mu}-405.6004\frac{m^4}{\mu^2}\right\}. \qquad (682)$$

If we reduce this equation to numbers, it will give

$$H=-0.003206483. \qquad (683)$$

And if we put for n the moon's mean motion in a Julian year, we shall obtain for the coefficient of $\int(e^2-e'^2_0)\,dt$ in the value of δv in equation (676)

$$2Hn=-111108''.5. \qquad (684)$$

If we multiply this coefficient by the numbers in the following table, we shall obtain the corresponding value of the secular inequality.

44

TABLE SHOWING THE VALUE OF $\int (e_0'^2 - e'^2)\,dt$.

i	$\int (e_0'^2 - e'^2)dt$	1st Diff.	2d Diff.	i	$\int (e_0'^2 - e'^2)dt$	1st Diff.	2d Diff.
0	0.000000			−32	0.068337		
−1	0.000071	+71	+138	33	0.072557	+4220	+118
2	0.000280	209	138	34	0.076895	4338	117
3	0.000627	347	138	35	0.081350	4455	115
4	0.001112	485	137	36	0.085920	4570	115
5	0.001734	622	138	37	0.090605	4685	115
6	0.002494	760	137	38	0.095405	4800	113
7	0.003391	897	135	39	0.100318	4913	113
8	0.004423	1032	135	40	0.105344	5026	112
9	0.005590	1167	136	41	0.110482	5138	113
10	0.006893	1303	134	42	0.115733	5251	111
11	0.008330	1437	133	43	0.121095	5362	109
12	0.009900	1570	133	44	0.126566	5471	109
13	0.011603	1703	133	45	0.132146	5580	107
14	0.013439	1836	132	46	0.137833	5687	107
15	0.015407	1968	130	47	0.143627	5794	106
16	0.017505	2098	130	48	0.149527	5900	105
17	0.019733	2228	131	49	0.155532	6005	104
18	0.022092	2359	129	50	0.161641	6109	103
19	0.024580	2488	128	51	0.167853	6212	102
20	0.027196	2616	127	52	0.174167	6314	100
21	0.029939	2743	128	53	0.180581	6414	102
22	0.032810	2871	126	54	0.187097	6516	99
23	0.035807	2997	125	55	0.193712	6615	97
24	0.038929	3122	125	56	0.200424	6712	97
25	0.042176	3247	124	57	0.207233	6809	98
26	0.045547	3371	123	58	0.214140	6907	96
27	0.049041	3494	123	59	0.221143	7003	93
28	0.052658	3617	121	60	0.228239	7096	93
29	0.056396	3738	122	61	0.235428	7189	93
30	0.060256	3860	120	62	0.242710	7282	91
31	0.064236	3980	121	63	0.250083	7373	+91
−32	0.068337	4101	+119	−64	0.257547	+7464	

52. The value of $\left(\frac{dR}{dr}\right)$ in equation (617) contains the term

$$+\tfrac{3}{2}\frac{m}{a}\mu\,\{\sin^2\epsilon_0-\sin^2\epsilon\}$$

arising from the oblateness of the earth; and this produces the secular inequality

$$\delta v=3mn\int\{\sin^2\epsilon_0-\sin^2\epsilon\}\,dt. \tag{685}$$

If we assume

$$\left.\begin{aligned}\epsilon_0&=23^\circ\ 27'\ 31''.0\\ \epsilon&=\epsilon_0-0''.48970t-0.0000012t^2,\end{aligned}\right\} \tag{686}$$

we shall find

$$\sin^2\epsilon_0-\sin^2\epsilon=0.0001733396i+0.00000000399i^2. \tag{687}$$

Whence we get

$$\int\{\sin^2\epsilon_0-\sin^2\epsilon\}\,dt=0.0086698i^2+0.000000133i^3. \tag{688}$$

The coefficient $3mn$ reduced to numbers becomes

$$3mn=22''.827; \tag{689}$$

therefore the secular inequality arising from the oblateness of the earth becomes

$$\delta v=0''.1979i^2+0''.000003036i^3, \tag{690}$$

in which i denotes the number of centuries before or after the year 1850.

We shall denote the secular inequality which is obtained by taking the sum of equations (676) and (690) by $\delta_i v$.

CHAPTER X.

PERTURBATIONS OF THE MOON'S MOTION ARISING FROM THE PERTURBATIONS OF THE SUN'S MOTION.

53. We shall now determine the perturbations of the moon's motion which depend on the perturbations of the sun's apparent motion. For this purpose we must take the partial differential coefficients of the forces $\left(\frac{dR}{dr}\right)$ and $\left(\frac{dR}{dv}\right)$, with respect to r' and v', and multiply them by the values of $\delta r'$ and $\delta v'$, in the same manner as we have already done in Chapter IV. We shall first consider the inequalities resulting from the perturbations of the earth's motion by the moon.

If we denote the moon's mass by m and neglect the inclination of the moon's orbit, the theory of the sun's motion gives the inequalities

$$\delta r' = \frac{m}{\mu} r \cos(v - v'), \qquad \delta v' = \frac{m}{\mu}\frac{r}{r'} \sin(v - v'). \tag{691}$$

We shall therefore obtain by means of equations (153), (154), and (691)

$$\left.\begin{aligned}\delta\left(\frac{dR}{dr}\right) = \frac{m}{\mu}\frac{\overline{m}^2}{a^2}\frac{a}{a'}\Big\{ & \tfrac{9}{4}\cos(nt - n't) - \tfrac{9}{2}e\cos(n't - \omega) + \tfrac{9}{4}e'\cos(nt - \omega') \\ & + \tfrac{21}{4}e'\cos(nt - 2n't + \omega') + \tfrac{15}{4}\cos 3(nt - n't) \\ & + \tfrac{15}{2}e\cos(4nt - 3n't - \omega) - 15e\cos(2nt - 3n't + \omega) \\ & + \tfrac{75}{4}e'\cos(3nt - 4n't + \omega') - \tfrac{15}{4}e'\cos(3nt - 2n't - \omega')\Big\}\end{aligned}\right\} . \tag{692}$$

$$\left.\begin{aligned}\delta\left(\frac{dR}{dv}\right) = \frac{m}{\mu}\frac{\overline{m}^2}{a}\frac{a}{a'}\Big\{ & -\tfrac{3}{4}\sin(nt - n't) + \tfrac{9}{8}e\sin(2nt - n't - \omega) \\ & - \tfrac{15}{8}e\sin(n't - \omega) - \tfrac{3}{4}e'\sin(nt - \omega') - \tfrac{9}{4}e'\sin(nt - 2n't + \omega') \\ & - \tfrac{15}{4}\sin 3(nt - n't) - \tfrac{45}{8}e\sin(4nt - 3n't - \omega) \\ & + \tfrac{135}{8}e\sin(2nt - 3n't + \omega) - \tfrac{75}{4}e'\sin(3nt - 4n't + \omega') \\ & + \tfrac{15}{4}e'\sin(3nt - 2n't - \omega')\Big\}\end{aligned}\right\} . \tag{693}$$

With these values and equations (228) and (234) we obtain the following values:

$$
\left.
\begin{aligned}
&c_2 \cos\beta\,\delta\left(\frac{dR}{dr}\right) = \\
&\frac{m}{\mu}\,\overline{m}^2\,\frac{a}{a'}\,ndt\Big\{ \mp \tfrac{9}{8}\cos(2nt-n't) \mp \tfrac{9}{8}\cos n't \mp \tfrac{15}{8}\cos(4nt-3n't) \\
&\qquad - \tfrac{15}{8}\cos(2nt-3n't) \pm \tfrac{9}{8}\,e\cos(nt+n't-\omega) \\
&\qquad \pm \tfrac{27}{8}e\cos(nt-n't+\omega) \mp \tfrac{9}{8}e\cos(3nt-n't-\omega) \\
&\qquad + \tfrac{9}{8}e\cos(nt-n't-\omega) \mp \tfrac{45}{8}e\cos(5nt-3n't-\omega) \\
&\qquad - \tfrac{15}{8}e\cos(3nt-3n't-\omega) \pm \tfrac{75}{8}e\cos(3nt-3n't+\omega) \\
&\qquad + \tfrac{45}{8}e\cos(nt-3n't+\omega) \mp \tfrac{9}{8}e'\cos(2nt-\omega') \mp \tfrac{9}{8}e'\cos\omega' \\
&\qquad \mp \tfrac{27}{8}e'\cos(2nt-2n't+\omega') \mp \tfrac{27}{8}e'\cos(2n't-\omega') \\
&\qquad \mp \tfrac{75}{8}e'\cos(4nt-4n't+\omega') - \tfrac{75}{8}e'\cos(2nt-4n't+\omega') \\
&\qquad \pm \tfrac{15}{8}e'\cos(4nt-2n't-\omega') + \tfrac{15}{8}e'\cos(2nt-2n't-\omega')\Big\}
\end{aligned}
\right\} . \quad (694)
$$

$$
\left.
\begin{aligned}
&c_3 \sin\beta\,\delta\left(\frac{dR}{dv}\right) = \\
&\frac{m}{\mu}\,\overline{m}^2\,\frac{a}{a'}\,ndt\Big\{ \pm \tfrac{3}{4}\cos(2nt-n't) \mp \tfrac{3}{4}\cos n't \pm \tfrac{15}{4}\cos(4nt-3n't) \\
&\qquad - \tfrac{15}{4}\cos(2nt-3n't) \pm \tfrac{15}{16}e\cos(nt+n't-\omega) \\
&\qquad \mp \tfrac{33}{16}e\cos(nt-n't+\omega) \pm \tfrac{9}{16}e\cos(3nt-n't-\omega) \\
&\qquad + \tfrac{9}{16}e\cos(nt-n't-\omega) \pm \tfrac{165}{16}e\cos(5nt-3n't-\omega) \\
&\qquad - \tfrac{75}{16}e\cos(3nt-3n't-\omega) \mp \tfrac{285}{16}e\cos(3nt-3n't+\omega) \\
&\qquad + \tfrac{195}{16}e\cos(nt-3n't+\omega) \pm \tfrac{9}{4}e'\cos(2nt-2n't+\omega') \\
&\qquad \mp \tfrac{9}{4}\,e'\cos(2n't-\omega') \pm \tfrac{75}{4}e'\cos(4nt-4n't+\omega') \\
&\qquad - \tfrac{75}{4}e'\cos(2nt-4n't+\omega) \mp \tfrac{15}{4}e'\cos(4nt-2n't-\omega') \\
&\qquad + \tfrac{15}{4}e'\cos(2nt-2n't-\omega') \pm \tfrac{3}{4}e'\cos(2nt-\omega') \mp \tfrac{3}{4}e'\cos\omega'\Big\}
\end{aligned}
\right\} . \quad (695)
$$

These two equations give

$$
\left.
\begin{aligned}
&c_2 \cos\beta\,\delta\left(\frac{dR}{dr}\right) + c_3 \sin\beta\,\delta\left(\frac{dR}{dv}\right) = \\
&\frac{m}{\mu}\,\overline{m}^2\,\frac{a}{a'}\,ndt\Big\{ \mp \tfrac{3}{8}\cos(2nt-n't) \mp \tfrac{15}{8}\cos n't \pm \tfrac{15}{8}\cos(4nt-3n't) \\
&\qquad - \tfrac{45}{8}\cos(2nt-3n't) \pm \tfrac{33}{16}e\cos(nt+n't-\omega)
\end{aligned}
\right\} . \quad (696)
$$

(Continued on the next page.)

$$\left.\begin{aligned}
&\pm \tfrac{21}{16}e \cos (nt - n't + \omega) \mp \tfrac{9}{16}e \cos (3nt - n't - \omega) \\
&+ \tfrac{27}{16}e \cos (nt - n't - \omega) \pm \tfrac{75}{16}e \cos (5nt - 3n't - \omega) \\
&- \tfrac{105}{16}e \cos (3nt - 3n't - \omega) \mp \tfrac{135}{16}e \cos (3nt - 3n't + \omega) \\
&+ \tfrac{285}{16}e \cos (nt - 3n't + \omega) \mp \tfrac{3}{8}e' \cos (2nt - \omega') \mp \tfrac{15}{8}e' \cos \omega' \\
&\mp \tfrac{9}{8}e' \cos (2nt - 2n't + \omega') \mp \tfrac{45}{8}e' \cos (2n't - \omega') \\
&\pm \tfrac{75}{4}e' \cos (4nt - 4n't + \omega') - \tfrac{225}{8}e' \cos (2nt - 4n't + \omega') \\
&\mp \tfrac{15}{8}e' \cos (4nt - 2n't - \omega') + \tfrac{45}{8}e' \cos (2nt - 2n't - \omega') \Big\}
\end{aligned}\right\} \quad (696)$$

This equation gives by integration,

$$\int \left\{ c_2 \cos \beta\, \delta \left(\frac{dR}{dr} \right) + c_3 \sin \beta\, \delta \left(\frac{dR}{dv} \right) \right\} =$$

$$\left.\begin{aligned}
&\frac{m}{\mu} \overline{m}^2 \frac{a}{a'} \Big\{ - (0.1947850) \sin (2nt - n't) - (25.06642) \sin n't \\
&\quad + (0.4966103) \sin (4nt - 3n't) \mp (3.167950) \sin (2nt - 3n't) \\
&\quad + e\,(1.918738) \sin (nt + n't - \omega) + e\,(1.418614) \sin (nt - n't + \omega) \\
&\quad - e\,(0.1922946) \sin (3nt - n't - \omega) \pm e\,(1.823933) \sin (nt - n't - \omega) \\
&\quad + e\,(0.9815530) \sin (5nt - 3n't - \omega) \\
&\quad \mp e\,(2.364357) \sin (3nt - 3n't - \omega) \\
&\quad - e\,(3.039888) \sin (3nt - 3n't + \omega) \pm e\,(22.96620) \sin (nt - 3n't + \omega) \\
&\quad - e'\,(0.1875) \sin (2nt - \omega') - e'\,(0.6079774) \sin (2nt - 2n't + \omega') \\
&\quad - e'\,(37.59979) \sin (2n't - \omega') + e'\,(2.533240) \sin (4nt - 4n't + \omega') \\
&\quad \mp e'\,(16.53638) \sin (2nt - 4n't + \omega') \\
&\quad - e'\,(0.4869630) \sin (4nt - 2n't - \omega') \\
&\quad \pm e'\,(3.039890) \sin (2nt - 2n't - \omega') \Big\}
\end{aligned}\right\} \quad (697)$$

Equations (240) and (697) will now give

$$\left.\begin{aligned}
&\delta \frac{d\delta_1 r}{dt} = a \frac{m}{\mu} \frac{\overline{m}^2}{\mu} \frac{a}{a'} n \Big\{ (24.87164) \sin (nt - n't) \\
&\quad - (2.671340) \sin 3(nt - n't) + e\,(25.76133) \sin (n't - \omega) \\
&\quad - e\,(5.047364) \sin (4nt - 3n't - \omega) + e\,(26.89284) \sin (2nt - n't - \omega) \\
&\quad + e\,(23.59087) \sin (2nt - 3n't + \omega) - e'\,(0.1875) \sin (nt - \omega') \\
&\quad + e'\,(36.99181) \sin (nt - 2n't + \omega') \\
&\quad + e'\,(14.00314) \sin (3nt - 4n't + \omega') \\
&\quad + e'\,(2.552927) \sin (3nt - 2n't - \omega') \Big\}
\end{aligned}\right\} \quad (698)$$

If we multiply equation (693) by dt and take the integral, we shall obtain

$$\left.\begin{aligned}&\int\delta\left(\frac{dR}{dv}\right)dt = \\ &a^2\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{(0.8106367)\cos(nt-n't)+(1.351061)\cos 3(nt-n't) \\ &\quad e\,(0.1947986)\cos(2nt-n't-\omega)+e\,(25.06642)\cos(n't-\omega) \\ &\quad +e'\,(0.75)\cos(n't-\omega')+e'\,(2.645823)\cos(nt-2n't+\omega') \\ &\quad +e\,(1.489831)\cos(4nt-3n't-\omega) \\ &\quad -e\,(9.502852)\cos(2nt-3n't+\omega) \\ &\quad +e'\,(6.94240)\cos(3nt-4n't+\omega') \\ &\quad -e'\,(1.315515)\cos(3nt-2n't-\omega')\Big\}\end{aligned}\right\} . \quad (699)$$

Equation (258) will now give

$$\left.\begin{aligned}&\delta\frac{d\delta_0 r}{dt}=a\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{+e\,(0.4053184)\sin(2nt-n't-\omega) \\ &\quad +e\,(0.4053184)\sin(n't-\omega)+e\,(0.6755306)\sin(4nt-3n't-\omega) \\ &\quad -e\,(0.6755306)\sin(2nt-3n't+\omega)\Big\}\end{aligned}\right\} . \quad (700)$$

Equation (261) will also give by means of (699)

$$\left.\begin{aligned}&\delta\frac{d\delta_0 v}{dt}=\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{-(0.8106367)\cos(nt-n't) \\ &\quad -(1.351061)\cos 3(nt-n't)-e\,(0.6158381)\cos(2nt-n't-\omega) \\ &\quad -e\,(25.87706)\cos(n't-\omega)-e\,(2.840892)\cos(4nt-3n't-\omega) \\ &\quad +e\,(8.151791)\cos(2nt-3n't+\omega)-e'\,(0.75)\cos(nt-\omega') \\ &\quad -e'\,(2.645820)\cos(nt-2n't+\omega') \\ &\quad -e'\,(6.942400)\cos(3nt-4n't+\omega') \\ &\quad +e'\,(1.315515)\cos(3nt-2n't-\omega')\Big\}\end{aligned}\right\} . \quad (701)$$

By means of the process explained in §23, we find

$$\left.\begin{aligned}&\delta\frac{d\delta_2 r}{dt}=a\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{+e\,(28.61781)\sin(nt-n't-\omega) \\ &\quad -e\,(28.61781)\sin(n't-\omega)-e\,(0.5901320)\sin(4nt-3n't-\omega) \\ &\quad -e\,(0.5901320)\sin(2nt-3n't+\omega)\Big\}\end{aligned}\right\} . \quad (702)$$

If we now take the sum of equations (698), (700), and (702), we get the complete value of $\delta\,\dfrac{d\delta r}{dt}$, as follows:

$$\left.\begin{aligned}\delta\frac{d\delta r}{dt} = a\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{&(24.87164)\sin(nt-n't)\\ &-(2.671340)\sin 3(nt-n't)+e\,(55.91597)\sin(2nt-n't-\omega)\\ &-e\,(2.45116)\sin(n't-\omega)-e\,(4.961965)\sin(4nt-3n't-\omega)\\ &+e\,(22.32521)\sin(2nt-3n't+\omega)-e'\,(0.1875)\sin(nt-\omega')\\ &+e'\,(36.99181)\sin(nt-2n't+\omega')\\ &+e'\,(14.003314)\sin(3nt-4n't+\omega')\\ &+e'\,(2.552927)\sin(3nt-2n't-\omega')\Big\}\end{aligned}\right\} . \quad (703)$$

This equation gives by integration,

$$\left.\begin{aligned}\delta^2 r = a\frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}\Big\{&-(26.88248)\cos(nt-n't)\\ &+(0.9624382)\cos 3(nt-n't)-e\,(29.05270)\cos(2nt-n't-\omega)\\ &+e\,(32.39101)\cos(n't-\omega)+e\,(1.315672)\cos(4nt-3n't-\omega)\\ &-e\,(12.43923)\cos(2nt-3n't+\omega)+e'\,(0.1875)\cos(nt-\omega')\\ &-e'\,(43.49940)\cos(nt-2n't+\omega')\\ &-e'\,(5.184820)\cos(3nt-4n't+\omega')\\ &-e'\,(0.8956392)\cos(3nt-2n't+\omega')\Big\}\end{aligned}\right\} . \quad (704)$$

This value of $\delta^2 r$ will give

$$\left.\begin{aligned}\delta\frac{d\delta_1 v}{dt} = \frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{&+(53.76496)\cos(nt-n't)\\ &-(1.9248764)\cos 3(nt-n't)+e\,(138.75284)\cos(2nt-n't-\omega)\\ &+e\,(15.86542)\cos(n't-\omega)-e\,(5.518659)\cos(4nt-3n't-\omega')\\ &+e\,(21.99115)\cos(2nt-3n't+\omega)-e'\,(0.375)\cos(nt-\omega')\\ &+e'\,(86.99880)\cos(nt-2n't+\omega')\\ &+e'\,(10.36964)\cos(3nt-4n't+\omega')\\ &+e'\,(1.7912784)\cos(3nt-2n't-\omega')\Big\}\end{aligned}\right\} . \quad (705)$$

If we now take the sum of equations (701) and (705) we shall obtain the complete value of $\delta \frac{d\delta v}{\delta t}$, as follows:

$$\left.\begin{aligned}\delta \frac{d\delta v}{dt} = \frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}n\Big\{ &+ (52.95432) \cos (nt - n't) \\ &- (3.275937) \cos 3(nt - n't) + e\,(138.1370) \cos (2nt - n't - \omega) \\ &- e\,(10.01164) \cos (n't - \omega) - e\,(8.359551) \cos (4nt - 3n't - \omega) \\ &+ e\,(30.14294) \cos (2nt - 3n't + \omega) - e'\,(1.125) \cos (nt - \omega') \\ &+ e'\,(84.35298) \cos (nt - 2n't + \omega') \\ &+ e'\,(3.427240) \cos (3nt - 4n't + \omega) \\ &+ e'\,(3.106793) \cos (3nt - 2n't - \omega')\Big\}\end{aligned}\right\} . \tag{706}$$

This gives by integration, the numbers in brackets being logarithms,

$$\left.\begin{aligned}\delta^2 v = \frac{m}{\mu}\frac{\overline{m}^2}{\mu}\frac{a}{a'}\Big\{ &+ [1.7576664] \sin (nt - n't) \\ &- [0.0719792] \sin 3(nt - n't) + e\,[1.8558345] \sin (2nt - n't - \omega) \\ &- e\,[2.1265962] \sin (n't - \omega) - e\,[0.3461975] \sin (4nt - 3n't - \omega) \\ &+ e\,[1.2298415] \sin (2nt - 3n't + \omega) - e'\,[0.0511525] \sin (nt - \omega') \\ &+ e'\,[1.9964781] \sin (nt - 2n't + \omega') \\ &+ e'\,[0.1034527] \sin (3nt - 4n't + \omega') \\ &+ e'\,[0.0374070] \sin (3nt - 2n't - \omega')\Big\}\end{aligned}\right\} . \tag{707}$$

53′. In the theory of the earth's motion there is an inequality of long period depending on the action of *Venus*. According to the calculations of AIRY and PONTÉCOULANT the effect of this inequality on the sun's radius vector is

$$\delta r' = a'\,[92.39127] \cos \alpha' t, \tag{707'}$$

the number in brackets being a logarithm, and α' being equal to eight times the mean motion of *Venus* minus thirteen times the mean motion of the *earth*. This produces in $\delta\left(\frac{dR}{dr}\right)$ the term

$$\delta\left(\frac{dR}{dr}\right) = \tfrac{3}{2}\,\frac{\overline{m}^2}{a^2}[92.39127] \cos \alpha' t. \tag{707''}$$

Putting $h' = \frac{3}{2}\,[92.39127]$ and $\log \alpha' = 96.49564$, the term $-2\,\frac{n^3 h'}{(n^2 - \alpha'^2)\alpha'} \sin (\alpha' t - \beta')$ of equation (307), gives

$$\delta v = 0''.2723 \sin (\alpha' t - \beta'). \tag{707'''}$$

CHAPTER XI.

NUMERICAL VALUES OF THE PERTURBATIONS OF THE MOON'S CO-ORDINATES.

54. IN order to reduce the perturbations of the moon's co-ordinates to numbers, we must first find the value of the coefficient $\frac{\overline{m}^2}{\mu}$. Now, according to the theory of elliptical motion we have, as in equation (88),

$$n^2 = \frac{\mu}{a^3}; \tag{708}$$

and if we designate the earth's mass by unity we shall also have

$$n'^2 = \frac{1 + m'}{a'^3}. \tag{709}$$

These two equations give

$$\frac{n'^2}{n^2} = \frac{m'}{\mu}\frac{a^3}{a'^3} + \frac{1}{\mu}\frac{a^3}{a'^3}. \tag{710}$$

If we substitute the value $\overline{m}^2$ given by equation (217), we shall find

$$\frac{\overline{m}^2}{\mu} = \frac{n'^2}{n^2} - \frac{1}{\mu}\frac{a^3}{a'^3}. \tag{711}$$

Now taking for n and n' the mean motions of the moon and sun as determined by observation, we shall find

$$\frac{n'}{n} = 0.07480133. \tag{712}$$

This gives

$$\frac{n'^2}{n^2} = 0.0055952389. \tag{713}$$

If we assume the sun's parallax to be $8''.75$ and the moon's mass as $\frac{1}{80}$, we shall find

$$\frac{1}{\mu}\frac{a^3}{a'^3} = 0.0000000166. \tag{714}$$

Substituting equations (713) and (714) in (711), we find

$$\frac{\overline{m}^2}{\mu} = 0.0055952223 \qquad \log 97.7478173. \tag{715}$$

If we multiply this by the radius in seconds, we shall get

$$\frac{\overline{m}^2}{\mu} = 1154''.0973 \qquad \log 3.0622424. \tag{716}$$

We also have

$$\left.\begin{aligned} e &= 0.05489930, & \gamma &= 0.09004560, \\ e' &= 0.01677120, & \frac{a}{a'} &= 0.002559084. \end{aligned}\right\}. \tag{717}$$

Equation (119) will give

$$\left.\begin{aligned} r = a\Big\{ & 1 + \tfrac{1}{2}e^2 - e\,\{1 - \tfrac{3}{8}e^2 + \tfrac{5}{192}e^4\} \cos(nt - \omega) \\ & - \tfrac{1}{2}e^2\,\{1 - \tfrac{2}{3}e^2\} \cos 2(nt - \omega) - \tfrac{3}{8}e^3\,\{1 - \tfrac{15}{16}e^2\} \cos 3(nt - \omega) \\ & - \tfrac{1}{3}e^4 \cos 4(nt - \omega) - \tfrac{125}{384}e^5 \cos 5(nt - \omega) \Big\} \end{aligned}\right\}. \tag{718}$$

Substituting the value of e in this equation, it becomes

$$\left.\begin{aligned} r = a\Big\{ & 1.001507 - 0.0548373 \cos(nt - \omega) - 0.0015039 \cos 2(nt - \omega) \\ & - 0.0000619 \cos 3(nt - \omega) - 0.0000030 \cos 4(nt - \omega) \\ & - 0.0000002 \cos 5(nt - \omega) \Big\} \end{aligned}\right\}. \tag{719}$$

This is the elliptical value of the moon's distance from the centre of the earth; and if we add the perturbations of the radius vector, we shall obtain the moon's true distance from the same point.

If we designate the earth's equatorial semidiameter by D, and the equatorial horizontal parallax by π, we shall have

$$\sin \pi = \frac{D}{r} = \frac{D}{a} \cdot \frac{a}{r}. \tag{720}$$

CHAPTER XI.

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Equation (119) will give

$$\left.\begin{aligned} r = a\Big\{ 1 &+ \tfrac{1}{2}e^2 - e\,\{1 - \tfrac{3}{8}e^2 + \tfrac{5}{192}e^4\}\cos(nt-\omega) \\ &- \tfrac{1}{2}e^2\,\{1 - \tfrac{2}{3}e^2\}\cos 2(nt-\omega) - \tfrac{3}{8}e^3\,\{1 - \tfrac{15}{16}e^2\}\cos 3(nt-\omega) \\ &- \tfrac{1}{3}e^4\cos 4(nt-\omega) - \tfrac{125}{384}e^5\cos 5(nt-\omega)\Big\} \end{aligned}\right\}. \tag{718}$$

Substituting the value of e in this equation, it becomes

$$\left.\begin{aligned} r = a\Big\{ & 1.001507 - 0.0548373\cos(nt-\omega) - 0.0015039\cos 2(nt-\omega) \\ & - 0.0000619\cos 3(nt-\omega) - 0.0000030\cos 4(nt-\omega) \\ & - 0.0000002\cos 5(nt-\omega)\Big\} \end{aligned}\right\}. \tag{719}$$

This is the elliptical value of the moon's distance from the centre of the earth; and if we add the perturbations of the radius vector, we shall obtain the moon's true distance from the same point.

If we designate the earth's equatorial semidiameter by D, and the equatorial horizontal parallax by π, we shall have

$$\sin\pi = \frac{D}{r} = \frac{D}{a}\cdot\frac{a}{r}. \tag{720}$$

Now we have, according to observation,

$$\frac{D}{a} = 0.01657671 = 3419''.19; \tag{721}$$

therefore we have

$$\sin \pi = 3419''.19 \frac{a}{r}. \tag{722}$$

If we increase this coefficient of $\frac{a}{r}$ by $0''.16$ we may take without sensible error

$$\pi = 3419''.35 \frac{a}{r}. \tag{723}$$

If we suppose r to be in error by one unit in the fifth decimal place, the error of the parallax would be less than $0''.04$.

55. We shall now form equations (246), and in the value of (δr) we shall express the numerical coefficients in units of the fifth decimal place of the moon's mean distance as the unit. We shall also omit all the terms of (δr) whose coefficients are less than 0.00001.

$$\left.\begin{aligned}
(\delta r) = &- (92 - 3 = 89) + (27 + 3 = 30) \cos (nt - \omega) \\
&+ (14 - 2 = 12) \cos (n't - \omega') + (1 + 1 = 2) \cos 2(nt - \omega) \\
&+ (18 + 8 = 26) \cos (nt + n't - \omega' - \omega') \\
&- (19 + 13 = 32) \cos (nt - n't - \omega + \omega') \\
&- (2 - 1 = 1) \cos 2(nt - \Omega) + (1) \cos (2nt + n't - 2\omega - \omega') \\
&- (1) \cos (2nt - n't - 2\omega + \omega') - (769 - 3 = 766) \cos 2(nt - n't) \\
&- (7) \cos (3nt - 2n't - \omega) - (863 + 23 = 886) \cos (nt - 2n't + \omega) \\
&+ (6 + 2 = 8) \cos (2nt - n't - \omega') \\
&- (50 + 3 = 53) \cos (2nt - 3n't + \omega') - (3) \cos (4nt - 2n't - 2\omega) \\
&+ (60 + 1 = 61) \cos 2(n't - \omega) - (2) \cos (2nt - 4n't + 2\omega') \\
&- (38 + 2 = 40) \cos (nt - 3n't + \omega + \omega') \\
&- (3) \cos (3nt - 3n't - \omega + \omega') \\
&+ (14 - 2 = 12) \cos (nt - n't + \omega - \omega') + (15) \cos 2(n't - \Omega) \\
&- (1) \cos (2nt - 2n't + 2\omega - 2\Omega) + (1) \cos (2nt - 2n't - 2\omega + 2\Omega)
\end{aligned}\right\} . \tag{724}$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+ (3)\cos(nt + 2n't - 3\omega) - (4)\cos(nt - 2n't - \omega + 2\Omega) \\
&- (2)\cos(nt - 2n't + 3\omega - 2\Omega) + (5)\cos(nt + 2n't - \omega - 2\Omega) \\
&- (1)\cos(nt - 4n't + \omega + 2\omega') + (3)\cos(3n't - 2\omega - \omega') \\
&- (1)\cos(n't - 2\omega + \omega') + (19 + 6 = 25)\cos(nt - n't) \\
&- (1)\cos 3(nt - n't) - (2)\cos 4(nt - n't) \\
&- (8)\cos(3nt - 4n't + \omega) + (1)\cos(2nt - n't - \omega) \\
&- (1)\cos(n't - \omega) + (1)\cos(nt - 2n't + \omega') \\
&- (1)\cos(3nt - 5n't + \omega + \omega')
\end{aligned}\right\} . \quad (724)$$

If we put $a = 1$ in equation (719), and add the value of (δr) given by equation (724), we shall obtain the complete value of r; and the substitution of r in equation (723) will give the moon's parallax with quite as great facility as the ordinary method of obtaining that co-ordinate.

We shall now determine the perturbations of the longitude which we have designated by (δv). The partial values of (δv) are given in equations (307), (400), (470), (551), (656), and (707). We shall omit all the inequalities whose coefficients are less than $0''.1$. The first term of the coefficients in the following equation is the value of the corresponding equation in the value of δv, and the second term is the sum of all the remaining equations, except for a few equations whose arguments depend wholly on the mean motions of the sun and moon, in which cases three or more terms of the coefficients are given. The parallactic equation also contains the term $+ (2''.1)$. This term was not calculated rigorously, but estimated by induction as being the sum of all the remaining terms of the series which make up the complete coefficient. By means of this arrangement we are able to estimate the convergency of the series and perceive at a glance just which equations require further approximations.

$$\left.\begin{aligned}
(\delta v) = &- \{46''.1 + 17''.6 = 63''.7\}\sin(nt - \omega) \\
&- \{3''.7 + 3''.1 = 6''.8\}\sin 2(nt - \omega) - 0''.3\sin 3(nt - \omega) \\
&- \{776''.5 - 120''.1 = 656''.4\}\sin(n't - \omega') \\
&- \{9''.7 - 2''6 = 7''.1\}\sin 2(n't - \omega') - 0''.2\sin 3(n't - \omega') \\
&- \{70''.9 + 31''.9 = 102''.8\}\sin(nt + n't - \omega - \omega') \\
&+ \{78''.1 + 56''.7\} = 134''.8\}\sin(nt - n't - \omega + \omega') \\
&+ \{0''.6 - 2''.8 = - 2''.2\}\sin 2(nt - \Omega) \\
&- 0''.9\sin(nt + 2n't - \omega - 2\omega') + 1''.1\sin(nt - 2n't - \omega + 2\omega')
\end{aligned}\right\} . \quad (725)$$

(Continued on the next page.)

$$\left.\begin{array}{l}
-4''.9 \sin(2nt+n't-2\omega-\omega') + 5''.4 \sin(2nt-n't-2\omega+\omega') \\
+0''.2 \sin(3nt-\omega-2\Omega) - 1''.4 \sin(nt+\omega-2\Omega) \\
+0''.6 \sin(2nt+n't-\omega'-2\Omega) - 0''.2 \sin(2nt-n't+\omega'-2\Omega) \\
-0''.4 \sin(3nt+n't-3\omega'-\omega') + 0''.4 \sin(3nt-n't-3\omega+\omega') \\
+0''.1 \sin(3nt+n't-\omega-\omega'-2\Omega) \\
-0''.2 \sin(3nt-n't-\omega+\omega'-2\Omega) \\
-0''.4 \sin(nt+n't+\omega-\omega'-2\Omega) \\
+0''.6 \sin(nt-n't+\omega+\omega'-2\Omega) \\
-\{82''.4+26''.8+10''.3+3''.9+(2''.1)-2''.1=123''.4\} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \sin(nt-n't) \\
+\{2353''.4+2''.5=2355''.9\} \sin 2(nt-n't) \\
+\{1''.6-0''.6-0''.2-0''.1=0''.7\} \sin 3(nt-n't) \\
+13''.4 \sin 4(nt-n't) \\
+\{191''.4-0''.9=190''.5\} \sin(3nt-2n't-\omega) \\
+0''.2 \sin(6nt-4n't-2\omega) \\
+\{4245''.2+122''.0=4367''.2\} \sin(nt-2n't+\omega) \\
+26''.5 \sin(2nt-4n't+2\omega) \\
-\{19''.8+7''.4=27''.2\} \sin(2nt-n't-\omega') \\
+\{156''.3+8''.2=164''.5\} \sin(2nt-3n't+\omega') \\
-5''.8 \sin(2nt-n't-\omega) + 14''.3 \sin(4nt-2n't-2\omega) \\
+11''.7 \sin(n't-\omega) - \{161''.8-17''.9=143''.9\} \sin 2(n't-\omega) \\
+\{7''.7+0''.7=8''.4\} \sin(2nt-4n't+2\omega') \\
-\{1''.5+1''.2=2''.7\} \sin(3nt-n't-\omega-\omega') \\
+\{197''.9+11''.6=209''.5\} \sin(nt-3n't+\omega+\omega') \\
+\{12''.7+1''.3=14''.0\} \sin(3nt-3n't-\omega+\omega') \\
-\{61''.3-7''.1=54''.2\} \sin(nt-n't+\omega-\omega') \\
+0''.2 \sin(5nt-5n't-\omega+\omega') + 3''.7 \sin(3nt-5n't+\omega+\omega') \\
-0''.7 \sin(3nt-3n't+\omega-\omega') - 5''.7 \sin(4nt-2n't-2\Omega) \\
-\{298''.1+7''.8=305''.9\} \sin 2(n't-\Omega)
\end{array}\right\} . \quad (725)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+5''.2\sin(2nt-2n't+2\omega-2\Omega)\\
&-3''.1\sin(2nt-2n't-2\omega+2\Omega)+1''.0\sin(5nt-2n't-3\omega)\\
&-11''.5\sin(nt+2n't-3\omega)+0''.3\sin(2nt-5n't+3\omega')\\
&-7''.8\sin(3nt-2n't+\omega-2\Omega)\\
&+13''.2\sin(nt-2n't-\omega+2\Omega)\\
&-14''.7\sin(nt+2n't-\omega-2\Omega)\\
&-0''.8\sin(5nt-2n't-\omega-2\Omega)\\
&-0''.3\sin(3nt-2n't-3\omega+2\Omega)\\
&+8''.7\sin(nt-2n't+3\omega-2\Omega)\\
&-0''.3\sin(4nt-3n't+\omega'-2\Omega)-9''.1\sin(3n't-\omega'-2\Omega)\\
&+9''.1\sin(n't+\omega'-2\Omega)+0''.3\sin(2nt-3n't+2\omega+\omega'-2\Omega)\\
&-0''.3\sin(2nt-3n't-2\omega+\omega'+2\Omega)\\
&+0''.6\sin(3nt-4n't-\omega+2\omega')\\
&+7''.4\sin(nt-4n't+\omega+2\omega')\\
&+0''.9\sin(4nt-3n't-2\omega+\omega')\\
&-0''.1\sin(4nt-n't-2\omega-\omega')-8''.1\sin(3n't-2\omega-\omega')\\
&+2''.5\sin(n't-2\omega+\omega')+0''.8\sin(nt+n't-2\omega)\\
&-0''.8\sin(2nt+2n't-4\omega)+0''.6\sin(2nt+2n't-4\Omega)\\
&+0''.1\sin(2nt-2n't+4\omega-4\Omega)-0''.6\sin(2n't-4\omega+2\Omega)\\
&-0''.5\sin(2nt+2n't-2\omega-2\Omega)-0''.2\sin(4n't-2\omega'-2\Omega)\\
&+2''.5\sin 2(\omega-\Omega)+108''.5\sin(\omega-\omega')\\
&+0''.2\sin(3nt-5n't-\omega+3\omega')+0''.2\sin(nt-5n't+\omega+3\omega')\\
&-0''.5\sin(nt+3n't-3\omega-\omega')+0''.2\sin(nt+n't-3\omega+\omega')\\
&-0''.3\sin(4n't-2\omega-2\omega')-0''.3\sin(3nt-3n't+\omega+\omega'-2\Omega)\\
&-0''.4\sin(nt-n't-\omega-\omega'+2\Omega)\\
&+0''.1\sin(3nt-n't+\omega-\omega'-2\Omega)\\
&+0''.3\sin(nt-3n't-\omega+\omega'+2\Omega)\\
&+0''.5\sin(nt+n't-\omega+\omega'-2\Omega)\\
&-0''.3\sin(nt+3n't-\omega-\omega'-2\Omega)
\end{aligned}\right\}. \quad (725)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&-0''.1 \sin(nt - n't + 3\omega - \omega' - 2\Omega) \\
&+0''.4 \sin(nt - 3n't + 3\omega + \omega' - 2\Omega) \\
&+0''.2 \sin(4nt - 3n't - \omega) - 1''.4 \sin(2nt - 3n't + \omega) \\
&-2''.4 \sin(nt - 2n't + \omega') + 0''.2 \sin(3nt - 4n't + \omega') \\
&-0''.4 \sin(3nt - n't - 2\omega) - 1''.1 \sin(nt - 3n't + 2\omega) \\
&-0''.2 \sin(2nt - 2n't - \omega + \omega') - 0''.1 \sin(2nt - 4n't + \omega + \omega') \\
&+0''.2 \sin(3nt - n't - 2\Omega) + 0''.3 \sin(nt + n't - 2\Omega) \\
&-0''.4 \sin(nt - 3n't + 2\Omega) + 3''.3 \sin(2nt - 4n't + 2\Omega) \\
&+1''.4 \sin(5nt - 4n't - \omega) + 32''.8 \sin(3nt - 4n't + \omega) \\
&-0''1 \sin(4nt - 3n't - \omega') + 1''.2 \sin(4nt - 5n't + \omega') \\
&+4''.4 \sin \Omega + 0''.4 \sin(2nt - \Omega) + \delta_s v
\end{aligned}\right\} . \quad (725)$$

The value of $(\delta\theta)$ is the sum of equations (311) and (405), except a single term depending on the oblateness of the earth, which is given by equation (657). It is readily seen that the expression which determines the latitude is more converging than the corresponding expression for the longitude; and it seems scarcely necessary to carry the approximations to terms of a higher order of magnitude, unless it be for the purpose of finding the effect of such terms on the expression of the perturbations in longitude.

$$\left.\begin{aligned}
(\delta\theta) = &\{73''.1 + 0''.9 = 74''.0\} \sin(nt - \Omega) + 17''.1 \sin(\omega - \Omega) \\
&-\{2''.6 + 1''.5 = 4''.1\} \sin(2nt - \omega - \Omega) \\
&-\{9''.7 - 5''.0 = 4''.7\} \sin(nt + n't - \omega' - \Omega) \\
&+\{7''.6 - 1''.3 = 6''.3\} \sin(nt - n't + \omega' - \Omega) \\
&-0''.2 \sin(nt - 2\omega + \Omega) - 0''.4 \sin(3nt - 2\omega - \Omega) \\
&+0''.2 \sin 3(nt - \Omega) + 0''.1 \sin(nt + 2n't - 2\omega - \Omega) \\
&-2''.7 \sin(n't + \omega - \omega' - \Omega) - 3''.1 \sin(n't - \omega - \omega' + \Omega) \\
&-3''.8 \sin(2nt + n't - \omega - \omega' - \Omega) \\
&+3''.9 \sin(2nt - n't - \omega + \omega' - \Omega) \\
&-0''.4 \sin(3nt + n't - 2\omega - \omega' - \Omega) \\
&+0''.4 \sin(3nt - n't - 2\omega + \omega' - \Omega) \\
&-0''.2 \sin(nt + n't - 2\omega - \omega' + \Omega)
\end{aligned}\right\} . \quad (726)$$

(Continued on the next page.)

$$\left.\begin{aligned}
&+0''.2\sin(nt-n't-2\omega+\omega'+\Omega)\\
&+\{116''.0-0''.2=115''.8\}\sin(3nt-2n't-\Omega)\\
&+\{672''.2-24''.9=647''.3\}\sin(nt-2n't+\Omega)\\
&+\{15''.1-0''.1=15''.0\}\sin(4nt-2n't-\omega-\Omega)\\
&+\{184''.5+5''.3=189''.8\}\sin(2nt-2n't+\omega-\Omega)\\
&-\{150''.8+14''.5=165''.3\}\sin(2n't-\omega-\Omega)\\
&+\{34''.5-1''.6=32''.9\}\sin(2nt-2n't-\omega+\Omega)\\
&-\{1''.0+0''.4=1''.4\}\sin(3nt-n't-\omega'-\Omega)\\
&-\{10''.4+3''.5=13''.9\}\sin(nt-n't-\omega'+\Omega)\\
&+\{7''.5+0''.4=7''.9\}\sin(3nt-3n't+\omega'-\Omega)\\
&+\{29''.9+0''.7=30''.6\}\sin(nt-3n't+\omega'+\Omega)\\
&+1''.4\sin(5nt-2n't-2\omega-\Omega)\\
&-14''.1\sin(nt+2n't-2\omega-\Omega)\\
&+0''.3\sin(3nt-4n't+2\omega'-\Omega)\\
&+1''.0\sin(nt-4n't+2\omega'+\Omega)-1''.0\sin(5nt-2n't-3\Omega)\\
&-13''.8\sin(nt+2n't-3\Omega)+0''.3\sin(3nt-2n't+2\omega-3\Omega)\\
&-0''.7\sin(nt-2n't-2\omega+3\Omega)\\
&-0''.1\sin(4nt-n't-\omega-\omega'-\Omega)\\
&+8''.6\sin(2nt-3n't+\omega+\omega'-\Omega)\\
&-7''.2\sin(3n't-\omega-\omega'-\Omega)\\
&+0''.9\sin(4nt-3n't-\omega+\omega'-\Omega)\\
&-2''.7\sin(2nt-n't+\omega-\omega'-\Omega)\\
&+2''.2\sin(n't-\omega+\omega'-\Omega)\\
&-0''.5\sin(2nt-n't-\omega-\omega'+\Omega)\\
&+1''.6\sin(2nt-3n't-\omega+\omega'+\Omega)\\
&+4''.7\sin(3nt-2n't-2\omega+\Omega)-2''.1\sin(nt-2n't+2\omega-\Omega)\\
&+0''.1\sin(6nt-2n't-3\omega-\Omega)\\
&+0''.1\sin(4nt-2n't-3\omega+\Omega)
\end{aligned}\right\}\ .\quad(726)$$

(Continued on the next page.)

$$
\left.\begin{aligned}
&- 1''.4 \sin (2nt + 2n't - 3\omega - \Omega) \\
&- 0''.1 \sin (4nt - 2n't + \omega - 3\Omega) - 0''.1 \sin (2n't + \omega - 3\Omega) \\
&- 1''.5 \sin (2nt + 2n't - \omega - 3\Omega) \\
&+ 0''.6 \sin (2nt - 2n't + 3\omega - 3\Omega) \\
&- 0''.4 \sin (nt + 3n't - \omega' - 3\Omega) + 0''.4 \sin (nt + n't + \omega' - 3\Omega) \\
&- 0''.7 \sin (nt + 3n't - 2\omega - \omega' - \Omega) \\
&+ 0''.2 \sin (nt + n't - 2\omega + \omega' - \Omega) \\
&+ 0''.3 \sin (2nt - 4n't + \omega + 2\omega' - \Omega) \\
&- 0''.3 \sin (4n't - \omega - 2\omega' - \Omega) + 0''.6 \sin (5nt - 4n't - \Omega) \\
&+ 3''.1 \sin (3nt - 4n't + \Omega) + 0''.1 \sin (6nt - 4n't - \omega - \Omega) \\
&+ 2''.4 \sin (4nt - 4n't + \omega - \Omega) + 0''.4 \sin (4nt - 4n't - \omega + \Omega) \\
&+ 6''.5 \sin (2nt - 4n't + \omega + \Omega) + 0''.3 \sin (3nt - 5n't + \omega' + \Omega) \\
&- 3''.6 \sin (2nt - n't - \Omega) + 3''.2 \sin (n't - \Omega) \\
&- 0''.1 \sin (2nt - 3n't + \Omega) - 0''.5 \sin (3nt - n't - \omega - \Omega) \\
&- 0''.6 \sin (nt - n't - \omega + \Omega) + 0''.2 \sin (nt + n't - \omega - \Omega) \\
&+ 0''.1 \sin (nt - n't + \omega - \Omega) - 0''.2 \sin (nt - 3n't + \omega + \Omega) \\
&- 0''.1 \sin (2nt - 2n't + \omega' - \Omega) - 8''2 \sin nt \\
&+ \gamma \delta_s v \cos (nt - \Omega)
\end{aligned}\right\} \quad . \quad (726)
$$

$\delta_s v$ being the secular inequality in the longitude.

If we add equation (725) to equation (132) we shall obtain the complete expression for the moon's longitude, referred to the fixed equinox of the epoch; and if we add equations (133) and (726) together we shall obtain the expression of the moon's true latitude.